THE PUPILS' ARITHMETIC

BOOK SIX

BYRNES-RICHMAN-ROBERTS

THE PUPILS' ARITHMETIC

BOOK SIX

THE COMPLETE ARITHMETIC

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO · DALLAS
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

THE PUPILS' ARITHMETIC

BOOK SIX

THE COMPLETE ARITHMETIC

BY

JAMES C. BYRNES, B.S., PH.M.

MEMBER BOARD OF EXAMINERS, DEPARTMENT OF EDUCATION, NEW YORK

JULIA RICHMAN

LATE DISTRICT SUPERINTENDENT OF SCHOOLS, NEW YORK

JOHN S. ROBERTS, A.M., PD.M.

DISTRICT SUPERINTENDENT OF SCHOOLS
NEW YORK

New York
THE MACMILLAN COMPANY
1913

Set up and electrotyped. Published September, 1913.

FERRIS PRINTING COMPANY
New York, N. Y., U. S. A.

PREFACE

Book Six, The Complete Arithmetic, of *The Pupils' Arithmetic* is designed for grammar grades with special emphasis upon the grade work of the seventh and eighth years. It harmonizes with the previous numbers of the series, preserving the terminology, the definitions, and the model forms of computation and of explanation that were developed and elaborated in the other volumes.

The chapters on fundamental rules, common fractions, and decimals, and their subdivisions, are adapted to classroom use in all grades, though relieved of the details of method appropriate to a first presentation.

Beginning with percentage, however, there is continued as from previous volumes the exposition and exemplification of modern methods of presentation which, though originally designed for the pupils' understanding, seem to have satisfied a need felt by teachers. Thus, the chapters on percentage, interest, longitude and time, the algebraic equation, and ratio and proportion are elaborated sufficiently to exhibit the divisions and subdivisions of each topic, the various types of drills, and the types of

problems which are real — not fictitious, practical — not academic, motivating — not stupefying.

The authors have kept steadily before them the initial aims of the series: first, to present a text usable in the several phases or activities of the arithmetic lesson, hence the explanatory matter for reading and discussion, the generalizations inductively derived, and the great number of problems; second, to present a text entirely modern in theory and practice, hence the elimination of obsolete terms, processes, and problems, and the correlation of schoolroom practice with life through the choice of problems, the adoption of model forms of solution for permanent use after school life, and the presentation of short processes sanctioned by commercial usage.

As in previous books, the logical arrangement of subject matter is adhered to, in preference to the spiral arrangement.

The model forms of bills and accounts and of other business papers are in accord with current accounting and banking practice.

CONTENTS

THE PUPILS' ARITHMETIC

BOOK SIX

THE COMPLETE ARITHMETIC

PUPILS' ARITHMETIC

BOOK VI

NUMERATION AND NOTATION

1. **Notation** is the writing of numbers by means of characters; *e.g.* eighty-one is written 81 or LXXXI.

2. **Numeration** is the naming or reading of numbers written by means of characters; *e.g.* 81 is read eighty-one.

ARABIC OR DECIMAL NOTATION

The common mode of writing numbers is called the **Arabic notation.** It was first introduced into Europe by the Mohammedan Arabs, who had acquired it from the Hindus of India. It has been in general use among Europeans for about four hundred years. It is sometimes called the **decimal system of notation.**

3. The Arabic or decimal notation is distinguished by four features:

(1) **Ten numerals, figures, or digits are used as characters;** *i.e.* 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

(2) **The zero or cipher, 0, is used to fill vacant places or orders.**

(3) **Each figure has a place-value**; *e.g.* 75 means 7 tens and 5 units.

(4) **The place-values or orders increase uniformly by powers of ten from right to left.**

NOTES. The power of a number is the number of times it is taken as a factor; *e.g.* the first power of 10 is 10; the second power of 10 is 10×10 or 100; the third power of 10 is $10 \times 10 \times 10$ or 1000, etc. Hence the orders of numbers in the Arabic notation as we progress from right to left in any whole number are *units, tens, hundreds, thousands, ten-thousands, hundred-thousands, millions,* etc. Each order has a value ten times as great as the order on its right.

The orders to the right of units are used in writing decimal fractions. They are *tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths,* etc.

4. The orders of decimal fractions are separated from the orders of whole numbers by a dot or period called the **decimal point.**

5. United States money being a decimal currency is expressed in the decimal notation.

ROMAN NOTATION

6. The **Roman notation** was generally used in Europe until about the year 1500. About that time merchants and statesmen began to realize the superiority of the Arabic notation and began to use it in business and to introduce it into schools. By

1600, the Arabic notation had supplanted the Roman notation almost completely. The use of the Roman system is now confined almost solely to numbering chapters in books, hours on clock dials, and inscriptions on monuments.

The Roman notation employs seven capital letters or numerals, namely:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

These letters are usually repeated or combined to represent all other numbers according to the following rules:

(1) Combinations of two letters represent the difference between their values when the letter of smaller value is written to the left; *e.g.* IV = 5 − 1 or 4; IX = 10 − 1 or 9; XL = 50 − 10 or 40; XC = 100 − 10 or 90; CD = 500 − 100 or 400.

(2) All other combinations represent sums of the values represented by the single letters; *e.g.* VII = 7; XVI = 16; LXXIX = 79; MDC = 1600.

(3) A bar over a letter or a combination of letters multiplies its value by 1000; *e.g.* $\overline{\text{V}}$ = 5000; $\overline{\text{C}}$ = 100,000; $\overline{\text{L}}$ = 50,000; $\overline{\text{IV}}$ = 4000; $\overline{\text{M}}$ = 1,000,000.

EXERCISE

Write in Roman notation:

1914, 1607, 3785, 23, 449, 67, 5982, 83, 131, 5540.

FUNDAMENTAL PROCESSES

7. The **fundamental processes** of arithmetic are **addition, subtraction, multiplication,** and **division.**

ADDITION OF WHOLE NUMBERS

8. **Addition** is the process of finding the number that is equal to the aggregate of two or more given numbers.

9. The numbers added are called **addends.**

10. The result of addition is the **sum** or **amount.**

11. **Like numbers** are numbers that consist of units of the same kind. 2 pounds and 3 pounds are like numbers.

12. **Unlike numbers** are numbers that consist of units of different kinds. $ 5 and 7 yards are unlike numbers.

PRINCIPLE: **Only like numbers can be added.**

13. The sign of addition + is called "plus."

14. The sign of equality = is read "equals" or "is equal to."

Types of Oral Drills for Frequent Practice

15. 1. Add by 9's to the number nearest 100. What is the number?

2. Add by 7's from 3 to the first total beyond 100. What is that total?

3. Beginning with 6, add 9 at each tap of the pencil.

4. Find the sum of: 41, 8, 8, 9, 12, 5, 9, 14.

5. What is the amount of 75 ct., 18 ct., 19 ct., 37 ct., 15 ct.?

6. Add 15 to each of the following: 18, 21, 35, 27, 39, 43, 56.

7. Add 31 and 16; 41 and 16; 81 and 16; 111 and 16.

8. Add 39 and 22; 59 and 22; 79 and 22; 49 and 22.

9. Add 28 and 15; 38 and 15; 48 and 15.

SIGHT EXERCISES

For Oral Drill from Text, Blackboard, or Chart, or for Seat Work in Limited Time

16. Add 47 and 35.

Hint. Add thus, 47, 77, 82.

Give sums quickly:

1.	46	49	51	57	53	39	54	48
	33	38	29	36	46	57	49	45

Add rapidly, both columns, if possible, at once:

2.	3.	4.	5.	6.
43	22	18	35	42
27	55	92	34	24
20	13	24	42	85
19	16	25	49	79
44	12	73	58	63
15	78	25	46	27

Prove examples 2, 3, and 6.

WRITTEN EXERCISES

17. Add, looking for combinations of 10.
Verify each answer by adding in reverse order.
Read the numbers aloud and then add silently.
Verify each answer as in example 1.

1.	2.	3.
16497	53754	3497016
8534	71895	8205767
7625	108407	14087128
908	7659	9453342
20654	33792	27908136
18998	89525	40075000
3941	875000	7745845
7075	98876	6078974
42	729302	3892474
39	10701	20967845
48	78844	3807049
39	204721	10007006
4	68973	30800049
84232		

Write answers:

4. $756 + 254 + 327 = ?$

5. $927 + 183 + 450 + 623 = ?$

6. $924 + 486 + 653 = ?$

7. $289 + 428 + 673 + 818 = ?$

8. $\$8.75 + 3.50 + 6.35 = ?$

9. $\$3.43 + 8 + 4.97 + 6.13 = ?$

10. $\$2.15 + 4.57 + 9.23 = ?$

11. $\$1.76 + 4.87 + 9 + .65 = ?$

12. $\$9.50 + .28 + 2.02 = ?$

13. $\$6 + .55 + 1.94 + 2.16 = ?$

14. $\$6.36 + .75 + .28 = ?$

15. $\$27 + 9.24 + 7.56 + 3.24 = ?$

16. $\$10.72 + 7.28 + 5 = ?$

17. $\$4.75 + 3.50 + 3.25 + 6.66 = ?$

18. $\$7.79 + 4.34 + 8.06 = ?$

19. $\$8.52 + 6.28 + 1.33 + 4 = ?$

20.

$256.84	$416.83	$125.75
318.19	219.44	264.82
420.08	377.25	194.38
294.65	240.80	227.55
301.98	329.65	203.46

Add vertically and horizontally and compare the totals.

WRITTEN PROBLEMS

18. 1. A girl about to enter college estimates that her expenses for the first year will be: tuition $125; board $250; clothing $175; traveling $47.50; sundries $83.50. Find the amount estimated.

2. A bride furnishing her home estimates the cost as follows: carpets $185; dining-room furniture $115.50; bedroom furniture $92.50; sitting-room furniture $217.25; kitchen furniture and utensils $87.60; linen $75; bedding $25.60; miscellaneous articles $57.35. What is the total estimate?

3. A barber about to fit up a new shop estimates the cost as follows: For wall fixtures $250; for 4 chairs at $36.50 each ——; for 15 doz. towels at $1.80 per dozen, ——; for 20 razors at $1.35 each ——; for china and glassware $15.75; for cosmetics $17.50; for a table $4.50; for 12 chairs at $2.50 ——; for sundries $25. How much money will be needed?

4. The cost of a wedding is estimated as follows: music $17.50; flowers $12.75; supper $23.50; favors $6.50; service $4.50; sundries $9.70. What is the estimate?

If to this expense there must be added the cost of a party dress $35.00, and slippers $6.50, how much will the party cost?

5. The number of pupils enrolled in the common schools of the several geographical divisions

of the United States was in 1910 as follows: North Atlantic States 4,149,381; South Atlantic States 2,550,107; North Central States 3,745,902; Western States 1,158,146. What was the total enrollment?

6. The number of immigrants admitted into the United States for the nine years ending June 30, 1912, was as follows: 1903, eight hundred fifty-seven thousand, forty-six; 1904, eight hundred twelve thousand, eight hundred seventy; 1905, one million, twenty-seven thousand, four hundred twenty-one; 1906, one million, one hundred thousand, seven hundred thirty-five; 1907, one million, two hundred eighty-five thousand, three hundred forty-nine; 1908, seven hundred eighty-two thousand, eight hundred seventy; 1909, seven hundred fifty-one thousand, seven hundred eighty-six; 1910, one million, forty-one thousand, five hundred seventy; 1911, eight hundred seventy-eight thousand, five hundred eighty-seven. Find the total immigration for these nine years.

7. The areas of the insular possessions of the United States given in square miles are: Philippines 115,026; Porto Rico 3606; Hawaii 6449; Tutuila, Samoa, 34; Guam 200. The area of the United States, including Alaska, is 3,617,673 sq. mi. What is the total area of the United States and its dependencies?

8. The several divisions of Continental United States had the following population according to the census of 1910: New England Division 6,552,681; Middle Atlantic Division 19,315,892; East North Central Division 18,250,621; West North Central Division 11,637,921; South Atlantic Division 12,194,895; East South Central Division 8,409,901; West South Central Division 8,784,534; Mountain Division 2,633,517; Pacific Division 4,192,304. What was the total population?

9. The areas of the public lands of the United States reported in 1911 were: Surveyed, one hundred eighty-eight million, eight hundred eighty-nine thousand, one hundred thirty-six acres; Unsurveyed, six hundred ninety-five million, four hundred one thousand, two hundred fifty-nine acres. Write the total number of acres in words.

10. On February 1, a merchant's bank account showed a balance of $867.58. On Feb. 5, he deposited $38.96; on Feb. 10, $185.50; on Feb. 14, $345.00; on Feb. 17, $89.96; Feb. 18, $638.47; Feb. 19, $841.33; Feb. 25, $3300; Feb. 28, $136.81. If he did not draw against this account, what was his balance on March 1?

11. Compose three problems in addition based on the register and attendance of all the classes in your school for one month.

Solve the problems.

SUBTRACTION OF WHOLE NUMBERS

19. **Subtraction** is the process of finding how much greater one number is than another; or the process of finding what number must be added to the second number in order to equal the first.

20. The **minuend** is the number from which the other number is subtracted.

21. The **subtrahend** is the number that is subtracted.

22. The result of the subtraction is the **difference** or **remainder.**

PRINCIPLE 1. **Only like numbers can be subtracted.**

PRINCIPLE 2. **The minuend is the sum of the subtrahend and the remainder.**

PRINCIPLE 3. **If the same number be added to or subtracted from both minuend and subtrahend, the difference or remainder will be the same.**

23. The sign of subtraction – is called "minus."

SUBTRACTION

Types of Oral Drills for Frequent Practice

24. 1. Beginning with 100, subtract 7 at each tap of the pencil. Give the remainder.

2. Beginning with 93, subtract 6 until the remainder next below 30 is reached. Write your answer.

3. Beginning with 95, subtract 8 until a remainder ending with the figure 5 is reached. Write your answer.

4. Subtract 9 from each of the following: 51, 42, 55, 67, 83, 94, 38.

5. I had $ 4.50. I spent $ 1.30, 25 ct., 20 ct., 18 ct., 22 ct. How much had I left?

WRITTEN EXERCISES

25. Verify the answers of examples 2, 5, and 7 by adding the remainder to the subtrahend; the sum should equal the minuend.

Subtract:

1.	2.	3.	4.
45832	63212	51924	28816
38948	19763	47976	17927

5.	6.	7.
3417642	7696112	5525417
1894876	3977875	3856729

8.	9.	10.	11.
$ 230.75	$ 357.26	$ 617.63	$ 941.81
178.98	184.98	438.58	682.97

12.	13.	14.	15.
$ 2614.50	$ 8888.70	$ 2037.55	$ 2076.03
1896.75	6975.82	878.66	1839.28

EXERCISES

26. Find the total of the results of the following equations: *

1. $3864 - 2767 + 5805 = ?$
2. $5675 - (1479 - 894) = ?$
3. $7671 - 5328 + 6296 = ?$
4. $9271 - (6284 - 1299) = ?$
5. $6463 - (3889 + 1941) = ?$
6. $5180 - (3972 - 2106) = ?$
7. $4131 - (2947 + 792) = ?$
8. $3149 - (1755 - 1431) = ?$
9. $2101 - 678 + 4190 = ?$
10. $2447 - (805 - 694) = ?$

ORAL EXERCISES

FOR ORAL DRILL OR FOR SEAT WORK IN LIMITED TIME

27. Find the change from a one-dollar bill of the following amounts:

1. 76 ct.	2. 91 ct.	3. 71 ct.	4. 35 ct.	5. 27 ct.
6. 18 ct.	7. 47 ct.	8. 57 ct.	9. 65 ct.	10. 76 ct.

From a two-dollar bill:

11. $1.44	12. $1.56	13. $1.13	14. $.87
15. $ .19	16. $1.39	17. $1.61	18. $.54

* In every example the operations in parentheses are to be done first.

From a five-dollar bill:

19. $3.25	20. $2.14	21. $4.26	22. $2.63
23. $3.37	24. $4.15	25. $3.72	26. $.89
27. $3.55	28. $2.97		

From a ten-dollar bill:

29. $7.25	30. $2.32	31. $7.12	32. $3.13
33. $3.03	34. $2.24	35. $3.29	36. $9.18
37. $4.44	38. $8.74		

From a double eagle:

39. $13.20	40. $12.12	41. $6.46	42. $11.11
43. $ 2.19	44. $ 8.60	45. $13.33	46. $8.29
47. $12.44	48. $10.01		

WRITTEN PROBLEMS

28. 1. How much greater or less is 900 m. than 1000 yd.? (A meter is 39.37 in.)

2. A freight car has a capacity 32,500 lb.; a second one has a capacity of half as much more. What is the joint capacity of the two cars?

3. A "local train" is moving along a railroad track at a uniform speed of 23 mi. an hour; an "express train" moves along a parallel track in the same direction at a uniform speed of 62 mi. per hour. How far ahead will the "express train" be 4 hr. and 15 min. after it passes the "local train"?

4. By how many inches does the length of your desk exceed its width?

5. Sound travels 1089 ft. per second; light travels 186,600 mi. per second. How much farther does light travel in a second than sound?

6. The earth is ninety-two million, eight hundred ninety-four thousand, eight hundred miles from the sun, and the moon is two hundred thirty-eight thousand, eight hundred fifty miles from the earth. How many miles nearer to the earth is the moon than the sun?

7. Mars is one hundred forty-one million, five hundred forty-two thousand miles from the sun, and Venus is sixty-seven million, one hundred ninety-three thousand miles from the sun. Mars is how much farther from the sun than Venus?

8. In 1912 the vote of New York for President was as follows: Mr. Wilson, 655,475; Mr. Taft, 455,428; Mr. Roosevelt, 390,021. How many more votes were cast for Mr. Wilson than for Mr. Taft? How many less were cast for Mr. Wilson than for Mr. Taft and Mr. Roosevelt together?

9. The total railway mileage in the United States for the eight years ending June 30, 1910, was reported as follows:

1903 — 207,977.22	1907 — 229,951.19
1904 — 213,904.34	1908 — 233,467.84
1905 — 218,101.04	1909 — 236,868.53
1906 — 224,363.17	1910 — 240,438.84

(*a*) What was the largest increase in any one year?

(*b*) The smallest increase?

(*c*) The total increase in the seven-year period?

(*d*) The average increase per year?

10. The imports of the United States for 1911 amounted to $ 1,527,226,105 and the exports amounted to $ 2,049,320,199. What was the balance of trade for the year?

11. The chief exports of the United States for the year 1911 were as follows:

Cotton, $ 585,318,869.
Breadstuffs, $ 108,221,416.
Iron and steel, $ 230,725,351.
Copper, $ 103,813,110.
Beef and pork, $ 129,387,520.
Mineral oil, $ 98,115,516.

(*a*) Compare the value of cotton exports with the total of the four other largest items.

(*b*) Compare the exports of breadstuffs with those of beef and pork.

(*c*) Compare the exports of copper with those of iron and steel.

12. On March 1 my bank balance was $181.13. During the month I made deposits aggregating $2809.78, and I drew checks amounting to $1316.99. Find the balance on April 1.

MULTIPLICATION OF WHOLE NUMBERS

29. **Multiplication** is the process of taking one number as many times as there are units in another.

30. The **multiplicand** is the number multiplied.

31. The **multiplier** is the number by which the multiplicand is multiplied.

32. The result of multiplication is called the **product.**

In 5×8 ft. $= 40$ ft., 5 is the multiplier, 8 ft. is the multiplicand, 40 ft. is the product.

The **multiplicand** and **multiplier** are called **factors** of the **product.**

A **concrete number** is one that refers to some particular kind of object or unit of measure.

Thus, 10 books, 3 yards, $ 5, are concrete numbers.

An **abstract number** is one that does not refer to any particular object or unit of measure.

Thus, 4, 75, 10,000, are abstract numbers.

PRINCIPLE 1. **The multiplier is regarded as an abstract number.**

PRINCIPLE 2. **The product must refer to the same kind of unit as the multiplicand.**

In saying 5 times 10 ft. equals 50 ft., we recognize (1) that the multiplier 5 is an abstract number;

and (2) that the product 50 must refer to the unit, foot, because the multiplicand is 10 ft.

PRINCIPLE 3. **The product is numerically the same whichever number is regarded as the multiplier.**

Thus, 3 times 2 is the same as 2 times 3.

The sign of multiplication × is read "times" when the multiplier precedes it and "multiplied by" when the multiplier follows it.

Thus, $4 \times \$7$ is read "4 times 7 dollars."

And $\$4 \times 7$ is read "4 dollars multiplied by 7."

Types of Oral Drills for Frequent Use

33. 1. Add by 15's to 100.

2. Add by 6's to 15 times 6.

3. Repeat the multiplication table of sevens.

4. Write the twelfth multiple of 3; of 6; of 9; of 12; of 5; of 10; of 7; of 4; of 8; of 11.

5. How much is 11×8? 5×13? 4×18? 7×12?

6. How many are 3 times 14? 6×15? 12×9?

7. How many are 3×15? 8×15? 16×15?

8. How many are 10 times 38? 5 times 38?

9. $3 \times 17 = 51$; $6 \times 17 = ?$ $9 \times 17 = ?$

10. $(5 \times 8) + (6 \times 8) = ?$ $(18 \times 5) - (3 \times 5) = ?$

11. $(4 \times 15) + (4 \times 15) = ?$ $(5 \times 16) + (5 \times 16) = ?$

12. $(9 \times 15) - (4 \times 15) = ?$ $(12 \times 16) - (4 \times 16) = ?$

13. How much is 15×15? 20×20?

Some Short Methods in Multiplication

34. 1. To multiply by 10, 100, 1000, etc., rewrite the multiplicand, and annex to the product as many ciphers as there are ciphers in the multiplier.

2. To multiply by 20, 400, 17,000, etc., multiply by all the figures in the multiplier except the final ciphers, and annex to the product as many ciphers as there are ciphers in the multiplier.

3. To multiply by 50, annex two ciphers to the multiplicand and then divide by 2.

4. To multiply by 25, annex two ciphers to the multiplicand and then divide by 4.

5. To multiply by 125, annex three ciphers to the multiplicand and then divide by 8.

SIGHT DRILLS ON SHORT METHODS IN MULTIPLICATION

35. Multiply, using the pencil only to write the answers:

1.	2.	3.	4.	5.
262	494	563	$719	$1225
50	50	50	50	50
6.	**7.**	**8.**	**9.**	**10.**
640	428	716	$944	$725
25	25	25	25	25
11.	**12.**	**13.**	**14.**	**15.**
88	400	24	$1.28	$3.60
125	125	125	125	125

ORAL PROBLEMS

For forms of explanations, see pages 26–29. Use short methods to save time and effort.

36. 1. At 60 lb. to the bushel, how much should 25 bu. of potatoes weigh?

2. A plumber charges 75 ct. an hour for his own labor and 50 ct. an hour for that of his helper. What is the proper charge for 8 hr. of their joint service?

3. At 9 ct. each, what is the cost of 16 bottles of ink?

4. At the rate of 3 for a quarter of a dollar, find the price of 18 grapefruits.

5. Collars are 8 for $ 1. How many can I buy for $ 15?

6. At 72 ct. a yard, what will 25 yd. of taffeta cost?

7. Bricks are selling at $ 12 a thousand. How much must I pay for 15,000?

8. A speculator's profit on eggs is 15 ct. a dozen. Calculate his profit on 100 crates of 30 doz. each.

9. A dealer bought 100 tons of coal by the long ton (2240 lb.) at $5 per ton and sold it by the short ton (2000 lb.) at $6 per ton. How much did he gain by the transaction?

10. How much will 125 casks contain, if each cask holds 24 gallons?

DIVISION OF WHOLE NUMBERS

37. Division has two meanings:

(1) It is the process of separating a given number into a definite number of equal parts in order to find the number of units in each part.

(2) It is the process of finding how many times one number contains another number.

The **dividend** is the number divided.

The **divisor** is the number by which the dividend is divided.

The result of division is called the **quotient.**

The **remainder** is the part of the dividend that remains when the division is not exact.

PRINCIPLE 1. **If the dividend is a concrete number and the divisor is an abstract number, the quotient and the dividendare like numbers.**

PRINCIPLE 2. **If the divisor and dividend are like numbers, the quotient is an abstract number.**

PRINCIPLE 3. **If the divisor and dividend be multiplied or divided by the same number, the quotient will not be changed.** Thus, $3)15 = 12)60$.

The sign of division $\div$ is read "divided by."

Division may also be indicated by writing the dividend above the divisor with a line between them. Thus, $8 \div 4$ may be written $\frac{8}{4}$.

DIVISION

Types of Oral Drills for Frequent Use

38. 1. Repeat the division table of twelves.

2. How much is $72 \div 12$; by 3; by 4; by 6?

3. $550 \div 10 = ?$ $1100 \div 10 = ?$ $2150 \div 10 = ?$

4. One factor of 45 is 5; what is the other factor?

5. Give the sets of factors of 24; 35; 36; 40.

6. How many 6's in 36; in 54; in 60; in 96?

7. $? \div 3 = 12$; $? \div 4 = 12$; $? \div 6 = 12$?

8. $15 \div 3 = 5$; $45 \div 3 = ?$; $90 \div 3 = ?$

9. Add the quotients of $48 \div 4$ and $28 \div 7$.

SIGHT EXERCISES

For Rapid Drill from Text or Blackboard; or for Seat Work in Limited Time

39. 1. Divide 100 by 10; by 5; by 20; by 4; by 25.

2. Divide 144 by 3, 4, 6, 8, 9, 12, 24.

3. Divide 250 by 5, 10, 20, 25, 50, 100.

4. Write the answers of the following:

$$3)1539; \quad 9)7569; \quad 73{,}514 \div 7.$$

5. How much are $(144 \div 12) + (120 \div 12)$?

6. What is the difference between $81 \div 9$ and $120 \div 10$?

7. $(96 \div 8) - (56 \div 8) = ?$ 8. $(3 \times 14) \div 2 = ?$

9. Divide the following numbers by 10: 330; 750; 7500; 5600; 14,000.

10. Divide the following numbers by 100: 42,400; 17,600; 244,900; 64,000.

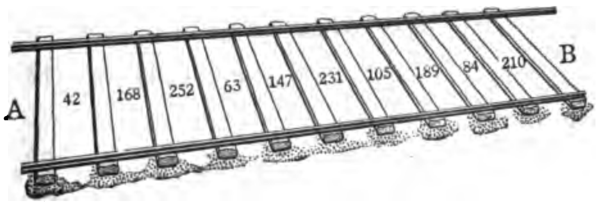
11. Divide the following numbers by 1000: 43,000; 642,000; 754,000.

ORAL DRILLS

"Railroad Ties"

40. Go from *A* to *B* as quickly as possible, and then back from *B* to *A*.

Divide by 7; by 21.



Some Short Methods in Division

41. 1. To divide by 10; 100; 1000, etc., cut off from the right of the dividend by a vertical line or by a decimal point as many figures as there are ciphers in the divisor.

2. To divide by 20; 400; 6000; 180,000, etc., cut off from the right of the dividend by a vertical line or by a decimal point as many figures as there are ciphers in the divisor; then divide the remain-

ing figures of the dividend by the remaining figures of the divisor.

3. To divide by 50, multiply the dividend by 2 and divide by 100 by pointing off two places from the right of the dividend.

4. To divide by 25, multiply the dividend by 4 and then divide by 100 by pointing off two places from the right of the dividend.

5. To divide by 125, multiply the dividend by 8 and divide by 1000 by pointing off three places from the right of the dividend.

SIGHT DRILLS

For Seat Work in Limited Time

42. Divide, using the pencil only to write the answer.

1.	50)6200	2.	50)961	3.	50)7225
4.	25)800	5.	25)700	6.	25)1200
7.	125)6000	8.	125)9000	9.	125)3000

Prove examples 2, 5, and 6.

10. $\frac{1}{1200}$ of $96,000 = ?$ 11. $\frac{1}{120}$ of $8400 = ?$

12. $\frac{1}{25}$ of $600 = ?$ 13. $\frac{1}{25}$ of $1600 = ?$

Prove examples 10 and 13.

14. Divide by 800: 32,000; 1440; 12,000.

15. Divide by 70: 1400; 2090; 1610.

16. Divide by 9000: 63,000; 94,500; 10,800.

SIGHT EXERCISES

For Seat Work in Limited Time

43. Write the answers as rapidly as possible:

1. $(100+20)\div 10=?$
2. $(120-24)\div 8=?$
3. $(72-12)\div 5=?$
4. $120\div(8+4)=?$
5. $480\div(5\times 6)=?$
6. $500\div(4\times 5)=?$
7. $(60+12)\div(8+1)=?$
8. $(250-50)\div(75-25)=?$
9. $(16\times 8)\div(32\times 1)=?$
10. $(20\times 4)\div(8\times 2)=?$
11. $20\times 4+72\div 12=?$
12. $48\div 8-25\div 5=?$

If there are no parentheses, the operations of multiplication and division are to be done first.

WRITTEN EXERCISES

44. Divide, using short methods to save time:

1. 80)450
2. 90)56280
3. 70)6580
4. 120)79080
5. 400)2700
6. 500)2650
7. 600)8240
8. 300)1690
9. 2184 by 560
10. $39.16 by 79
11. $20.18 by 35
12. 6081 by 37
13. 9074 by 45
14. 6760 by 83
15. $120.83 by 224
16. $480.96 by 588
17. $730.14 by 373
18. $920.04 by 828
19. $177.76 by 677
20. $589.88 by 909
21. 84,600 by 2150
22. 27,665 by 3750
23. 36,544 by 4769

ANALYSIS OF PROBLEMS

Forms of statement and forms of analysis in the solution of problems in multiplication and division.

45. The complete solution of a problem contains (1) **the statement,** (2) **the analysis,** (3) **the computation** (or **algorism**), and, possibly, (4) **the proof or verification.** Some solutions may properly be accompanied by a diagram.

1. (Multiplication.) At $ 6.25 per barrel, how much will 9 bbl. of flour cost?

Statement	Analysis
1 bbl. . . . . . $ 6.25 9 bbl. . . . . . ?	The cost of 9 bbl. is 9 times the cost of 1 barrel. Therefore, 9 bbl. will cost 9 × $ 6.25, or $ 56.25. *Ans.*

2. (Multiplication.) How many collars can be bought for $ 15, if 8 collars cost $ 1?

Statement	Analysis
$ 1 . . . . 8 col. $ 15 . . . . ?	$ 15 will buy 15 times as much as $ 1. Therefore, $ 15 will buy 15 × 8 collars, or 120 collars. *Ans.*

Note to Teachers. Pupils should be trained to set down problems in a clear but brief form. The statements presented herewith make conspicuous the **correspondence** (*i e.* the ratios) of the given terms and the required term. The required term, indicated by ? or *x*, is preferably placed last. Mere correspondence is indicated by the (. . . . .), and not by the sign of equality (=).

3. (Partition.) If 16 yd. of cloth cost $40, what is the cost of 1 yd.?

Statement	Analysis
16 yd. . . . . . $40 1 yd. . . . . . ?	The cost of 1 yd. is $\frac{1}{16}$ of the value of 16 yd. Therefore, the cost of 1 yd. is $\frac{1}{16}$ of $40 = $2.50. *Ans.*

4. (Partition.) If $18 will buy 144 handkerchiefs, how many will $1 buy?

Statement	Analysis
$18 . . . . . 144 h. $1 . . . . . ?	$1 will buy $\frac{1}{18}$ as many as $18. Therefore, $1 will buy $\frac{1}{18}$ of 144 handkerchiefs, or 8 handkerchiefs. *Ans.*

5. (Division.) At 75 ct. per yard, how many yards may be bought for $15?

Statement	Analysis
$.75 . . . . 1 yd. $15 . . . . ?	The number of yards will equal the number of times that 75 ct. is contained in $15. Therefore, the number of yards is 1500 ÷ 75 (15 ÷ $\frac{3}{4}$), or 20 yd. *Ans.*

6. (Partition and Multiplication.) If 24 books cost $18, how much will 100 books cost?

Statement	Analysis
24 b. . . . . . $18 100 b. . . . . . ?	Use Form 3 to find the cost of 1 book and then Form 1 to find the cost of 100 books.

7. If the third term (100) in example 6 were changed to some number that is an *exact multiple* of the first term (24), Form 1 may be used to advantage, *e.g.*:

If 24 books cost \$18, how much will 96 books cost?

Statement	Analysis
24 b. . . . . . \$18 96 b. . . . . . ?	96 books will cost 4 times as much as 24 books. Therefore, 96 books will cost $4 \times \$18$, or \$72. *Ans.*

8. If the third term (100) in example 6 were changed to some number that is a *factor* of the first term (24), then Form 3 may be used to advantage, *e.g.*:

If 24 books cost \$18, how much will 4 books cost?

Statement	Analysis
24 b. . . . . . \$18 4 b. . . . . . ?	The cost of 4 books is $\frac{1}{6}$ of the cost of 24 books. Therefore, 4 books will cost $\frac{1}{6}$ of \$18, or \$3. *Ans.*

9. (Partition and Division.) If 25 yd. cost \$7.50, how many yards can be bought for \$69?

Statement	Analysis
\$7.50 . . . . 25 yd. \$69 . . . . ?	Use Form 3 to find the cost of 1 yd. and then Form 5 to find the required number.

10. If the third term ($ 69) in example 9 were changed to some number that is a multiple of the first term ($ 7.50), Form 2 may be used to advantage, *e.g.*:

If 25 yd. cost $ 7.50, how many yards can be bought for $ 75?

Statement	**Analysis**
$ 7.50 . . . . 25 yd. $ 75 . . . . ?	$ 75 will buy 10 times as much as $ 7.50. Therefore, $ 75 will buy 10×25 yd., or 250 yd. *Ans.*

11. If the third term ($ 69) in example 9 were changed to a number that is a factor of the first term ($ 7.50), then Form 4 may be used to advantage, *e.g.*:

If 25 yd. cost $ 7.50, how many yards can be bought for $ 2.50?

Statement	**Analysis**
$ 7.50 . . . . 25 yd. $ 2.50 . . . . ?	$ 2.50 will buy $\frac{1}{3}$ as much as $ 7.50. Therefore, $ 2.50 will buy $\frac{1}{3}$ of 25 yd., or $8\frac{1}{3}$ yd. *Ans.*

ORAL PROBLEMS

For forms of explanation, see pages 26–29. Use short methods to save time and effort.

46. 1. A regiment of 1000 men are arranged in ranks of 50. How many ranks are there?

2. A train is scheduled to travel 640 mi. in 16 hr. What average speed must the engineer maintain in order to keep the train on schedule?

3. Box lunches for 15 girls will cost $3.75. How much should each girl pay?

4. By saving 15 ct. a day, a boy acquired the price of the bicycle he wanted, $21. How long did he have to save?

5. A student finds his pocket money reduced to $7.20, which must last him the remaining 18 da. of the month. To how much a day must he limit his expenditures?

6. A ship's crew of 16 men finds itself 10 da. from port with only 60 lb. of meat aboard. What limit must be put to the daily meat ration for each man?

7. A speculator made a profit of $180 on a purchase and sale of 3000 lb. of butter. Find his profit per pound.

8. At 25 ct. apiece, how many tickets must be sold to yield $20?

9. At $1.25 each, how many dictionaries can be bought for $40?

10. How many gowns can be made from 750 yd. of material, if 15 yd. are required for each gown?

WRITTEN PROBLEMS

47. Give statement and analysis of each of the following:

Given			To Find	
	Quantity Bought	Total Cost	Cost Of	How Much For
1.	18 gal.	$ 27.00	1 gal.	——
2.	40 rings	$140.00	10 rings	——
3.	25 yd.	$162.50	——	$300.00
4.	1 A.	$335.00	25 A.	——
5.	72 hats	$118.00	144 hats	——
6.	27 doz.	$ 90.00	9 doz.	——
7.	48 books	$ 36.00	1 book	——
8.	120 sq. ft.	$ 15.00	——	$ 62.50
9.	24 pkg.	$ 1.00	——	$ 5.00
10.	18 qt.	$ 1.75	50 qt.	——
11.	35 lb.	$ 75.00	25 lb.	——
12.	150 boxes	$ 22.50	50 boxes	——
13.	16 yd.	$ 1.20	——	$ 6.00
14.	32 collars	$ 4.00	——	$ 1.00
15.	20 chairs	$125.00	——	$ 50.00
16.	1 shirt	$ 1.75	——	$ 56.00
17.	15 M bricks	$112.50	100 M	——
18.	3 pr. hose	$ 1.00	——	$ 15.00
19.	1 cap	$ .88	24	——
20.	60 lamps	$ 24.00	——	$ 96.00

MISCELLANEOUS PROBLEMS

48. 1. A cashier began a day's business with $45.73 on hand. His cash receipts during the day were as follows:

$3.50	$2.14	$5.47	$16.75
2.50	7.67	6.25	8.24
.75	10.55	5.10	9.38
7.25	3.29	.48	5.05
6.00	6.78	1.50	17.49
1.38	.89	6.49	21.16

He paid out the following amounts: postage, \$5.50; wages, \$15.00; gas bill, \$17.80; wrapping-paper, \$8.20.

Find the cash balance.

2. A merchant's bank balance on Feb. 1 was \$14,957. His deposits during the month were: Feb. 4, \$367.40; Feb. 6, \$459.82; Feb. 9, \$314.70; Feb. 11, \$187.67; Feb. 13, \$234.15; Feb. 16, \$542.68; Feb. 20, \$816.00; Feb. 23, \$208.90; Feb. 25, \$477.69; Feb. 27, \$718.70.

He drew checks on the bank as follows: Feb. 2, \$425.00; Feb. 8, \$584.50; Feb. 10, \$250.00; Feb. 16, \$1980.72; Feb. 23, \$2679.29; Feb. 27, \$5167.47. Find his bank balance of March 1.

3. A stenographer earning \$1000 per year computed her expenses for next year as follows: board \$450, clothing \$250, vacation \$50, lunches and carfare \$105, miscellaneous expenses \$100. How much may she save?

4. If 25 yd. of silk cost \$52, how many yards can be bought for \$75?

5. How much must I pay for 8000 copies of a book at the rate of 16 for \$1?

6. What will 150 gross of pens cost at 75 ct. per gross?

7. If bricks are worth \$12.50 per thousand, what will 200,000 bricks cost?

8. A certain grade of men's hose is retailed at the price of $ 1 for 3 pr. Find the cost of 2120 pr.

9. At 40 ct. a dozen, what is the price of 50 crates of eggs, each crate containing 30 doz.?

10. If 18 handkerchiefs cost $ 5.40, what will 25 cost?

11. How long will 12 bbl. of flour supply a company of 98 men, allowing 2 lb. per man a day?

12. The following is the Hightide Table for New York Harbor for the first five days in July, 1912:

July	1	2	3	4	5
A.M.	9.25	9.50	10.20	10.53	11.33
P.M.	9.20	9.52	10.22	11.00	11.38

(*a*) What was the longest interval between tides?
(*b*) The shortest interval?
(*c*) The average interval?

13. A commission merchant sold for A. Vale: 260 lb. of butter at 36 ct., 320 lb. of cheese at 30 ct., 300 doz. eggs at 35 ct., and 720 lb. chicken at 23 ct. His charges were: freight $ 21.60, storage $ 2.75, cartage $ 6.20, commission $ 23.01. How much should he remit to Vale?

14. A speculator bought a building for $ 26,500, spent $ 2150 in altering the building and sold it for $ 31,250. How much was his profit?

15. A tailor bought 6 pieces of goods containing 42 yd. each; he used 38 yd. of the first piece, 39 yd. of the second, 35 yd. of the third, 37 yd. of the fourth, 41 yd. of the fifth, and 23 yd. of the sixth.

(*a*) How many yards did he have left in the remnants?

(*b*) He received $1.75 per yard for the goods sold to customers, and 90 ct. per yard for the remnants. If the goods cost $1.25 per yard, how much did he gain by the entire transaction?

16. A commercial traveler purchased a thousand-mile ticket and used it on the following trips: 315 mi., 48 mi., 96 mi., 28 mi., 47 mi., 116 mi., 237 mi., 14 mi., 29 mi., 56 mi. How much mileage had he left out of the thousand-mile ticket?

17. The mileage book cost $20. The regular fares for the trips mentioned were respectively: $9.10, $1.50, $3, $.90, $1.35, $3.50, $8, $.35, $.85, $1.60. How much was saved by using the mileage book?

18. A traveling salesman sent the following expense account to his firm for one week: railroad fares $19.72, hotel bills $18, telegraph and telephone $7.60, excess baggage $9.80, carriage hire $2.50, sundries $6.50. Find the total amount.

PROPERTIES OF WHOLE NUMBERS

FACTORING

49. The **factors of a number** are the whole numbers which, multiplied together, produce the number.

Therefore every product is exactly divisible by each of its factors.

Every exact divisor of a number is a factor of the number. The quotient is the other factor, or the product of the other factors if there are more than two.

For example: 2 is a factor of 12; then $12 \div 2$, or 6, is the other factor. Again, 2 and 3 are factors of 12; then $12 \div (2 \times 3)$, or 2, is the third factor of 12.

50. Separating a number into its factors is called **factoring**; *e.g.* $30 = 2 \times 3 \times 5$.

51. A **prime number** is a number that has no factors except itself and 1; *e.g.* 2, 5, 7, 13, 17 are prime numbers.

52. A **composite number** is a number that has factors other than itself and 1; *e.g.* 4, 6, 9, 10, 12, are composite numbers.

53. A **prime factor** is a factor that is a prime number; *e.g.* 7 is a prime factor of 28.

54. An **even number** is a number that contains

2 as a factor. All numbers that have 2, 4, 6, 8, or 0 in units' place are even numbers.

55. **An odd number** is a number that does not contain 2 as a factor. All numbers that have 1, 3, 5, 7, or 9 in units' place are odd numbers.

56. The product of two or more numbers equals the product of all the prime factors of the numbers.

$$420 = 10 \times 42 = (5 \times 2) \times (2 \times 3 \times 7) = 5 \times 2 \times 2 \times 3 \times 7$$

Tests of Divisibility

57. Learn and apply the following:

A number is exactly divisible:

By 2, if it ends in 0, 2, 4, 6, 8.

By 3, if the sum of its digits is divisible by 3.

By 4, if the number represented by the two digits at the right is divisible by 4.

By 5, if it ends in 0 or 5.

By 6, if divisible by 2 and 3.

By 8, if the number represented by the three digits at the right is divisible by 8.

By 9, if the sum of its digits is divisible by 9.

By 10, if it ends in 0.

By 12, if it is divisible by 3 and by 4.

Also, if the number represented by the two digits at the right is divisible by any factor of 100, then the whole number is divisible by the same factor.

And if the number represented by the three digits at the right is divisible by any factor of 1000, then the whole number is divisible by the same factor.

Finding Prime Factors

58. Find the prime factors of 420.

$$\begin{array}{r|r} 5 & 420 \\ 2 & 84 \\ 2 & 42 \\ 3 & 21 \\ & 7 \end{array}$$

420 ends in 0; divide by 5, which is prime.
84 is even; divide by 2, which is prime.
42 is even; divide by 2.
21 contains 3 and 7, both prime.
$420 = 5 \times 2 \times 2 \times 3 \times 7$. Hence the prime factors are 2, 2, 3, 5, 7.

Find the prime factors of these numbers:

1. 712	2. 216	3. 621	4. 1624
5. 918	6. 930	7. 435	8. 936
9. 640	10. 648	11. 842	12. 144
13. 120	14. 900	15. 1056	16. 1764
17. 360	18. 572	19. 2200	20. 975
21. 981	22. 749	23. 1080	24. 2496

GREATEST COMMON DIVISOR

59. A **common divisor** or **common factor** of two or more numbers is a number that exactly divides each of the numbers. For example, 5 is a common factor of 15 and 25.

60. The **greatest common divisor (g. c. d.)** of two or more numbers is the greatest number that exactly divides each of them. For example, 6 is the g. c. d. of 12 and 18.

Finding the Greatest Common Divisor

61. Find the greatest common divisor of 630 and 540.

$$630 = 2 \times 3 \times 3 \times 5 \times 7$$

$$540 = 2 \times 3 \times 3 \times 5 \times 2 \times 3$$

$$2 \times 3 \times 3 \times 5 = 90, \text{ g. c. d.}$$

RULE. **Resolve both numbers into their prime factors. The product of the prime factors that are common to both numbers is the greatest common divisor.**

EXERCISES

62. Find the greatest common divisor in each example:

1. 35, 49
2. 25, 50
3. 81, 135
4. 70, 105
5. 60, 140
6. 168, 189
7. 540, 450
8. 384, 432
9. 252, 288, 324
10. 720, 864, 1008
11. 490, 588, 735
12. 3150, 4050, 1350.

63. When it is required to find the greatest common divisor of two or more numbers whose factors are not easily discoverable by inspection, the following process may be employed:

To find the g. c. d. of 474, 790, and 1185.

1°. Divide one number by a smaller number.

2°. Divide the last divisor by the remainder.

3°. Continue this procedure till an exact divisor is found, which must be the g. c. d. of the two numbers taken.

4°. Find in the same way the g. c. d. of the third number and of the g. c. d. just obtained.

For practice, examples 6 to 12 of the last exercise may be done by this method.

NOTE. The explanation of this process belongs to the high school course in algebra.

```
474)790(1
    474
    316)474(1
        316
        158)316(2
            316

158)1185(7
    1106
      79)158(2
         158
```

79 is the g. c. d. *Ans.*

CANCELATION

For Reading and Discussion

64. Prove by trial that statements 1, 2, 3, and 4 are true.

1. $20)\underline{500} = 2)\underline{50}$
2. $4)\underline{12} = 2)\underline{6}$
3. $12)\underline{180} = 3)\underline{45}$
4. $25)\underline{150} = 5)\underline{30}$

We have the following important

Principle. **In any division, both dividend and divisor may be divided by the same number without changing the quotient.**

Statements 1, 2, 3, 4 may be written as follows:

1. $2 \times 10)\underline{50 \times 10} = 2)\underline{50}$
2. $2 \times 2)\underline{6 \times 2} = 2)\underline{6}$
3. $3 \times 4)\underline{45 \times 4} = 3)\underline{45}$
4. $5 \times 5)\underline{30 \times 5} =)\underline{30}$

In 1, the factor 10 is omitted, or **canceled**, from both dividend and divisor; in 2, the factor 2 is canceled; in 3, the factor 4; in 4, the factor 5.

65. Cancelation is the process of casting out equal factors from the dividend and divisor.

ORAL EXERCISES

66. Give the results of the following examples:

1. $2 \times 5)\underline{6 \times 5}$
2. $4 \times 7)\underline{12 \times 7}$
3. $5 \times 10)\underline{15 \times 10}$
4. $3 \times 8)\underline{12 \times 8}$
5. $5 \times 17)\underline{25 \times 17}$
6. $4 \times 21)\underline{20 \times 21}$
7. $2 \times 13)\underline{4 \times 13}$
8. $3 \times 9)\underline{18 \times 9}$
9. $7 \times 65)\underline{14 \times 65}$
10. $4 \times 16)\underline{20 \times 16}$

WRITTEN EXERCISES

67. Divide 10×12 by 6×5.

We write the dividend above the divisor with a line between, thus:

$$\frac{10 \times 12}{6 \times 5}$$

Dividing both dividend and divisor by 6, we have:

$$\frac{10 \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}} \times 5}$$

Dividing by 5, we have:

$$\frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{5}}}$$

The answer is $\frac{2 \times 2}{1 \times 1}$, or 4.

Divide, using cancelation:

1. 24×30 by 24×10
2. 32×42 by 8×14
3. 50×18 by 15×30
4. 60×11 by 22×5
5. 78×15 by 65×9
6. 45×40 by 60×9

Divide by canceling common factors:

7. 1260 by 180
8. 1224 by 136
9. 3600 by 225
10. 3600 by 144
11. 1350 by 75
12. 280 by 35

Use cancelation in doing the following:

13. $(12 \times 27 \times 10) \div (9 \times 24 \times 25)$
14. $(16 \times 21) \div (35 \times 12)$
15. $(28 \times 36 \times 25) \div (100 \times 21 \times 18)$
16. $(150 \times 27) \div 225$
17. $(84 \times 96 \times 144) \div (1728 \times 14)$
18. $(400 \times 54) \div 1125$

19. Divide 175 times 256 by 25 times 72.

20. How many bricks each 8 in. by 4 in. by 2 in. will be needed to build a brick wall 24 ft. long, 4 ft. high, and 2 ft. thick?

21. Find the cost of 32,600 bricks at $12.50 per M.

22. At 49 ct. a dozen, what will 600 blank-books cost?

23. At $1.50 a gross, what will 1200 pencils cost?

LEAST COMMON MULTIPLE

68. The **least common multiple (l. c. m.)** of two or more numbers is the least number that contains each of them an exact number of times.

For example, 12 is the least common multiple of 4 and 6.

Finding the Least Common Multiple

Method of Inspection

EXERCISES

69. Find the l. c. m. of 4, 6, 9, and 12.

The multiples of the largest given number are 12, 24, 36, 48, etc.

Try 12. 12 is a multiple of 4 and 6, but not of 9.

Try 24. 24 is not a multiple of 9.

Try 36. 36 is a multiple of 9; and therefore 36 is the l. c. m. of 4, 6, 9, and 12.

Find the least common multiple:

1. 3, 4, 6
2. 10, 15, 9
3. 8, 10, 16
4. 6, 24, 18

5. 32, 24, 12
6. 15, 25, 5
7. 15, 20, 30, 8
8. 36, 18, 9, 6
9. 6, 4, 15, 12
10. 12, 14, 4, 21

Name three numbers whose least common multiple is 72; 90; 100; 128.

The Division Method

EXERCISES

70. Find the l. c. m. of 18, 24, 30, 45.

When the given numbers are large, it is usually easier to find the l. c. m. by the "division method," as follows:

2	18	24	30	45
3	9	12	15	45
3	3	4	5	15
5	1	4	5	5
	1	4	1	1

Find all the prime factors by dividing by any prime number that will exactly divide two or more of the given numbers. The product of the prime factors is the l. c. m.

$2 \times 3 \times 3 \times 5 \times 4 = 360$, l. c. m.

Find the least common multiple as in example 1:

1. 15, 24, 40
2. 30, 35, 42
3. 15, 16, 20
4. 18, 27, 30
5. 16, 24, 48
6. 12, 24, 30
7. 12, 48, 72
8. 10, 25, 50
9. 8, 12, 9, 18
10. 24, 27, 36, 54
11. 35, 56, 64, 120
12. 20, 35, 40, 49

COMMON FRACTIONS

71. Numbers like 1, 6, 15, 100, are called **whole numbers** or **integers.**

72. Numbers like $\frac{1}{2}$, $\frac{3}{4}$, \$ $\frac{5}{8}$, $\frac{2}{3}$ yd., are called **fractions**. These are read: one half, three fourths, five eighths of a dollar, two thirds of a yard, respectively.

73. A **unit** is one; as 1 apple, \$ 1, 1 ft., 1 doz.

74. A **fraction** is one or more of the equal parts of a unit.

For example, $\frac{3}{4}$ represents 3 of the 4 equal parts into which the unit is divided.

75. A fraction has two **terms**, a **numerator** and a **denominator**.

For example, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

76. The **denominator** of a fraction shows into how many equal parts the unit is divided.

It is written below the line. In the fraction $\frac{3}{4}$, the denominator 4 denotes that the unit is divided into 4 equal parts.

77. The **numerator** of a fraction shows how many of the equal parts are taken to form the fraction.

It is written above the line. In the fraction $\frac{3}{4}$,

the numerator 3 denotes how many fourths are taken. Hence $\frac{3}{4}$ means $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, or 3 times $\frac{1}{4}$.

78. Common fractions are **Proper** or **Improper**.

79. A **proper fraction** is a fraction whose numerator is less than its denominator. For example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{99}{100}$, are proper fractions.

The value of a proper fraction is less than 1 (unit).

80. An **improper fraction** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to 1 or is greater than 1.

For example, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{12}{7}$, $\frac{125}{124}$, are improper fractions.

81. A **fractional unit** is a fraction whose numerator is 1; as, $\frac{1}{4}$, $\frac{1}{25}$, $\frac{1}{100}$.

82. A **mixed number** is a whole number and a proper fraction written together. It represents their sum.

For example, the mixed number \$ $5\frac{3}{4}$ represents \$ $5+$ \$ $\frac{3}{4}$. It is read "5 dollars and 3 quarters," or "5 and 3 quarter dollars."

83. Fractions are **simple, compound,** or **complex.**

84. A **simple fraction** is a single fraction with integral terms; *e.g.* $\frac{2}{3}$, $\frac{13}{24}$, $\frac{209}{3000}$.

85. A **compound fraction** is a fraction of a fraction; *e.g.* $\frac{2}{3}$ of $\frac{5}{8}$, $\frac{3}{4}\times\frac{5}{6}$ of 25.

86. A **complex fraction** is a fraction whose numerator or denominator contains a fraction; *e.g.*

$$\frac{\frac{1}{2}}{5},\ \frac{3}{\frac{2}{3}},\ \frac{2\frac{1}{2}}{5\frac{2}{5}}.$$

87. All operations with fractions depend upon the following important principles:

1. **A fraction is an indicated division with the dividend above the line and the divisor below it;** ***e.g.*** $\mathbf{\frac{2}{3} = 2 \div 3}$**.**

2. **Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.**

These principles are also expressed by the fundamental axiom: $1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{100}{100} = \frac{20}{20}$.

REDUCTION OF FRACTIONS

88. Reduction is the process of changing the form of a number without changing its value.

For example: $2 may be reduced to 200 ct.; 2 lb. to 32 oz.; 9 ft. to 3 yd.; $\frac{1}{2}$ to $\frac{2}{4}$; $\frac{6}{8}$ to $\frac{3}{4}$; $\frac{5}{10}$ to .5; $2\frac{1}{4}$ to $\frac{9}{4}$.

89. The reduction of a fraction to **higher terms** means making its terms larger without changing its value; *e.g.* $\frac{1}{3}$ may be reduced to $\frac{2}{6}$.

90. The reduction of a fraction to **lower terms** means making its terms smaller without changing its value; *e.g.* $\frac{6}{8}$ may be reduced to $\frac{3}{4}$.

91. A fraction is in its lowest terms when the terms of the fraction contain no common divisor.

92. Cancelation is the process of casting out equal factors from the numerator and denom-

inator (or from the dividend and divisor); *e.g.* $\frac{18}{24}=\frac{18\div 6}{24\div 6}=\frac{3}{4}$. Thus the factor 6 is canceled in order to reduce $\frac{18}{24}$ to lowest terms.

93. Fractions that have the same denominator are said to have a common denominator, and are called **similar fractions.**

For example, $\frac{3}{8}$ and $\frac{7}{8}$ are similar fractions. Their common denominator is 8.

94. The **least common denominator (l. c. d.)** of two or more fractions is the smallest common denominator to which they can be reduced; it is the least common multiple of their denominators.

Two methods of finding the least common multiple of two or more numbers are described in arts. 69 and 70. For all, or nearly all, of the examples in this book the l. c. d. can be found by the method of inspection.

95. Reducing a whole number or a mixed number to an improper fraction.

How many fifths in 24?

$$\begin{array}{r} 24 \\ 5 \\ \hline 120 \end{array}$$

The answer is $\frac{120}{5}$.

We say: There are 5 fifths in 1; in 24 there are 24 times 5 fifths, or 120 fifths.

How many fifths in $24\frac{3}{5}$?

$$\begin{array}{r} 24^{3} \\ 5 \\ \hline 123 \end{array}$$

The answer is $\frac{123}{5}$.

We say: In 24 units there are 24 times 5 fifths, or $\frac{120}{5}$; and in $24\frac{3}{5}$ there are 120 fifths plus 3 fifths, or $\frac{123}{5}$.

Express the rule in clear and exact language.

96. Reducing an improper fraction to a whole number or to a mixed number.

Reduce $\frac{123}{5}$ to a whole number or to a mixed number.

$\frac{123}{5} = 123 \div 5$
$= 24\frac{3}{5}$

Explanation. $\frac{5}{5} = 1$; $\frac{123}{5} =$ as many 1's as $123 \div 5 = 24\frac{3}{5}$. *Ans.*

Formulate the rule.

97. Reducing a fraction to higher terms.

Reduce $\frac{7}{8}$ to twenty-fourths.

$\frac{7}{8} = \frac{21}{24}$ *Ans.*

Explanation. To make the denominator 24, multiply 8 by 3. Multiplying both terms by 3, we have

$\frac{3 \times 7}{3 \times 8} = \frac{21}{24}$. *Ans.*

Formulate the rule.

98. Reducing a fraction to lowest terms.

Reduce $\frac{18}{24}$ to lowest terms.

$\frac{18}{24} = \frac{9}{12} = \frac{3}{4}$ *Ans.*
or
$\frac{18}{24} = \frac{3}{4}$ *Ans.*

Explanation. Dividing both terms by 2 gives $\frac{9}{12}$; and dividing both terms of $\frac{9}{12}$ by 3 gives $\frac{3}{4}$.

Or, by canceling from both terms their greatest common divisor, 6, we obtain the answer by one reduction.

Formulate the rule.

99. Reducing two or more fractions to a common denominator.

Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{5}{8}$ to the least common denominator.

Explanation. To find the l. c. d., try the multiples of the greatest denominator.

$\frac{2}{3}=\frac{16}{24}$ These are 8, 16, 24, 32, etc.

$\frac{5}{6}=\frac{20}{24}$ 8 is a multiple of 4, but not of 3 or 6.

$\frac{3}{4}=\frac{18}{24}$ 16 is a multiple of 4, but not of 3 or 6.

$\frac{5}{8}=\frac{15}{24}$ 24 is a multiple of 4, 3, and 6.

24 is the l. c. d.

Reduce each fraction to twenty-fourths.

Formulate the rule.

100. Reducing a compound fraction to a simple fraction.

Reduce $\frac{1}{2}$ of $\frac{3}{10}$ of $\frac{5}{12}$ of $166\frac{2}{3}$ to a simple fraction.

Process. $$\frac{1\times\cancel{3}\times5\times\overset{\overset{25}{\cancel{50}}}{\cancel{500}}}{\cancel{2}\times\cancel{10}\times12\times\cancel{3}}=\frac{125}{12}=10\tfrac{5}{12}\quad \textit{Ans.}$$

101. Reducing a complex fraction to a simple fraction.

1. Reduce $\dfrac{2\frac{1}{2}}{3\frac{2}{3}}$ to a simple fraction.

Process. $$\frac{2\frac{1}{2}}{3\frac{2}{3}}=2\tfrac{1}{2}\div3\tfrac{2}{3}=\frac{5}{2}\times\frac{3}{11}=\frac{15}{22}\quad \textit{Ans.}$$

2. Reduce $\dfrac{5\frac{1}{3}}{7}$ to a simple fraction.

Explanation. Multiplying both terms by 3 gives

$$\frac{3\times5\frac{1}{3}}{3\times7}=\frac{16}{21}\quad \textit{Ans.}$$

Note. If the denominator of a complex fraction is a whole number, the fraction can usually be simplified at sight, by multiplying both terms by the denominator of the fraction in the given numerator.

EXERCISES

102. 1. How many tenths in 5 units? In 8 units? In 10?

2. How many fifths in $4\frac{2}{5}$? In $6\frac{1}{5}$? In $10\frac{3}{5}$?

Reduce to improper fractions:

3. $12\frac{5}{6}$. $11\frac{8}{9}$ $12\frac{4}{7}$ $6\frac{1}{4}$ $12\frac{3}{5}$.

4. $16\frac{2}{3}$ $10\frac{9}{10}$ $18\frac{3}{4}$ $26\frac{2}{3}$ $15\frac{5}{8}$

5. How many units in 65 twelfths? In 40 sixths?

Reduce to whole or mixed numbers:

6. $\frac{56}{9}$ $\frac{90}{7}$ $\frac{120}{9}$ $\frac{75}{8}$ $\frac{60}{11}$

7. $\frac{45}{8}$ $\frac{33}{5}$ $\frac{28}{3}$ $\frac{41}{4}$ $\frac{50}{6}$

8. How many 24ths in $\frac{2}{3}$? In $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{7}{8}$?

9. How many 48ths in $\frac{1}{2}$? In $\frac{2}{3}$? $\frac{5}{6}$? $\frac{7}{3}$? $\frac{7}{12}$?

10. Reduce $\frac{5}{8}$ to 40ths. To 72ds. To 80ths. To 200ths.

Reduce to lowest terms:

11. $\frac{20}{24}$ $\frac{48}{60}$ $\frac{80}{120}$ $\frac{75}{90}$ $\frac{144}{180}$

12. $\frac{15}{24}$ $\frac{25}{60}$ $\frac{36}{60}$ $\frac{18}{24}$ $\frac{72}{128}$

Reduce to the least common denominator:

13. $\frac{3}{4}$, $\frac{2}{3}$ 14. $\frac{5}{6}$, $\frac{3}{4}$ 15. $\frac{3}{8}$, $\frac{7}{10}$

16. $\frac{4}{5}$, $\frac{9}{20}$ 17. $\frac{5}{8}$, $\frac{5}{12}$ 18. $\frac{5}{9}$, $\frac{1}{6}$, $\frac{3}{4}$

19. $\frac{2}{5}$, $\frac{9}{10}$, $\frac{1}{2}$ 20. $\frac{15}{24}$, $\frac{11}{18}$ 21. $\frac{4}{7}$, $\frac{2}{3}$, $\frac{1}{6}$

22. $\frac{4}{11}$, $\frac{2}{3}$ 23. $\frac{7}{10}$, $\frac{7}{12}$ 24. $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{4}$

Reduce to simple fractions:

25. $\frac{1}{2}$ of $\frac{5}{8}$ 26. $\frac{2}{3}$ of $\frac{9}{10}$ 27. $\frac{3}{5}$ of $\frac{8}{9}$

28. $\frac{3}{4}$ of $2\frac{2}{3}$ 29. $\frac{4}{5}$ of $\frac{15}{16}$ 30. $\frac{7}{9}$ of $5\frac{2}{5}$

31. $\frac{3}{8}$ of $4\frac{4}{15}$ 32. $\frac{4}{9}$ of $8\frac{1}{10}$

Simplify: 33. $\dfrac{3\frac{1}{3}}{5}$ 34. $\dfrac{2\frac{2}{3}}{8}$ 35. $\dfrac{1\frac{1}{2}}{4}$ 36. $\dfrac{2\frac{1}{2}}{3}$ 37. $\dfrac{4\frac{4}{5}}{12}$

ADDITION OF FRACTIONS

103. Before adding or subtracting fractions, we must reduce them to similar fractions; *i.e.* to fractions having the same denominator. It is usually best to reduce them to their *least* common denominator. (See art. 105.)

1. Add $14\frac{2}{3}$, $21\frac{5}{6}$, $18\frac{3}{4}$, $56\frac{5}{8}$.

You may arrange your work in this way:

$$\begin{aligned} 14\tfrac{2}{3} &= 14\tfrac{16}{24} \\ 21\tfrac{5}{6} &= 21\tfrac{20}{24} \\ 18\tfrac{3}{4} &= 18\tfrac{18}{24} \\ 56\tfrac{5}{8} &= \underline{56\tfrac{15}{24}} \\ & \quad 109\tfrac{69}{24} = 111\tfrac{7}{8} \quad \textit{Ans.} \end{aligned}$$

Or you may arrange your work in this way:

$$\begin{array}{r|l} & 24 \\ \hline 14\frac{2}{3} & 16 \\ 21\frac{5}{6} & 20 \\ 18\frac{3}{4} & 18 \\ 56\frac{5}{8} & 15 \\ \hline 109 & \dfrac{69}{24} = 111\frac{7}{8} \quad \textit{Ans.} \end{array}$$

2. Add $56\frac{1}{4}$
$18\frac{5}{9}$
$\underline{48\frac{7}{12}}$

3. Add $124\frac{7}{10}$
$258\frac{5}{12}$
$\underline{141\frac{4}{15}}$

4. Add $\begin{array}{r} 278\frac{5}{8} \\ 160\frac{1}{6} \\ \underline{243\frac{9}{16}} \end{array}$

5. Add $\begin{array}{r} 12\frac{3}{4} \\ 17\frac{3}{10} \\ \underline{36\frac{5}{8}} \end{array}$

6. Add $13\frac{2}{5}$, $24\frac{3}{8}$, $51\frac{1}{6}$.

7. Add $98\frac{1}{6}$, $31\frac{1}{4}$, $25\frac{1}{3}$.

8. Add $16\frac{5}{6}$, $71\frac{1}{2}$, $33\frac{1}{3}$.

9. Add $296\frac{1}{4}$, $45\frac{11}{15}$, $6\frac{19}{24}$.

10. Add $45\frac{5}{7}$, $150\frac{1}{10}$, $35\frac{4}{5}$.

11. Add $53\frac{2}{9}$, $36\frac{4}{15}$, $21\frac{3}{10}$.

12. Add $25\frac{2}{3}$, $56\frac{3}{4}$, $28\frac{5}{9}$, $15\frac{7}{8}$, $36\frac{7}{12}$, $48\frac{5}{6}$.

SUBTRACTION OF FRACTIONS

104. 1. Subtract $9\frac{4}{5}$ from $16\frac{2}{3}$.

You may arrange your work according to either model:

$$\begin{array}{rcl} 16\frac{2}{3} & = & 16\frac{10}{15} \\ 9\frac{4}{5} & = & \underline{9\frac{12}{15}} \\ & & 6\frac{13}{15} \quad \textit{Ans.} \end{array}$$

$$\begin{array}{r|l} & 15 \\ \hline 16\frac{2}{3} & 10 \\ 9\frac{4}{5} & 12 \\ \hline 6 & \frac{13}{15} \quad \textit{Ans.} \end{array}$$

Find remainders:

2. $\begin{array}{r} 48\frac{9}{10} \\ \underline{24\frac{3}{4}} \end{array}$

3. $\begin{array}{r} 294\frac{5}{16} \\ \underline{150\frac{11}{20}} \end{array}$

4. $\begin{array}{r} 45\frac{2}{3} \\ \underline{28\frac{1}{2}} \end{array}$

5. $\begin{array}{r} 122\frac{3}{4} \\ \underline{67\frac{2}{3}} \end{array}$

6. $\begin{array}{r} 118\frac{3}{4} \\ \underline{67\frac{8}{9}} \end{array}$

7. $\begin{array}{r} 62\frac{1}{2} \\ \underline{45\frac{7}{8}} \end{array}$

8. $\begin{array}{r} 400 \\ \underline{184\frac{5}{6}} \end{array}$

9. $\begin{array}{r} 210\frac{1}{10} \\ \underline{45\frac{5}{8}} \end{array}$

10. $\begin{array}{r} 743\frac{4}{5} \\ \underline{279\frac{3}{4}} \end{array}$

Perform the operations indicated:

11. $140\frac{1}{8} - 76\frac{5}{6}$

12. $128\frac{1}{4} - 59\frac{3}{5}$

13. $147\frac{2}{7} - 49\frac{3}{10}$

14. $286 - 1\frac{4}{15}$

15. $56\frac{2}{3} + 35\frac{3}{4} - 12\frac{5}{6}$

16. $7\frac{1}{2} - 3\frac{9}{10} + 2\frac{1}{5}$

17. $(5\frac{5}{8} + 2\frac{1}{3}) - (3\frac{1}{2} + 6\frac{3}{4})$ *

18. $(3\frac{1}{3} - 2\frac{1}{5}) + (6\frac{1}{4} - 3\frac{3}{8})$

19. $12\frac{1}{2} - 6\frac{2}{3} + 24\frac{7}{8} - 15\frac{3}{4}$

20. $(16\frac{2}{3} + 18\frac{3}{4}) - (25\frac{5}{9} + 7\frac{1}{5})$

ORAL EXERCISES WITH SHORT METHODS

105. 1. $\frac{3}{4} + \frac{2}{3} = ?$

Think this: $\overset{9}{\frac{3}{4}} + \overset{8}{\frac{2}{3}} = \frac{17}{12}$.

Say this: 3 times 3 are **9**; 4 times 2 are **8**; **9** + **8** = **17**; 4 × 3 = **12**. The answer is $\frac{17}{12}$, or $1\frac{5}{12}$.

2. $\frac{3}{4} - \frac{2}{3} = ?$

Think this: $\overset{9}{\frac{3}{4}} - \overset{8}{\frac{2}{3}} = \frac{1}{12}$.

Say this: 3 times 3 are **9**; 4 times 2 are **8**; **9** − **8** = **1**; 4 × 3 = **12**. The answer is $\frac{1}{12}$.

3. $\frac{5}{8} + \frac{2}{3} = ?$

4. $\frac{3}{4} - \frac{1}{5} = ?$

5. $\frac{5}{6} + \frac{3}{5} = ?$

6. $\frac{5}{9} - \frac{1}{4} = ?$

7. $\frac{1}{8} + \frac{7}{10} = ?$

8. $\frac{5}{7} - \frac{1}{2} = ?$

9. $\frac{1}{8} + \frac{1}{5} = ?$

10. $\frac{1}{5} - \frac{1}{8} = ?$

NOTE. When both fractions have 1 for a numerator, *add* the *denominators* to get the numerator of the sum and multiply the denominators for the denominator of the sum. In example 9, we say $8 + 5 = 13$; 8 times $5 = 40$; $\frac{13}{40}$ is the answer.

* The operations within a parenthesis are to be performed first and the result treated as a single number.

Similarly, to subtract one fractional unit from another, subtract the denominators to get the numerator of the difference and multiply the denominators to get the denominator of the difference. In example 10, we say $8-5=3$; $8\times 5=40$; $\frac{3}{40}$ is the answer.

11. $\frac{1}{5}+\frac{1}{12}=?$
12. $\frac{1}{4}-\frac{1}{5}=?$
13. $\frac{1}{9}+\frac{1}{7}=?$
14. $\frac{1}{9}-\frac{1}{10}=?$
15. $\frac{1}{10}+\frac{1}{25}=?$
16. $\frac{1}{15}-\frac{1}{20}=?$
17. $\frac{1}{8}+\frac{1}{10}=?$
18. $\frac{1}{8}-\frac{1}{10}=?$

MULTIPLICATION OF FRACTIONS

106. RULE. **Multiply the numerators together for a new numerator and denominators for a new denominator. Cancel if possible.**

This rule applies to the three cases of multiplication as follows:

107. Case 1. Product of a whole number and a fraction.

Multiply $\$\frac{5}{16}$ by 100.

$$\overset{25}{\cancel{100}}\times\frac{5}{\underset{4}{\cancel{16}}}=\frac{125}{4}=31\tfrac{1}{4}\ \textit{Ans.}$$

How much is $\frac{11}{24}$ of $\$84$?

$$\frac{11}{\underset{2}{\cancel{24}}}\text{ of }\$\overset{7}{\cancel{84}}=\frac{77}{2}=\$38\tfrac{1}{2}\ \textit{Ans.}$$

In Case 1, the whole number is regarded as the numerator of a fraction having 1 for a denominator, and the rule is applied.

108. Case 2. Product of a whole number and a mixed number.

Multiply $27\frac{2}{3}$ by 31.

Since $27\frac{2}{3}\times 31=31\times 27\frac{2}{3}$, we may take either number for the multiplicand. Hence there are two ways to find the product.

Learn both ways.

A mixed number multiplied by a whole number.

$$\begin{array}{lr}
 & 27\frac{2}{3}\\
 & \underline{31}\\
31\times\frac{2}{3}=62\times\frac{1}{3}= & 3\overline{)62}\\
 & 20\frac{2}{3}\\
1\times 27=\ \ 27 & 27\\
\underline{30\times 27=810} & \underline{81}\\
31\times 27\frac{2}{3}= & 857\frac{2}{3}\ \textit{Ans.}
\end{array}$$

A whole number multiplied by a mixed number.

$$\begin{array}{lr}
 & 31\\
 & \underline{27\frac{2}{3}}\\
\frac{2}{3}\text{ of }31=\frac{1}{3}\text{ of }62= & 3\overline{)62}\\
 & 20\frac{2}{3}\\
7\times 31=217 & 217\\
\underline{20\times 31=620} & \underline{62}\\
27\frac{2}{3}\times 31= & 857\frac{2}{3}\ \textit{Ans.}
\end{array}$$

In Case 2, the *fractional* part of the mixed number is first multiplied by the whole number as in Case 1; and to this product is added the product of the *integral* part of the mixed number by the whole number.

109. Case 3. Product of two or more fractions, proper or improper.

Multiply $\frac{8}{9}$, $\frac{3}{10}$, and $\frac{5}{12}$.

$$\frac{\overset{2}{\cancel{8}}}{9}\times\frac{\cancel{3}}{\underset{\cancel{2}}{\cancel{10}}}\times\frac{\cancel{5}}{\cancel{12}}=\frac{1}{9}.\quad \textit{Ans.}$$

Multiply $18\frac{3}{4}$ by $12\frac{2}{5}$.

$$\frac{\overset{15}{\cancel{75}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{31}{\cancel{62}}}{\cancel{5}}=\frac{465}{2}=232\tfrac{1}{2}.\quad \textit{Ans.}$$

In Case 3, all mixed numbers are reduced to improper fractions before applying the rule.

EXERCISES

110. 1. Find $\frac{15}{16}$ of 80; $\frac{7}{25}$ of 125; $\frac{11}{12}$ of 240.

2. How much is 18 times $1\frac{5}{9}$? 24 times $4\frac{5}{6}$?

3. Solve: $\frac{2}{3}$ of $\frac{9}{10}$ of 25.

4. Multiply $12\frac{1}{2}$ by $6\frac{2}{3}$; $18\frac{3}{4}$ by $4\frac{4}{9}$.

5. What is the product of $\frac{3\frac{3}{4}}{12}$ and $\frac{8}{9}$ of $3\frac{3}{4}$?

Multiply, using short methods:

6. $\frac{25}{27}$ by $1\frac{4}{5}$ **7.** $15\frac{3}{4}$ by 20 **8.** $\frac{14}{15}$ by $\frac{3}{7}$

9. $8\frac{1}{3}$ by $\frac{4}{5}$ **10.** $16\frac{2}{3}$ by $3\frac{3}{5}$ **11.** $24\frac{3}{4}$ by 36

12. $5\frac{5}{8}$ by $4\frac{4}{5}$ **13.** $\frac{24}{25}$ by $\frac{30}{7}$ **14.** $\frac{5}{12}$ by $\frac{9}{10}$

Simplify, using cancelation:

15. $\frac{5}{6}$ of $\frac{12}{25}$ of $\frac{3}{8}$ of $\frac{2}{5}$ **16.** $\frac{2}{3} \times \frac{5}{8} \times \frac{24}{25}$ of 100

17. $\frac{8}{15}$ of $\frac{25}{3} \times \frac{5}{2}$ of $3\frac{1}{8}$ **18.** $\frac{3}{4}$ of $\frac{2\frac{1}{2}}{3} + \frac{3}{8}$ of $\frac{1}{6}$

19. $2\frac{1}{2} \times 3 + 3\frac{1}{3} \times 5$ **20.** $3\frac{3}{4} \times 6 - 2\frac{1}{5} \times 3$

21. $\frac{5}{9} \times \frac{12}{25}$ of $18\frac{3}{4} + \frac{3}{8}$ of $16\frac{2}{3} \div \frac{1}{2}$

22. $\frac{14}{15}$ of $5\frac{5}{8} \div \frac{7}{12}$ of $4\frac{4}{5}$

DIVISION OF FRACTIONS

111. RULE. **Multiply the dividend by the reciprocal of the divisor. (Or invert the divisor and multiply.)**

NOTE. In applying this rule, a whole number is regarded as the numerator of the fraction with denominator 1; and mixed numbers are reduced to improper fractions. The most frequent cause of error in division of fractions is the failure to distinguish which number is the divisor. Of course, if the dividend is inverted, the answer will be wrong.

112. Case 1. A fraction divided by a whole number.

Divide $\frac{12}{25}$ by 80.

Process

$$\frac{12}{25} \div 80 = \frac{\overset{3}{\cancel{12}}}{25} \times \frac{1}{\underset{20}{\cancel{80}}} = \frac{3}{500}. \quad \textit{Ans.}$$

113. Case 2. A whole number divided by a fraction.

Divide 12 by $\frac{9}{200}$.

Process

$$12 \div \frac{9}{200} = \overset{4}{\cancel{12}} \times \frac{200}{\underset{3}{\cancel{9}}} = \frac{800}{3} = 266\tfrac{2}{3}. \quad \textit{Ans.}$$

114. Case 3. A fraction divided by a fraction.

Divide $\frac{12}{25}$ by $\frac{18}{35}$.

Process

$$\frac{12}{25} \div \frac{18}{35} = \frac{\overset{2}{\cancel{12}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{7}{\cancel{35}}}{\underset{3}{\cancel{18}}} = \frac{14}{15}. \quad \textit{Ans.}$$

Divide $18\frac{3}{4}$ by $3\frac{1}{3}$.

Process

$$\frac{75}{4} \div \frac{10}{3} = \frac{\overset{15}{\cancel{75}}}{4} \times \frac{3}{\underset{2}{\cancel{10}}} = \frac{45}{8} = 5\tfrac{5}{8}. \quad \textit{Ans.}$$

SPECIAL CASES OF DIVISION

115. Case 4. If dividend and divisor are similar fractions, their quotient is the quotient of their numerators.

Divide $36\frac{4}{5}$ by $1\frac{3}{5}$.

Process.* $\frac{184}{5} \div \frac{8}{5} = 184 \div 8 = 23$. *Ans.*

116. Case 5. A mixed number may be divided by a smaller whole number, as in the division of whole numbers, and the complex fraction formed from the remainder may be simplified:

*The principle of art. 63 underlies this process and also the process of Case 6, page 57.

Divide $2008\frac{4}{5}$ by 27.

Process

$$\begin{array}{r@{}l} & 74\frac{2}{5} \\ 27) & \overline{2008\frac{4}{5}} \\ & \underline{189} \\ & 118 \\ & \underline{108} \\ & 10\frac{4}{5} \end{array}$$

$$\frac{10\frac{4}{5}}{27} = \frac{54}{135} = \frac{2}{5}$$

Explanation

Dividing $2008\frac{4}{5}$ by 27, we obtain 74 for a quotient and $10\frac{4}{5}$ for a remainder; reducing $\frac{10\frac{4}{5}}{27}$ to a simple fraction, our answer is $74\frac{2}{5}$.

117. Case 6. A whole number may be divided by a smaller mixed number by multiplying both dividend and divisor by the denominator of the mixed number and then dividing as with whole numbers.

Divide 2587 by $8\frac{3}{4}$.

Process

$$\begin{array}{r|l} 8\frac{3}{4} & \underline{2587} \\ 35 & 10348(295\frac{23}{35}. \quad \textit{Ans.} \\ & \underline{70} \\ & 334 \\ & \underline{315} \\ & 198 \\ & \underline{175} \\ & 23 \end{array}$$

Explanation

Multiplying both divisor and dividend by 4, the example becomes $10{,}348 \div 35$, the quotient of which is $295\frac{23}{35}$.*

SIGHT EXERCISES

118. Divide:

1. $\frac{5}{8}$ by $\frac{1}{4}$	2. $\frac{4}{5}$ by $\frac{2}{3}$	3. $\frac{9}{10}$ by $\frac{2}{3}$
4. $\frac{24}{25}$ by $\frac{8}{15}$	5. $\frac{15}{16}$ by $\frac{5}{8}$	6. $\frac{15}{24}$ by $\frac{25}{72}$
7. 10 by $2\frac{2}{3}$	8. 20 by $3\frac{1}{4}$	9. 35 by $4\frac{1}{5}$
10. $\frac{8}{9}$ by 5	11. $\frac{9}{20}$ by 36	12. $\frac{14}{15}$ by 8

* The principle of art. 63 underlies this process and also the process of Case 4.

How much is:

13. $3\frac{3}{5} \div 9$?	14. $4\frac{4}{5} \div 15$?	15. $5\frac{2}{5} \div 18$?
16. $6\frac{2}{3} \div 12\frac{1}{2}$?	17. $5\frac{2}{5} \div 12\frac{1}{10}$?	18. $50 \div 8\frac{1}{3}$?
19. $\frac{25}{4} \div \frac{3}{4}$?	20. $6\frac{3}{10} \div \frac{9}{10}$	21. $9\frac{3}{8} \div 1\frac{7}{8}$?

WRITTEN EXERCISES

119. 1. How many times is $8\frac{3}{4}$ contained in 100?

2. How many times can $4\frac{5}{7}$ be subtracted from 60?

3. What is the quotient of $120 \div 3\frac{3}{4}$?

4. What number must $3\frac{1}{8}$ be multiplied by to produce $18\frac{3}{4}$?

5. If the dividend was $66\frac{2}{3}$ and the quotient $2\frac{2}{5}$, what was the divisor?

Find quotients:

6. $5\frac{2}{15} \div \frac{7}{30}$	7. $25\frac{2}{3} \div 44\frac{2}{5}$	8. $17\frac{3}{5} \div 36\frac{3}{5}$
9. $250 \div 8\frac{1}{3}$	10. $25\frac{5}{9} \div 2\frac{7}{9}$	11. $400 \div 4\frac{3}{8}$
12. $116\frac{2}{3} \div 10$	13. $341\frac{5}{7} \div 16$	14. $1248\frac{3}{5} \div 24$
15. $27\frac{4}{5} \div 78\frac{2}{3}$	16. $2741\frac{1}{9} \div 18$	17. $172\frac{4}{5} \div 428\frac{1}{2}$

TYPE PROBLEMS INVOLVING FRACTIONAL RELATIONS OF NUMBERS

First Type Problem. To find a Part of a Number

120. A man earns $18 a week. What does he earn in $\frac{2}{3}$ of a week?

Explanation. The amount earned in $\frac{2}{3}$ of a week is $\frac{2}{3}$ of the amount earned in a week. Therefore, the amount is $\frac{2}{3}$ of $\$18 = \$12$. *Ans.*

Second Type Problem. To find what Fraction One Number is of Another Number

121. A farmer had 60 sheep. He sold 15 sheep. What part of his flock did he sell?

Explanation. The part sold is 15 sheep out of a flock of 60 sheep, or $\frac{15}{60}$ of the flock. Therefore, the part sold is $\frac{15}{60} = \frac{1}{4}$. *Ans.*

Third Type Problem. Finding the Whole when a Fractional Part of it is Given

First Case

(*A*) To find a number when a fractional part of it is given.

122. $\frac{3}{5}$ of Mary's age is 12 yr. How old is Mary?

Explanation. $\frac{3}{5}$ of Mary's age is 12 yr. Therefore, $\frac{1}{5}$ of her age equals $\frac{1}{3}$ of 12. Her full age equals

$$5 \text{ times } \left(\frac{1}{3} \text{ of } 12\right) \text{ or } \frac{5}{\cancel{3}} \text{ of } \overset{4}{\cancel{12}} = 20 \text{ yr.} \quad \textit{Ans.}$$

A shorter explanation is: $\frac{3}{5}$ of her age equals 12; then her full age equals $\frac{5}{3}$ of $12 = 20$ yr. *Ans.*

Second Case

(*B*) To find a number when the number minus a fractional part is given.

123. A boy loses $\frac{1}{3}$ of his marbles. He then has 30 marbles. How many marbles had he at first?

Explanation. Since he lost $\frac{1}{3}$ of his marbles, 30 marbles is $\frac{2}{3}$ of the original number he had.

$\frac{2}{3}$ of the original number = 30 marbles.

$\frac{1}{3}$ of the original number = $\frac{1}{2}$ of 30 = 15 marbles.

$\frac{3}{3}$ of the original number $= 3 \times 15 = 45$ marbles.
Or, a shorter explanation is:
$\frac{2}{3}$ of the number of marbles $= 30$.
All the marbles $= 30 \div \frac{2}{3}$, or $\frac{3}{2}$ of $30 = 45$. *Ans.*

Third Case

(*C*) To find a number, when the number plus a fractional part is given.

124. A boy added to his savings $\frac{2}{5}$ of the original amount. He then had \$21 in his bank. What was the original amount of his savings?

Explanation. Since he added $\frac{2}{5}$ to the original $\frac{5}{5}$, his savings are $\frac{7}{5}$ of the original amount.

$\frac{7}{5}$ of the original amount $=$ \$21.
$\frac{1}{5}$ of the original amount $= (\frac{1}{7}$ of \$21$) =$ \$3.
$\frac{5}{5}$ or the original amount $= 5 \times 3 =$ \$15. *Ans.*
Or, a shorter explanation is:
$\frac{7}{5}$ of the original amount $=$ \$21.
The original amount $=$ \$21 $\div \frac{7}{5}$, or $\frac{5}{7}$ of \$21 $=$ \$15. *Ans.*

MISCELLANEOUS PROBLEMS IN FRACTIONS

ORAL PROBLEMS

125. 1. If $\frac{3}{5}$ of the amount of flour in a barrel is $16\frac{1}{2}$ lb., how many pounds are there in the barrel?

2. At \$$2\frac{1}{4}$ a day, how much will a man earn in $7\frac{1}{2}$ da.?

3. A tub of butter contains 53 lb. From it the following quantities were sold: $3\frac{1}{4}$ lb., $7\frac{1}{2}$ lb., $6\frac{1}{4}$ lb., $\frac{1}{2}$ lb., $9\frac{3}{4}$ lb. How many pounds were sold? How many pounds remained in the tub?

4. How much do 9 lb. of butter cost at $\$\frac{3}{5}$ a pound? How much change should I receive from \$10?

5. A hook $6\frac{1}{2}$ in. long is driven $4\frac{3}{4}$ in. into a wall. How much of the hook projects from the wall?

6. Find the cost of $6\frac{3}{4}$ lb. of fish at 12 ct. a pound.

7. Find the cost of $8\frac{1}{4}$ doz. eggs at 36 ct. a dozen.

8. $\frac{1}{4}$ lb. of meat costs $7\frac{1}{2}$ ct. Find the cost of $\frac{3}{4}$ lb.

9. $\frac{1}{4}$ lb. of meat costs 8 ct. How much meat may be bought for 64 ct.?

10. $\frac{1}{2}$ yd. of ribbon costs 14 ct. Find the cost of $3\frac{1}{2}$ yd.

11. $\frac{1}{2}$ yd. of ribbon costs 14 ct. How much ribbon may be bought for 70 ct.?

12. A boy earns $\$\frac{3}{5}$ a day. In how many days will he earn $\$1\frac{4}{5}$? $\$3\frac{3}{5}$?

13. 6 yd. of cloth cost \$14. How much do 12 yd. cost?

14. 3 lb. of tea cost $\$1\frac{3}{4}$. Find the cost of 9 lb.

15. I spend 48 ct. for candies at 2 for 1 ct. How many candies should I receive for the same money at the rate of 3 for 1 ct.?

16. 48 T. of coal were divided among a number of families. Each family received $\frac{3}{4}$ T. How many families were supplied?

WRITTEN PROBLEMS

(Use paper and pencil only when the numbers are too large to be carried in the mind.)

126. 1. A man who is walking from Philadelphia to New York covers the following distances: $18\frac{7}{8}$ mi., $19\frac{5}{6}$ mi., $16\frac{3}{4}$ mi., $17\frac{1}{2}$ mi. How far has he walked?

2. 12 bbl. of flour weighing 196 lb. each were bought at the rate of $4\frac{3}{4}$ ct. a pound and sold at the rate of $6\frac{1}{2}$ ct. a pound. Find the profit.

3. An express company delivered 96 packages, each weighing $7\frac{3}{4}$ lb. It charged $\frac{3}{4}$ ct. per pound. How much was received?

4. From a piece of cloth containing $68\frac{1}{4}$ yd., the following pieces were cut: $7\frac{3}{8}$ yd., $3\frac{3}{4}$ yd., $8\frac{1}{2}$ yd., 14 yd., $6\frac{1}{4}$ yd. What is the value of the remaining piece if sold at $\$1\frac{3}{4}$ a yard?

5. $\frac{3}{5}$ of the number of boys in a class is 27. How many boys are in the class?

6. If a man earns $\$4\frac{1}{4}$ a day, and spends $\$3\frac{1}{2}$ a day, how much will he save in 1 year? (365 da.)

7. A train that has been going at the rate of $18\frac{3}{4}$ mi. an hour increases its speed by $\frac{3}{8}$ of its former rate. At what rate per hour does it travel after the increase?

8. A train has a distance of 162 mi. to go. It travels at the rate of 36 mi. an hour for $2\frac{3}{4}$ hr. Then it decreases its speed by $\frac{1}{4}$ of its former rate.

How long will it take the train to go the rest of the journey?

9. A man sells a horse that cost him \$460 at a gain of $\frac{5}{8}$ of the cost. How much does he receive for the horse? How much does he gain?

10. A school contains 3024 pupils. 5 out of every 9 were born in New York. How many were born in New York?

11. An agent agrees to take all the apples from Mr. Brown's orchard, at the rate of \$ $2\frac{2}{5}$ per barrel. Mr. Brown sends the agent 475 bbl.

(*a*) If the agent pays \$ $\frac{3}{10}$ per barrel for freight, how much do the apples cost him?

(*b*) For how much per barrel must the agent sell the apples to gain \$ $1\frac{1}{4}$ on each barrel?

12. A publisher mails 216 packages of magazines. Each package contains 18 magazines. Each magazine weighs $1\frac{1}{4}$ lb. The post office charges 1 ct. a pound for postage. How much money must be paid for postage on the magazines?

13. The cargo of a boat consisted of: 56 boxes, weighing $27\frac{1}{2}$ lb. each; 38 bales, weighing $108\frac{3}{4}$ lb. each; 75 bbl., weighing $124\frac{1}{4}$ lb. each. Find the total weight of the cargo.

14. A boy is employed to address envelopes. He receives \$ $\frac{4}{5}$ for every 1000 envelopes he addresses. How much will he receive for addressing 13,750 envelopes?

15. In a large business there are the following employees: 375 persons at $\$ 8\frac{3}{4}$ a week; 120 persons at $\$ 12\frac{1}{2}$ a week; 94 persons at $\$ 16\frac{1}{4}$ a week. How much money is paid in wages each week?

16. Find the cost in each of the following:

(*a*) 234 bu. wheat at $96\frac{3}{8}$ ct. a bushel.
(*b*) 375 bu. corn at $37\frac{3}{4}$ ct. a bushel.
(*c*) 586 bu. oats at $40\frac{5}{8}$ ct. a bushel.
(*d*) 945 lb. turkey at $26\frac{1}{4}$ ct. a pound.
(*e*) 892 lb. chicken at $20\frac{3}{4}$ ct. a pound.

17. A wholesale dealer bought 1450 lb. of chicken at $15\frac{3}{4}$ ct. a pound. He paid an average of $2\frac{1}{2}$ ct. a pound for freight. He sold the chickens at 24 ct. a pound. Find his profit.

18. A dealer bought 876 bbl. of apples at $\$ 3\frac{3}{4}$ a barrel. He sold $\frac{1}{2}$ of them at $\$ 5\frac{1}{4}$ a barrel, $\frac{1}{4}$ of them at $\$ 5\frac{1}{2}$ a barrel, and the remainder at $\$ 4\frac{3}{4}$ a barrel. Find the dealer's profits if his shipping expenses were $375.

19. 260 bbl. of onions were bought at $\$ 2\frac{3}{5}$ a barrel. One half of them were sold at $\$ 3\frac{4}{5}$ a barrel; one half of the remainder were sold at $\$ 4\frac{1}{2}$ a barrel; and the rest were sold at cost. Find the profit.

20. A grocer puts sugar up in packages, each package containing $3\frac{1}{2}$ lb. If the grocer makes a profit of $3\frac{1}{4}$ ct. on each package, how much profit will he make on 966 lb. of sugar?

21. If $3\frac{3}{4}$ yd. of cloth cost \$ $4\frac{1}{2}$, what is the cost of $18\frac{1}{2}$ yd.?

22. $\frac{5}{8}$ of the distance between two cities is $170\frac{3}{8}$ mi. Find the entire distance.

23. A river is 375 mi. long. This is $\frac{3}{5}$ of the length of a second river. Find the length of the second river.

24. $\frac{5}{9}$ of the pupils in a school are girls. If there are 585 girls, how many pupils are in the school? How many boys?

25. How many barrels of cement may be purchased for \$ $57\frac{1}{2}$, if a barrel of cement costs \$ $2\frac{7}{8}$?

26. How many pieces of rope each $1\frac{3}{4}$ ft. long may be cut from a piece 84 ft. long?

27. How long will it take a man to earn \$ $79\frac{3}{4}$ at the rate of \$ $2\frac{3}{4}$ per day?

28. A vessel sailed $\frac{1}{4}$ of the distance between 2 ports the first day; $\frac{1}{5}$ of the distance the second day; $\frac{1}{2}$ of the remainder the third day. There were 99 mi. left. Find the total distance.

29. A board is cut into 12 equal pieces. Then one of the pieces is cut into 3 equal parts. If the length of one of the smaller pieces is $\frac{3}{8}$ of a foot, what was the length of the original board?

30. A and B are 58 mi. apart, and are walking toward each other. A walks at the rate of $3\frac{1}{2}$ mi. an hour; B at the rate of $3\frac{3}{4}$ mi. an hour. How long will it take them to meet each other?

31. (*a*) A can do a piece of work in 9 hr. What part of it can he do in 1 hr.?

(*b*) B can do a piece of work in $7\frac{1}{2}$ hr. What part of it can he do in 1 hr.?

32. (*a*) A pipe fills a tank in 8 hr. What part of the tank will be filled in 1 hr.?

(*b*) A pipe empties a tank in 10 hr. What part of the tank will be emptied in 1 hr.?

33. A tank has 2 pipes. One will fill the tank in 4 hr., the other will empty it in 6 hr. How long will it take to fill the tank if both pipes are opened at the same time?

34. (*a*) The distance between A and B is $3\frac{1}{3}$ times the distance between C and D. If $\frac{3}{5}$ of the distance between C and D is $38\frac{1}{4}$ mi., what is the distance between A and B? (Draw a diagram.)

(*b*) How many more miles in the distance between A and B than in the distance between C and D?

35. The electric light used in a factory costs $\$\frac{3}{5}$ every $4\frac{1}{2}$ hr. How much does it cost every 6 da. if light is used for $2\frac{3}{4}$ hr. a day?

36. The population of a city 3 yr. ago was 560,000. Each year during the last 3 yr. the population has increased $\frac{1}{20}$ of what it was the preceding year. Find the population?

37. If an aëroplane flies 56 mi. in $2\frac{2}{3}$ hr., in how many hours will it fly 96 mi.?

38. A merchant has two qualities of cloth. The better quality costs $3\frac{1}{3}$ times as much as the poorer quality. If $\frac{3}{4}$ yd. of the poorer quality costs \$ $1\frac{1}{2}$, find the cost of $7\frac{1}{2}$ yd. of the better quality.

39. $\frac{3}{4}$ lb. of coffee cost \$ $\frac{3}{5}$.

(*a*) Find the cost of $\frac{1}{2}$ lb.

(*b*) Find the cost of $6\frac{1}{2}$ lb.

(*c*) Find how many pounds may be bought for \$ $2\frac{4}{5}$.

40. $\frac{1}{2}$ yd. of ribbon cost 18 ct.

(*a*) What part of a yard may be bought for 9 ct.?

(*b*) How many yards may be bought for \$ 3.78?

(*c*) Find the cost of $3\frac{1}{4}$ yd.

41. A contractor completed $\frac{2}{5}$ of a job in $12\frac{1}{2}$ da. How much longer should it take to finish the job?

42. Two trains leave St. Louis at the same time. One goes in a westerly direction at the rate of $23\frac{7}{8}$ mi. an hour, the other goes in an easterly direction at the rate of $25\frac{2}{3}$ mi. an hour. How far apart will the trains be after traveling $6\frac{3}{4}$ hr.?

43. The distance from New York to Albany is $\frac{3}{20}$ of the distance from New York to Chicago. Assuming that Albany is 147 mi. from New York, what is the distance from New York to Chicago?

44. The register of a school is 480. All except 80 were promoted. What fractional part was promoted?

45. A train runs 45 mi. the first hour, and $1\frac{1}{2}$ times as much the second hour; the third hour its speed is $\frac{3}{4}$ of that of the second hour. Find the number of miles the train travels in the three hours.

46. A man invests his money in Railroad bonds and real estate. His income from the latter is \$360, or $\frac{3}{5}$ of his entire income. Find the income from bonds.

47. Find the amount of the following bill:

115 yd. of Ribbon, Style X, @ $8\frac{1}{2}$ ct.
$127\frac{1}{2}$ yd. of Ribbon, Style XX, @ 6 ct.
$134\frac{3}{4}$ yd. of Ribbon, Style XXX, @ $7\frac{1}{2}$ ct.

48. A man's pay was increased by $\frac{1}{20}$, after which he received \$4.20 a day. What did he receive per day before the increase?

49. The Twentieth Century Express runs at an average speed of 45 mi. an hour. This is $2\frac{7}{8}$ times the speed of Stephenson's first locomotive of 1829. Find the rate of the first locomotive.

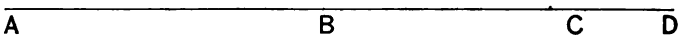
50. On a map a line $2\frac{3}{4}$ in. long represents a distance of 132 mi. What distance is represented by a line $8\frac{1}{2}$ in. long?

51. A piece of glass $6\frac{3}{4}$ in. by $5\frac{1}{2}$ in. is cut from a pane $18\frac{1}{2}$ in. by $15\frac{1}{2}$ in. Find the area of the part of the glass left over. (Draw a diagram.)

52. The distance from A to D is $197\frac{3}{5}$ mi., the distance from A to B is $71\frac{3}{8}$ mi., from C to D is $32\frac{3}{4}$ mi.

(*a*) Find the distance from B to C.

(*b*) The distance from A to C is what part of the distance from A to D?



53. A grocer sells 3465 lb. of sugar, put up in $3\frac{1}{2}$ lb. packages, at 19 ct. per package. How much does he receive for the sugar?

54. Find the amount of the following bill:

347 yd.	Serge	@	$38\frac{1}{2}$ ct.
189 yd.	Alpaca	@	\$ $1.12\frac{1}{2}$.
74 yd.	Silk	@	\$ $3.37\frac{1}{2}$.

DECIMAL FRACTIONS

127. A **decimal fraction** is a fraction whose denominator is ten or a power of ten; *e.g.* 10, 100, 1000, etc.

The first power of $10 = 10$.
The second power of $10 = 10 \times 10 = 100$.
The third power of $10 = 10 \times 10 \times 10 = 1000$.
The fifth power of $10 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$.

The denominator of a decimal fraction is rarely expressed in figures. It is understood and read by its position in relation to the decimal point.

Ten orders of whole numbers and eight orders of decimals are shown in the following table:

Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units		Ten*ths*	Hundred*ths*	Thousand*ths*	Ten-thousand*ths*	Hundred-thousand*ths*	Million*ths*	Ten-million*ths*	Hundred-million*ths*
4	9	7	6	3	5	8	2	4	6	•	9	7	5	3	8	2	4	6

ORDERS OF WHOLE NUMBERS ORDERS OF DECIMALS

The denominator of a decimal fraction is the name of the order to the right of the decimal point occupied by the last figure of the numerator; *e.g.* .025 is read 25 thousandths, the numerator being 25 and the denominator being thousandths.

128. PRINCIPLES. **1. The value of the orders of decimals decreases from left to right and increases from right to left in the same manner as the orders of whole numbers.**

2. The removal of a decimal figure one place to the left multiplies its value by 10, and its removal one place to the right divides its value by ten; *e.g.* .40 multiplied by 10 becomes 4.0; and 0.5 divided by 10 becomes .05.

NOTE. Moving the decimal point to the right produces the same result as moving the figure to the left; and moving the decimal point to the left produces the same result as moving the figure to the right.

129. A **mixed decimal** is a whole number and a decimal written together.

130. A **complex decimal** is a decimal containing a common fraction; *e.g.* $.5\frac{3}{4}$ is read five and three-fourths tenths; $.0785\frac{1}{3}$ is read seven hundred eighty-five and one third ten-thousandths.

NOTATION AND NUMERATION

131. Read aloud:

1. .1234	2. .12345	3. .123456	4. 200.0020
.7068	.20640	.260450	4060.00065
.9600	.03090	.100900	900.000900
.0745	.00781	.084010	84 00084
.0109	.42008	.100604	27.0087

Read the same numbers, after moving the decimal points one place to the right. Two places. Three places.

Read the fractions given in the next exercises.

WRITTEN EXERCISES

132. Write decimally:

1. $\frac{56}{10000}$	2. $\frac{703}{10000}$	3. $\frac{2064}{10000}$	4. $\frac{86}{100000}$
$\frac{450}{100000}$	$\frac{703}{1000000}$	$\frac{2064}{10}$	$\frac{7}{10000}$
$\frac{906}{1000000}$	$\frac{703}{10}$	$\frac{2064}{1000000}$	$\frac{3285}{10000}$
$\frac{2400}{100000}$	$\frac{703}{100000}$	$\frac{2064}{1000}$	$\frac{7060}{100000}$

Seventy hundred-thousandths; seven ten-thousandths; seven hundred millionths; seven hundred hundred-thousandths.

Eleven thousandths; sixty-four hundred-thou-

sandths; four hundred eight thousandths; three thousand ten-thousandths.

76 hundred-thousandths; 7600 thousandths; 207 ten-thousandths; 6020 millionths.

REDUCTION OF DECIMALS

133. To reduce a decimal to a lower order.

$.5 = .50 = .500 = .5000 = .50000 = .500000$, etc.
$6.03 = 6.030 = 6.0300 = 6.03000 = 6.030000$, etc.

PRINCIPLE. **Annexing ciphers to a decimal fraction multiplies its numerator and denominator by the same number* and reduces the fraction to a *lower* order without changing its value.**

134. To reduce a decimal to a higher order.

$.250 = .25$	$.08400 = .084$	$.5000 = .5$
$8.0140 = 8.014$	$6.00300 = 6.003$	$21.90000 = 21.9$

PRINCIPLE. **Striking out ciphers from the right of a decimal fraction divides the numerator and denominator by the same number and reduces the fraction to a *higher* order without changing its value.**

135. To reduce a decimal to an equivalent common fraction.

$$.75 = \frac{75}{100} = \frac{3}{4} \qquad 2.75 = 2\tfrac{75}{100} = 2\tfrac{3}{4}$$

$$.066\tfrac{2}{3} = \frac{66\frac{2}{3}}{1000} = \frac{200}{3000} = \frac{1}{15}$$

RULE. **To reduce a decimal to a common fraction, write the decimal as a common fraction and reduce to lowest terms.**

* This number is always some power of 10.

136. To reduce a common fraction to an equivalent decimal.

Reduce $\frac{9}{32}$ to a decimal.

$$\begin{array}{l} \quad .28125 \ \textit{Ans.} \\ 32)\overline{9.00000} \\ \quad \underline{6\,4} \\ \quad 2\,60 \\ \quad \underline{2\,56} \\ \qquad 40 \\ \qquad \underline{32} \\ \qquad\ 80 \\ \qquad\ \underline{64} \\ \qquad\ 160 \\ \qquad\ \underline{160} \end{array}$$

$\frac{9}{32} = \frac{1}{32}$ of 9.0, or $\frac{1}{32}$ of 9.00, etc.
9.0 ÷ 32 gives .2 and a remainder.
9.00 ÷ 32 gives .28 and a remainder.
9.000 ÷ 32 gives .281 and a remainder.
9.0000 ÷ 32 gives .2812 and a remainder.
9.00000 ÷ 32 gives .28125 and no remainder.

Reduce $\frac{5}{12}$ to a decimal.

$$\begin{array}{l} 12)\underline{5.000} \\ \quad .416\frac{2}{3}, \text{ or } .417 \ \textit{Ans.} \end{array}$$

Usually, thousandths is exact enough.

RULE. **To reduce a common fraction to an equivalent decimal, regard the denominator as a divisor and the numerator as a dividend and divide, annexing as many ciphers to the dividend as are needed.**

Decimal Equivalents of Business Fractions

137. Copy and memorize the following equivalents:

$\frac{1}{2} = .5$	$\frac{1}{4} = .25$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{8} = .125$
$\frac{1}{5} = .2$	$\frac{3}{4} = .75$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{3}{8} = .375$
$\frac{2}{5} = .4$	$\frac{1}{20} = .05$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{5}{8} = .625$
$\frac{3}{5} = .6$	$\frac{1}{25} = .04$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{7}{8} = .875$
$\frac{4}{5} = .8$	$\frac{1}{50} = .02$	$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{40} = .025$

Miscellaneous Examples in Reduction of Decimals

SIGHT EXERCISES

138. 1. Reduce to tenths: .60; .400; .7000; 25.; .625.

2. Reduce to hundredths: .4; .3; .7; $.3\frac{1}{3}$; $.2\frac{1}{5}$; .375.

3. Reduce to highest decimal order: .400; .35000; .016000.

4. Reduce to thousandths: .9; $.8\frac{1}{4}$; .0325; $.00\frac{1}{2}$; $.0\frac{3}{4}$.

5. Change to hundredths: .060; .0100; .755; $.2\frac{2}{3}$.

6. Change to thousandths: .1500; .68; .25000; $.3\frac{1}{4}$.

7. Change to ten-thousandths: .07560; .08500; .001000.

8. Change to hundred-thousandths: .7506200; .640000.

9. Change to millionths:

.28346500; .777690000.

10. Express as ten-thousandths: .4; $.14\frac{1}{4}$; $.1\frac{7}{8}$; $.0\frac{3}{5}$.

WRITTEN EXERCISES

139. Reduce to common fractions or to mixed numbers in lowest terms:

1. .0325	2. 1.62500	3. 8.0720
4. 18.1875	5. .24	6. .0400

7. $6.194\frac{4}{9}$	8. $5.11\frac{1}{2}$	9. $6.0066\frac{2}{3}$
10. $12.0042\frac{2}{5}$	11. $1.014\frac{2}{7}$	12. $10.016\frac{2}{3}$
13. $.02\frac{1}{2}$	14. $.01\frac{1}{4}$	15. $.428\frac{4}{7}$

EXERCISES

140. Reduce to decimal form:

1. $\frac{81}{125}$	2. $\frac{16}{25}$	3. $\frac{41}{50}$	4. $\frac{7}{25}$
5. $\frac{9}{16}$	6. $\frac{11}{16}$	7. $\frac{56}{125}$	8. $\frac{18}{70}$
9. $7\frac{47}{60}$	10. $14\frac{51}{120}$	11. $25\frac{1}{80}$	12. $66\frac{1}{60}$
13. $\frac{7}{800}$	14. $\frac{7}{8000}$	15. $\frac{7}{80000}$	16. $25\frac{15}{32}$
17. $40\frac{127}{625}$	18. $12\frac{31}{75}$		

19. Compare: .487 with $\frac{5}{12}$; .387 with $\frac{9}{25}$; .925 with $\frac{15}{16}$.

20. Which is greater, .287 or $\frac{2}{7}$? .435 or $\frac{4}{9}$? $\frac{4}{15}$ or .2345?

21. Which is less, $\frac{84}{125}$ or .6385? 1.167 or $1\frac{1}{6}$? .0325 or $\frac{1}{30}$?

ADDITION OF DECIMALS

141. Add 285.035, 16.7086, .92701, 26.7.

Process	Explanation
285.035 16.7086 .92701 26.7 329.37061	Since only like things can be added, the figures of the same order must be written in the same column. Since ten units in any order equal one unit in the next higher order, we may begin at the right and add as with integers.

SUBTRACTION OF DECIMALS

142. Subtract 47.2806 from 104.15.

First Process	Second Process
104.1500	104.15
47.2806	47.2806
56.8694	56.8694

Since only like things can be subtracted, the figures of the same order must be written in the same column. Since ten units in any order equal one unit in the next higher order, we begin at the right and subtract as with integers.

Instead of actually writing ciphers to make both decimals of the same order, as in the First Process, we may imagine the ciphers to be written, as in the Second Process.

RULE. **To add or subtract decimals, write like orders under one another and proceed as with whole numbers.**

EXERCISES

143. Find sums:

1. 3.098; 42.2706; 108.5; 907.08067; 16.09.
2. 250.0786; 2.1909; 749.009; 841.0176.
3. 2078.7; 170.07; 200.02; 747.700; 264.0981
4. 56204.341; 9.0007; 281.403; 1010.; 25.042.
5. 808.00095; 6.872354; .298017; .6731429; 2.10.
6. 27.0702; 5600.; .08040; 18.; 47.3987; .080195.
7. 94.72; 94567.09; 1200.; 365.; 40.0008; 2.0000027.

EXERCISES

144. Find remainders:

1. $267.018 - 148.84$
2. $26.17 - 18.049$
3. $2000.10 - 1476.0804$
4. $47.0035 - 25.764$
5. $941.076 - 279.4091$
6. $2846.5 - 1009.60754$
7. $6.6 - .002578$
8. $90675.207 - 84072.004$
9. $2,000,000 - 1,284,750.68$
10. $4,619,584.57 - 3,984,631.89$

EXERCISES

145. Find results, correct to thousandths:

1. $25\frac{3}{8} + 16.08 - 3\frac{2}{5}$
2. $18\frac{3}{4} - 16.075 + 9\frac{1}{3}$
3. $106 - 41\frac{5}{6} + 57.018$
4. $14\frac{1}{7} + 21\frac{1}{2} - 5.021$
5. $200 - 74\frac{3}{4} - 26.0475$
6. $20.175 + 56\frac{1}{9} + 45\frac{2}{3}$
7. $65\frac{7}{10} - 42.625 + 6$
8. $16\frac{5}{16} + 35\frac{5}{12} - 42.025$

EXERCISES

146. Find results according to the following method: (1) Add to the first number all the other numbers that are preceded by the plus sign. (2) Add together all the numbers preceded by the minus sign. (3) Subtract the second sum from the first sum.

1. $24.18 + 281.374 - 106.87 - 48.2089 + 17.3 - 2.0075$

2. $64.289 - 14.75 + 28.333 + 71.556 - 18.35 - 2.571$

3. $9.095 + 28.385 - 17.066 - 41.833 - 75.625 + 115.5$

MULTIPLICATION

147. Multiply 8.478 by .52

$$\begin{array}{r} 8.478 \\ .52 \\ \hline 16956 \\ 42390 \\ \hline 4.40856 \end{array}$$

$8478 \times 52 = 440856$.

There are 3 decimal places in the multiplicand and 2 in the multiplier Therefore, $3 + 2$, or 5 decimal places must be pointed off in the product.

Proof. $\frac{8478}{1000} \times \frac{52}{100} = \frac{440856}{100000} = 4\frac{40856}{100000}$.

148. RULE. **To multiply one decimal by another, multiply as with whole numbers and point off as many places from the right of the product as the sum of the decimal places in multiplicand and multiplier.**

NOTES. 1. Since a whole number may be regarded as a decimal number containing 0 decimal places, the same rule applies to the multiplication of a whole number and a decimal; *e.g.* $8.478 \times 52 = 440.856$, since $(3 + 0)$ decimal places $= 3$.

2. In pointing off, it is sometimes necessary to prefix ciphers to the product in order to get the proper number of decimal places; *e.g.* $.012 \times .06 = .00072$.

EXERCISES

Find products:

1. $32.5 \times .75$
2. 45.62×14.7
3. 167.085×45
4. 17.058×2.009

5. $2000 \times .0625$ 6. 4.50×2.080604

7. 7894.008×35.64 8. 16.909×725

9. 605.40×27.00748

Short Methods in Multiplication of Decimals

Multiplication by 10, 100, 1000, etc.

149. 1. Multiply 8.56 by 10; by 100.

$8.56 \times 10 = 85.6.$ $8.56 \times 100 = 856.$

2. Multiply 24.0768 by 1000; by 1,000,000.

$24.0768 \times 1000 = 24{,}076.8.$

$24.0768 \times 1{,}000{,}000 = 24{,}076{,}800.$

150. Rule. **To multiply a decimal by a power of 10, move the decimal point as many places to the RIGHT as there are ciphers in the multiplier. Annex ciphers whenever necessary to give the product the proper number of decimal places.**

EXERCISES

1. Multiply by 100: 2.75; 6.8; 12 34; 32; 6.0078; .000085.

2. Multiply by 1000: 17.568; 13.2; 57.96; 175; 3.0042; .000621.

3. Multiply by 10,000: 53.4728; 7.5; 28.52; 1728; 2.00056; .000075.

4. Multiply by 100,000: 5.62896; 2.2; 1.00725; 38; .00000125.

5. Multiply by 1,000,000: 4.625018; 1.75; 15; 6.2; .000000050.

To multiply by a Whole Number ending in One or More Ciphers

151. Multiply 25.625 by 80; by 800; by 80,000.

$$25.625 \times 80 = 256.25 \times 8 = 2050.$$
$$25.625 \times 800 = 2562.5 \times 8 = 20{,}500.$$
$$25.625 \times 80{,}000 = 256{,}250 \times 8 = 2{,}050{,}000.$$

152. RULE. **To multiply a decimal by a whole number ending in one or more ciphers, move the decimal point in the multiplicand as many places to the right as there are ciphers in the multiplier and multiply by the significant * figures of the multiplier.**

EXERCISES

153. 1. Multiply by 20: 16.50; .081; 3.7; 156; 8.957.

2. Multiply by 600: 35.76; 28.925; 9.367; 250; .007.

3. Multiply by 4000: 2.5; 76.03; .087; 200.0065.

4. Multiply by 90,000: 5.6; 37.50; 1.218; 9.00054.

5. Multiply by 47,000: 9.5; 46.13; 7.485; 3.000675.

6. Multiply by 8,000,000: 7.8; 12.63; 17.086.

7. Multiply by 7600: 1.8; 46.25; 9.086; 200.0081.

* The significant figures of a number are the figures that remain after the ciphers adjacent to the decimal point are struck out; *e.g.* in 20,500 the significant figures are 2, 0, and 5.

8. Multiply by 325,000: 6.3; 66.33; 9.111; 17.14280.

A SHORT METHOD WITH MULTIPLIERS EQUIVALENT TO EASY COMMON FRACTIONS

154. Multiply 42.15 by .5; by .25; by .625; by $.83\frac{1}{3}$.

$$42.15 \times .5 = 42.15 \times \frac{1}{2} = 21.075$$

$$42.15 \times .25 = 42.15 \times \frac{1}{4} = 10.5375$$

$$42.15 \times .625 = 42.15 \times \frac{5}{8} = \frac{210.75}{8} = 26.35625$$

$$42.15 \times .83\tfrac{1}{3} = 42.15 \times \frac{5}{6} = \frac{210.75}{6} = 35.125$$

NOTE. Time may sometimes be saved by reducing a decimal multiplier to the equivalent common fraction.

SIGHT EXERCISES

155. 1. Multiply by .50, giving answers at once:

166	2.72	1.980	.8248
4.36	28.50	2.742	.82568

2. Multiply by .25:

160	328	6.4	.1728
7.20	2.564	24.368	.47960

3. Multiply by .125:

8000	160	2.4	.0144

4. Multiply $.33\frac{1}{3}$:

900	1.50	9.6	.06246
2.34	28.176	125.073	.957612

5. Multiply by $.16\frac{2}{3}$:

180	3.60	14.10	.081
378	5.07	7.2	.00144

6. Multiply by $.14\frac{2}{7}$:

700	1.4	.2842	.5607

7. Multiply by $.06\frac{2}{3}$:

2400	4.8	.0144	.56808

8. Multiply by .20:

1500	3.5	.085	.00625

9. Multiply by .025:

4000	2.8	16.28	.0024

10. Multiply by $.03\frac{1}{3}$:

1200	3.6	1.59	.2952

WRITTEN EXERCISES

156. 1. Multiply by .75: 228; 14.48; 2.04; 12.0452.

2. Multiply by .375: 160; 1728; 2.1672; .98548.

3. Multiply by .625: 3200; 745.6; 23.88; .2964.

4. Multiply by .875: 720; 384.8; 3.672; .6984.

5. Multiply by $.66\frac{2}{3}$: 450; 2.16; 4.0056; .4575.

6. Multiply by $.83\frac{1}{3}$: 1200; 21.6; 49.08; .07122.

MISCELLANEOUS WRITTEN EXERCISES

1. How much is $6\frac{1}{4}$ hundredths of \$3200?
2. Find .125 of \$24.30.
3. What is $.83\frac{1}{3}$ of \$72.50?
4. Multiply 3.0385 by 250.8.
5. Solve: $.33\frac{1}{3}$ of 28.45×16.00345.
6. What is the product of $\frac{5}{6}$ of 1.008×291?
7. How much is 25 thousandths of 2.325 times \$5.145?
8. Multiply 8 ten-millionths by 12 thousandths.
9. Find: .75 of 150 times 12.025 ft.
10. How much is 4.85 lb. multiplied by $.06\frac{1}{4}$?

DIVISION

157. Divide .87024 by .24

Process

```
      3.626
 x24.)x87.024
      72
      150
      144
        62
        48
        144
        144
```

Explanation

The divisor being a decimal, the division is not simple.

To make the division simple, the divisor must be changed to a whole number.

The divisor may be changed to a whole number without changing the quotient by multiplying both dividend and divisor by 100, or by moving the decimal point two places to the right in both dividend and divisor.

The division is now simple.

158. RULE. **To divide by a decimal.**

1. Change the divisor to a whole number by moving the decimal point to the right of the last figure.

2. Move the decimal point in the dividend as many places to the right as the decimal point was moved in the divisor; if necessary, annex ciphers to the dividend.

3. Divide as with whole numbers.

4. Place the decimal point in the quotient directly above the decimal point in the dividend.

159. Special Cases: 1. Ciphers annexed in the dividend. 2. Ciphers prefixed in the quotient.

WRITTEN EXERCISES

1. Divide 1.75 by .30125

```
           56.
03125.)1x75000.
        15 625
         18750
         18750
```

Explanation. Since the dividend has fewer decimal places than the divisor, it is necessary to add ciphers to the dividend when moving the decimal point to the right.

2. Divide .475 by 7.6

```
       .0625
7x6.)x4.7500
       4 56
        190
        152
         380
         380
```

Explanation. The quotient must have the same number of decimal places as the dividend: hence, a cipher must be put between the decimal point and the first partial quotient (6). Sometimes two or more ciphers are necessary.

WRITTEN EXERCISES

160. Divide

1. .923 by .671	2. 8.89 by 1.49
3. .864 by 1235	4. 108.4 by 63.13
5. 19.3666 by 12.84	6. 31.94 by 6.104
7. 51323 by .1315	8. 693.45 by 8.412
9. 89.61 by 1.108	10. 395.16 by 4.8015

EXERCISES

1. $12.76 \div 784.009 = ?$
2. $.34 \div .7286 = ?$
3. $213 \div 1.8 = ?$
4. $18.104 \div 8.40 = ?$
5. $8.615 \div .68 = ?$
6. $16.1406 \div 1.16 = ?$
7. $87.48 \div 8.10 = ?$
8. $41.738 \div 9.04 = ?$
9. $3.75 \div .62572 = ?$
10. $.0012 \div 7.25 = ?$

EXERCISES

Find quotients:

1. $4.98 \div .63654$
2. $671 \div 5.41941$
3. $68.54 \div 9.41743$
4. $31.1 \div .31832$
5. $18.76 \div .391838$
6. $579 \div 1.18723$
7. $6.71 \div 1.12929$
8. $3.70 \div 7.61614$
9. $34.9 \div 381010$
10. $497 \div 1.85595$

EXERCISES

Find the quotients:

1. $1.36\frac{1}{2} \div .714$
2. $78.1\frac{1}{10} \div 41.1\frac{1}{2}$
3. $67.3\frac{1}{5} \div 7.834$
4. $.341\frac{1}{4} \div 7.85\frac{1}{2}$
5. $49.6 \div 3.41$
6. $78.9 \div 4.31$
7. $3.81\frac{1}{2} \div 19.8\frac{1}{5}$
8. $84\frac{1}{2} \div 78.1\frac{1}{10}$
9. $63.41\frac{1}{4} \div 76\frac{1}{8}$
10. $17.48 \div 376.54$

Short Methods in the Division of Decimals

161. Division by 10, 100, 1000, etc.

1. Divide 28.54 by 10; by 100.

$$28.54 \div 10 = 2.854$$
$$28.54 \div 100 = .2854$$

2. Divide 7.08 by 1000; by 1,000,000.

$$7.08 \div 1000 = .00708$$
$$7.08 \div 1,000,000 = .00000708$$

162. RULE. **To divide a decimal by a power of 10, move the decimal point as many places to the left as there are ciphers in the divisor. Ciphers are prefixed whenever necessary to give the quotient the proper number of decimal places.**

ORAL OR SIGHT EXERCISES

163. 1. Divide by 10: 3.2; 416.5; .625; .0425.

2. Divide by 100: 6.25; 28.754; 1.0096; .0009.

3. Divide by 1000: 2856.5; 2.874; 72.75; .000045.

4. Divide by 10,000: 75,265.5; 29.3756; 2.8.

5. Divide by 100,000: 6,295,321.8; 7654.75; 1.5.

6. Divide by 1,000,000: 21,289,361.25; 18,250.75.

Division by a Whole Number ending in One or More Ciphers

164. Divide 78.096 by 60; by 600; by 60,000.

$$78.096 \div 60 = 7.8096 \div 6 = 1.3016$$
$$78.096 \div 600 = .78096 \div 6 = .13016$$
$$78.096 \div 60,000 = .0078096 \div 6 = .0013016$$

RULE. **To divide a decimal by a whole number ending in one or more ciphers, move the decimal point in the dividend as many places to the left as there are ciphers in the divisor and divide by the significant figures of the divisor.**

EXERCISES

165. 1. Divide by 70: 14.42; 6.3; .287; .0098.

2. Divide by 900: 18.9; 6.309; 75.15; .018.

3. Divide by 8000: 456.8; 29.04; 7.008; .056.

4. Divide by 40,000: 2868.72; 3.12; 96.88; .092.

5. Divide by 300,000: 6000; 18.51; 8.463; 5.202.

6. Divide by 5,000,000: 1000; 7.25; .35.

7. Divide by 6200: 3.1; .434; 1.178; 99.2.

8. Divide by 25000: 3.6; .185; .7; .0009.

Division by a Decimal Equivalent to an Easy Common Fraction

166. Divide 3.016 by .5; by .25; by $.66\frac{2}{3}$

$$3.016 \div .5 = 3.016 \div \tfrac{1}{2} = 3.016 \times 2 = 6.032$$
$$3.016 \div .25 = 3.016 \div \tfrac{1}{4} = 3.016 \times 4 = 12.064$$
$$3.016 \div .66\tfrac{2}{3} = 3.016 \div \tfrac{2}{3} = \overset{1.508}{\cancel{3.016}} \times \tfrac{3}{2} = 4.524$$

Time may sometimes be saved by reducing a decimal divisor to the equivalent common fraction.

EXERCISES

167. Find quotients:

1. $41.3 \div .5$
2. $.125 \div .25$
3. $17.2 \div .33\frac{1}{3}$
4. $2.5 \div .83\frac{1}{3}$
5. $24.75 \div .75$
6. $.48 \div .66\frac{2}{3}$

7. $.72 \div .125$

8. $.15 \div .375$

9. $60 \div .625$

10. $.14 \div .875$

11. $1.2 \div .14\frac{2}{7}$

12. $.25 \div .025$

13. $11 \div .025$

14. $.9 \div .08\frac{1}{3}$

15. $.07 \div .16\frac{2}{3}$

16. $.072 \div .5$

17. $6.5 \div .25$

18. $4.1 \div .11\frac{1}{9}$

19. $90 \div .125$

20. $35 \div .16\frac{2}{3}$

21. $2.1 \div .375$

22. $5.7 \div .375$

23. $.63 \div .875$

24. $1200 \div .08\frac{1}{3}$

MISCELLANEOUS WRITTEN EXERCISES

168. 1. How much is 96 ten-thousandths divided by 3 tenths?

2. One factor of .1805 is .025. What is the other factor?

3. Solve: $2.0016 \div .54$

4. What is the quotient of 4.8 divided by .000076?

5. Solve: $(21.6 + .034) \div (1.2 \times .018)$

6. How much is .025 of $.33\frac{1}{3} \times 7.5$?

7. Solve: $1.5(2.04 - .0068) + .83\frac{1}{3}$ of 2.16

8. Solve: $\dfrac{3\frac{1}{3}}{15}$ times $.018 \div 7.2$

9. Solve: $2.5(.098 + 1.272) - .66\frac{2}{3}$ of 1.5

10. Solve: $.08(.004 - .0025) + \frac{2}{5}$ of .20

TYPE PROBLEMS IN DECIMAL FRACTIONS

First Type Problem. To find a Part of a Number

169. A statue weighing 350 lb. is made of copper and zinc. .95 of the statue is copper. How many pounds of copper are in the statue?

Process

.95 of 350 lb. = 224 lb. Weight of copper. *Ans.*

Second Type Problem. To find what (Decimal) Part One Number is of Another Number

A tailor had 350 overcoats. He sold 250 coats. What decimal part of his stock did he sell?

Process

$\frac{250}{350} = \frac{5}{7} = .71\frac{3}{7}$ *Ans.*

HINT. Solve the problems by the use of common fractions whenever the decimal fraction can be easily reduced to a common fraction.

Third Type Problem. To find a Number when a Part of it is Given

First Case

.82 of the population of a town is 4059. What is the population?

Process

.82 of pop. = 4059
.01 of pop. = $\frac{1}{82}$ of 4059
Total pop. = $100 \times \frac{1}{82}$ of 4059
Total pop. = $\frac{100}{82}$ of 4059 = 4950. *Ans.*

A shorter form is:

.82 of the population equals 4059. Therefore, the total population equals

$$4059 \div .82 = .4950. \quad \textit{Ans.}$$

Second Case

To find a Number when the Number plus a Part of it is Given

One building is .16 higher than another building. If the first building is 290 ft. high, how high is the second building?

Represent the height of the second building by 1.

Then the height of the first building = 1.16 of the height of second building.

Therefore, the height of the first building = 290 ÷ 1.16 = 250 ft. *Ans.*

Third Case

To find a Number when the Number minus a Part of it is Given

I spent .24 of my money and had $1406 left. How much had I at first?

Represent the original amount of money by 1.

Then the amount spent = .24 of the original amount.

The amount left = .76 of the original amount.

Original amount = $1406 ÷ .76 = 1850. *Ans.*

MISCELLANEOUS PROBLEMS IN DECIMALS

170. 1. 950 T. of coal were shipped to New York. .72 of it was hard coal, the rest was soft coal. How many tons of each kind were there?

2. A manufacturer bought 250 T. of coal. He used .52 of it.

(*a*) How many tons were used?

(*b*) How many tons were not used?

3. A book which cost $2.50 was sold at a gain of .45 of its cost.

(*a*) Find the gain.

(*b*) Find the selling price.

4. A wagon which cost $125 was sold at a loss of .24 of the cost.

(*a*) Find the loss.

(*b*) Find the selling price.

5. 1500 ft. is what decimal part of 1 mi. (5280 ft.)?

6. 1750 lb. is what decimal part of a ton?

7. A grocer had 240 doz. oranges. He sold 200 doz. What part of the total amount (expressed in decimals) was sold? What part was not sold?

8. An automobile marked at $1800 was reduced to $1375. What reduction (expressed decimally) was made in the price of the automobile?

9. The distance from New York to Pittsburg is 440.5 mi. The distance between New York and Philadelphia is .208 of that distance.

(*a*) How far is it from New York to Philadelphia?

(*b*) How far is it from Philadelphia to Pittsburg?

10. A steamboat that is making a trip of 875 mi. has gone 675 mi. What part of the entire distance (expressed decimally) has been covered?

11. .37 of the height of the Woolworth Building, New York, is 277.5 ft. How high is the building?

12. The population of a city in 1910 was .21 greater than the population in 1900. The actual increase was 147,000 persons. Find the population in 1900, and the population in 1910.

13. .095 of the amount of money received for an entertainment is $261.25. What was the total amount received?

14. 500 bbl. of apples were sent to a wholesale dealer. .46 of them were sold at $2.50 per barrel. The rest were sold at $2.75 per barrel. How much was received for all the apples?

15. The wages of the employees in a factory were increased .12. The manufacturer paid $636.06 more than formerly on account of the increase. Find the total amount formerly paid for wages.

16. The population of Boston is 670,500; the population of Massachusetts is 3,366,000. The population of Boston is what decimal part of the population of Massachusetts?

17. Harry weighs 160 lb. This is .71 as much as William weighs. How much do both boys weigh?

18. One river is 950 mi. long. Another is 1.4 as long. How long is the second river?

Estimate the answer. Is it more or less than 950 mi.? Why?

19. A horse is sold at a gain of .28 of its cost. The gain is $175. Find the cost.

20. A number of boxes of lemons weighed 1352 lb. .02 of the weight was due to the wooden boxes. Find the actual weight of the lemons; of the boxes.

21. A nickel (5-cent piece) is made of copper and nickel. 3 parts are copper and 1 part is nickel. What part of the coin (express as a decimal fraction) is copper?

22. $24\frac{1}{2}$ lb. is what part (express as a decimal fraction) of 196 lb.?

23. If a boat travels .33 of its journey in 39.6 hr., how many hours are required for the entire journey?

24. If a train travels .06 of its journey in 3.6 hr., how many hours will be required for the rest of the journey?

25. A rectangular plot of ground is 75.5 ft. long and 23.5 ft. wide. Find the area.

° Railroad Time Table

DISTANCE FROM PENNA. STA.	MAIN LINE EXPRESS Read Down	STATIONS			EASTERN EXPRESS Read Up
.0	9.00 A.M.	Lv.	New York	Ar.	7.00 A.M.
19.1	9.22	Lv.	Newark	Lv.	6.37
91.7	11.00	Ar.	Philadelphia	Lv.	4.33
91.7	11.35 A.M.	Lv.	Philadelphia	Ar.	4.13
195.6	2.35 P.M.	Ar.	Harrisburg	Lv.	1.15
195.6	2.50	Lv.	Harrisburg	Ar.	1.00 A.M.
326.7	6.30	Ar.	Altoona	Lv.	9.00 P.M.
326.7	6.52	Lv.	Altoona	Ar.	8.50
364.1	8.12	Lv.	Johnstown	Ar.	7.12
440.5	10.25 P.M.	Ar.	Pittsburg	Lv.	4.55 P.M.

° Note that the ⊙ in column 1 is a decimal point; but that in columns 2 and 4, it separates hours and minutes.

26. At what time does the Main Line Express leave Newark?

27. At what time does it arrive at Philadelphia?

28. How long does it remain at Altoona?

29. At what time does the Eastern Express leave Pittsburg?

Find the distance between

30. Newark and Philadelphia.

31. Philadelphia and Altoona.

PROBLEMS IN QUANTITY AND COST

BUSINESS FRACTIONS

171. Express in cents the following fractional parts of a dollar: $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{9}{8}$.

How many cents in \$ $\frac{1}{3}$? \$ $\frac{2}{3}$? \$ $\frac{1}{6}$? \$ $\frac{5}{6}$? \$ $\frac{4}{3}$?

Reduce to cents: \$ $\frac{1}{5}$, \$ $\frac{2}{5}$, \$ $\frac{3}{5}$, \$ $\frac{4}{5}$, \$ $\frac{1}{10}$, \$ $\frac{3}{10}$.

How many cents in \$ $\frac{1}{20}$? \$ $\frac{1}{16}$? \$ $\frac{1}{12}$? \$ $\frac{1}{9}$?

Express as a fractional part of a dollar $12\frac{1}{2}$ ct., $37\frac{1}{2}$ ct., $33\frac{1}{3}$ ct., $66\frac{2}{3}$ ct., $87\frac{1}{2}$ ct., $62\frac{1}{2}$ ct., 75 ct. 85 ct., 99 ct.

TABLE OF EQUIVALENTS

Copy and memorize the following:

50 ct. = \$ $\frac{1}{2}$	20 ct. = \$ $\frac{1}{5}$	30 ct. = \$ $\frac{3}{10}$	$62\frac{1}{2}$ ct. = \$ $\frac{5}{8}$
25 ct. = $\frac{1}{4}$	40 ct. = $\frac{2}{5}$	70 ct. = $\frac{7}{10}$	$87\frac{1}{2}$ ct. = $\frac{7}{8}$
$33\frac{1}{3}$ ct. = $\frac{1}{3}$	60 ct. = $\frac{3}{5}$	90 ct. = $\frac{9}{10}$	$16\frac{2}{3}$ ct. = $\frac{1}{6}$
$63\frac{2}{3}$ ct. = $\frac{2}{3}$	80 ct. = $\frac{4}{5}$	$12\frac{1}{2}$ ct. = $\frac{1}{8}$	5 ct. = $\frac{1}{20}$
75 ct. = $\frac{3}{4}$	10 ct. = $\frac{1}{10}$	$37\frac{1}{2}$ ct. = $\frac{3}{8}$	4 ct. = $\frac{1}{25}$

172. 1. How much will 36 readers cost at $62\frac{1}{2}$ ct. each?

Process. At \$ 1 each, 36 readers would cost \$ 36.

Therefore, at $62\frac{1}{2}$ ct. each, 36 readers will cost $\frac{5}{8}$ of \$ 36 or \$ 22.50.

EXERCISES

Find the cost of 48 books at:

1. 10 ct. each.
2. 20 ct. each.
3. 25 ct. each.
4. $12\frac{1}{2}$ ct. each.
5. 30 ct. each.
6. $33\frac{1}{3}$ ct. each.
7. 50 ct. each.
8. 75 ct. each.
9. $37\frac{1}{2}$ ct. each.
10. $66\frac{2}{3}$ ct. each.
11. 80 ct. each.
12. $87\frac{1}{2}$ ct. each.

173. Find the cost of 50 bu. of turnips at 38 ct. a bushel.

Process. 100 bu. would cost \$ 38.
Therefore, 50 bu. cost $\frac{1}{2}$ of \$ 38 or \$ 19.50.

EXERCISES

Find the cost of the following at 24 ct. each:

1. 50 articles.
2. 25 articles.
3. 40 articles.
4. 75 articles.
5. 60 articles.
6. 20 articles.

Find the cost of the following at 36 ct. per dozen:

7. $33\frac{1}{3}$ doz.
8. $16\frac{2}{3}$ doz.
9. $66\frac{2}{3}$ doz.
10. $37\frac{1}{2}$ doz.
11. $12\frac{1}{2}$ doz.
12. 60 doz.

174. At $12\frac{1}{2}$ ct. each, how many tablets can be bought for \$ 10.

Process. $12\frac{1}{2}$ ct. is $\frac{1}{8}$ of a dollar.
Therefore, \$ 1 will buy 8 tablets, and \$ 10 will buy (10×8) or 80 tablets.

EXERCISES

How many articles may be bought for $ 72, if one costs:

1. 25 ct.?	4. 50 ct.?	7. $33\frac{1}{3}$ ct.?
2. $12\frac{1}{2}$ ct.?	5. 5 ct.?	8. $66\frac{2}{3}$ ct.?
3. 30 ct.?	6. 10 ct.?	9. 75 ct.?

175. At $ $1.87\frac{1}{2}$ per yard, how many yards of silk can be bought for $ 500?

Process

$$500 \times \frac{8}{15} = 266\frac{2}{3} \text{ yd.}$$

EXERCISES

How many articles may be bought for $ 150 if one costs:

1. $ $1.12\frac{1}{2}$	3. $ 1.40	5. $ 2.50
2. $ 2.50	4. $ $1.37\frac{1}{2}$	6. $ $2.62\frac{1}{2}$

7. How many weeks' board can be paid for with $ 300, if board costs $ $12\frac{1}{2}$ a week?

8. How many packages of sugar each weighing $3\frac{1}{2}$ lb. can be made up from 350 lb.?

176. Find the cost of carpet as follows:

60 yd. at $ 1.25 per yard.

Explanation	Process
60 yd. at $ 1	$ 60
60 yd. at $\frac{1}{4}$	15
60 yd. at $ 1.25 . .	$ 75

80 yd. at $ $1.33\frac{1}{3}$ per yard.

Explanation	Process
80 yd. at $ 1 . .	$ 80
80 yd. at $ $\frac{1}{3}$ . .	$26\frac{2}{3}$
80 yd. at $ $1.33\frac{1}{3}$	$ $106\frac{2}{3}$

Find the cost of 45 qt. of olive oil

@ 75 ct.		@ $83\frac{1}{3}$ ct.	
Explanation	**Process**	**Explanation**	**Process**
45 qt. @ \$1 . .	\$45	45 qt. @ \$1 . .	\$45
45 qt. @ \$$\frac{1}{4}$ . .	$-11\frac{1}{4}$	45 qt. @ \$$\frac{1}{6}$ . .	$-7\frac{1}{2}$
45 qt. @ \$$\frac{3}{4}$ . .	\$$33\frac{3}{4}$	45 qt. @ \$$\frac{5}{6}$ . .	\$$37\frac{1}{2}$

WRITTEN PROBLEMS

Find the cost of:

1. 62 books @ $87\frac{1}{2}$ ct.
2. 48 combs @ \$$1.12\frac{1}{2}$.
3. 72 fountain pens @ \$$2.37\frac{1}{2}$.
4. 30 chairs @ \$5.75.
5. 36 desks at \$12.75.
6. 24 hats @ \$$4.87\frac{1}{2}$.
7. 46 clocks @ \$$1.66\frac{2}{3}$.
8. 80 chimneys @ $37\frac{1}{2}$ ct.
9. 4 doz. table covers @ \$$3.62\frac{1}{2}$ per dozen.
10. 36 pr. gloves @ $83\frac{1}{3}$ ct.
11. Make and solve five problems similar to those given above.

DENOMINATE NUMBERS

177. To measure a quantity is to find how many times it contains a smaller quantity of its own kind called **a unit of measure.**

For example, the length of a desk is said to be 3 ft. because its length contains the length of a foot rule, 3 times.

178. A **defined unit** of measure is a measure whose quantity is fixed by law or custom; *e.g.* yard, inch, pound, quart, degree, dollar.

Cupful, pace, load, etc., are variable quantities and therefore are called undefined units of measure.

The names given to the defined units of measure are called denominations; *e.g.* the denominations of the table of length are inch, foot, yard, rod, and mile.

179. The **standard unit** of a table is so called because its meaning is fixed first in the law and from it the other measures of the table are derived; *e.g.* yard, bushel, day, and dollar are standard units.

180. A **denominate number** is a number composed of defined units of measure; *e.g.* 3 ft., 5 qt.

181. A **simple denominate number** is a denominate number that is composed of measures of one denomination; *e.g.* 7 ft., 4 lb.

182. A **compound denominate number** is a denominate number that is composed of related meas-

ures of two or more denominations; *e.g.* 3 lb. and 7 oz. is a compound denominate number.

REDUCTION

183. Reducing a denominate number to units of smaller denomination is called **Reduction descending,** *e.g.* 6 ft. may be reduced to 72 in.

184. **Reduction ascending** is the reduction of a denominate number to units of larger denomination; *e.g.* 6 ft. may be reduced to 2 yd.

NOTE. Tables of Denominate Numbers are printed in the Appendix.

REDUCTION DESCENDING

185. (*a*) Reduce 6 bu. 1 pk. 3 qt. to quarts.

Short Process	**Explanation***
6	
4	1 bu. = 4 pk.
24	6 bu. = 24 pk.
1	
25	24 pk. + 1 pk. = 25 pk.
8	1 pk. = 8 qt.
200	25 pk. = 200 qt.
3	200 qt. + 3 qt. = 203 qt.
203 qt. *Ans.*	*Ans.*

* NOTE TO TEACHER. Explanations may preferably begin with a statement of the value of the given denominate unit in terms of the required denominate units as in the explanations of (*a*) and (*d*). The advantages of a uniform method of departure for all examples in reduction are obvious.

(*b*) Change $\frac{5}{9}$ of a rod to feet and inches.

Process

$$\frac{\overset{}{5}}{\underset{3}{\cancel{9}}} \text{ of } \frac{\overset{11}{\cancel{33}}}{2} \text{ ft.} = \frac{55}{6} = 9\tfrac{1}{6} \text{ ft.}$$

$\frac{1}{6}$ of 12 in. = 2 in.

9 ft. 2 in. *Ans.*

(*c*) Reduce £.6085 to lower denominations.

Process

20*s.*	12*d.*
.6085	.17
12.1700*s.*	2.04*d.*

12*s.* 2*d.* *Ans.*

REDUCTION ASCENDING

186. (*d*) Reduce 422 dry quarts to higher denominations.

Process

8)442
4) 55 pk. 2 qt.
13 bu. 3 pk.

13 bu. 3 pk. 2 qt. *Ans.*

Explanation

1 qt. = $\frac{1}{8}$ pk.
442 qt. = $\frac{442}{8}$ = 55 pk. 2 qt.
1 pk. = $\frac{1}{4}$ bu.
55 pk. = $\frac{55}{4}$ bu. = 13 bu. 3 pk.
422 qt. = 13 bu. 3 pk. 2 qt. *Ans.*

(*e*) Reduce 78*s.* 9*d.* to pounds sterling.

Process

12) 9. 0*d.*
2$\cancel{0}$)78.75*s.*
£)3.9375 *Ans.*

Explanation

$$1d. = \frac{1}{12}s.$$

$$9d. = \frac{9}{12}s. = .75s.$$

$$1s. = £\,\frac{1}{20}.$$

$$78.75s. = £\,\frac{78.75}{20} = £\,3.9375$$

Ans.

(*f*) What part of a working day of 9 hr. is 3 hr. and 45 min.?

Process

3 hr. 45 min. $= 3\frac{45}{60}$ hr. $= 3\frac{3}{4}$ hr.

1 hr. $= \frac{1}{9}$ of the working day.

$3\frac{3}{4}$ hr. $= \frac{3\frac{3}{4}}{9}$ or $\frac{15}{36}$ or $\frac{5}{12}$ of the working day. *Ans.*

WRITTEN EXERCISES

187. 1. How many feet in $\frac{5}{8}$ of a mile?

2. Reduce $\frac{3}{4}$ sq. ft. to inches.

3. How many feet in 32.18 chains?

4. What is the length in feet of 21.8 fathoms?

5. How many cubic feet should 12 cords of wood measure?

6. What part of an ounce is $3\frac{1}{3}$ pwt. of gold?

7. Two thirds of an hour is what part of a day?

8. Reduce 275 lb. to the decimal of a ton.

9. What decimal part of a gallon is $.66\frac{2}{3}$ of a pint?

10. How many grains in .25 of a pound avoirdupois?

11. Express 72 lb. of wheat in terms of bushels.

12. What fractional part of a quadrant is $18° 43'$?

13. Express 18*s.* 5*d.* as the decimal fraction of £1.

14. How many grains of gold in a nugget weighing 1 oz. 7 dwt.?

15. What is the weight of $\frac{2}{5}$ of a barrel of flour?

COMPOUND DENOMINATE NUMBERS

Addition

188. 1. Add £3 10*s.* 8*d.*, £1 8*s.* 4*d.*, £5 3*s.*, £10 8*d.*, 14*s.* 11*d.*, £1 18*s.* 9*d.*

Process

£	*s.*	*d.*
3	10	8
1	8	4
5	3	
10		8
	14	11
1	18	9
22	16	4

Explanation

Since only like numbers can be added, the numbers of the same denomination are to be written in the same column. Beginning at the right hand, we add, reduce, and carry. 40*d.* = 3*s.* 4*d.* Write 4 and carry 3. 56*s.* = £3 16*s.* Write 16 and carry 2. The answer is £22 16*s.* 4*d.*

EXERCISES

Find sums:

2.

2 cwt.	10 lb.
1	50
18	78
75	80
12	3

3.

10 gal.	3 qt.	
4	1	1 pt.
12	2	1
3	0	1
16	3	

4.

5 yd.	2 ft.	10 in.
3	1	4
1	1	8
6	0	5
12	2	7

5.

1 yr.	8 mo.	12 da.
3	5	18
1	2	28
	7	15
3		6

6.

£3	8*s*.	2*d*.
4	10	6
1	9	10
5		3
6	7	9

7.

1 lb.	3 oz.	18 pwt.
2	10	16
7		12
	11	9

Subtraction

189. **1.** From 12 bu. 1 pk. 6 qt. take 5 bu. 3 pk. 4 qt.

Process

bu.	pk.	qt.
14	1	7
5	3	6
8	2	1

Say: $7 - 6 = 1$, write down 1.

1 bu. + 1 pk. = 5 pk.; $5 - 3 = 2$, write down 2.

$13 - 5 = 8$, write down 8.

The answer is 8 bu. 2 pk. 1 qt.

Teacher's Note. If the Austrian method of Subtraction is used by the pupils, the oral expression of the process should be as follows: 6 qt. and 1 qt. are 7 qt., write down 1 qt.; 3 pk. and 2 pk. are 5 pk. (*i.e.* 1 bu. 1 pk.), write down 2 pk.; 1 bu. and 5 bu. are 6 bu.; 6 bu. and 8 bu. are 14 bu., write down 8 bu.

EXERCISES

Find remainders:

2.

10 bu.	3 pk.	5 qt.
8	2	7

3.

92°	15′	20″
75	45	45

4.

yr.	mo.	da.
1913	4	15
1910	8	20

5.

yr.	mo.	da.
1913	6	25
1911	11	12

6.

£75	6*s*.	3*d*.
55	10	10

7.

1 lb.	6 oz.	
	11	17 pwt.

Multiplication

190. Multiply 8 lb. 6 oz. avoirdupois by 25.

Process	
LB.	OZ.
8	6
	25
209	6

Explanation

Beginning at the right, say: 25 × 6 oz. = 150 oz. = 9 lb. 6 oz., write down 6.

25 × 8 lb. = 200 lb.

200 lb. + 9 lb. = 209 lb., write down 209.

The answer is 209 lb. 6 oz., or 2 cwt. 9 lb. 6 oz.

PROBLEMS

1. How much wine will 10 casks hold, each having a capacity of 18 gal. 3 qt. 1 pt.?

2. Find the weight of a dozen silver plates, allowing 7 oz. 8 dwt. for each plate.

3. A train moves 80 ft. per second. What is its rate expressed in miles per hour?

4. A wheat barge had 60 bins with a capacity of 150 bu. 3 pk. each. What is the total capacity expressed in bushels? What is the capacity expressed in pounds?

5. A coal wagon carries when filled 3 T. 2 cwt. 50 lb. of coal; how much will it convey in 20 trips, if filled on each trip?

6. A tile layer laid 150 tiles in a day, each tile having a surface 16 sq. in. How many square feet of surface can he cover in 18 da.?

7. What is the cost of 200 yd. of Scotch Tweed at 3*s*. $4\frac{1}{2}$*d*. per yard?

Division

191. Divide 14 pk. 6 qt. by 4.

Process

	Pk.	Qt.	Pt.
4)	14	$^{16}+6$	$^{4}+0$
	3^{+2}	5^{+2}	1

Explanation

$\frac{1}{4}$ of 14 pk. = 3 pk. + 2 pk. remainder. Change the remainder to the next denomination. 2 pk. = 16 qt. 16 qt. + the 6 qt. in the dividend = 22 qt. $\frac{1}{4}$ of 22 qt. = 5 qt. + 2 qt. remainder. Change the remainder to the next denomination. 2 qt. = 4 pt. $\frac{1}{4}$ of 4 pt. = 1 pt. 3 pk. 5 qt. 1 pt. *Ans.*

PROBLEMS

1. A goldsmith used 4 oz. 7 pwt. 12 gr. of gold in making 25 bracelets. How much gold did each bracelet contain?

2. A ship sailing along the 40th parallel of latitude traveled 68° 6′ in 12 da. How far did it sail on an average each day?

3. A canal barge contains 320 T. of coal. How many bucket loads of 150 lb. each will it take to remove the cargo?

4. How many bottles containing $\frac{7}{8}$ of a quart can be filled from a barrel of vinegar containing $31\frac{1}{2}$ gal.?

5. An invoice for 250 yd. of carpet costs £412 10*s.* What is the price per yard in English money? In U. S. money?

6. In covering a half mile, the wheels of my bicycle revolved 510 times. Find the circumference of the wheels.

7. How many square feet in a plot, if there are 18 plots in an acre?

8. Divide 280 bu. 3 qt. by 35.

FOREIGN MONEYS

192. The units of the money systems of England, Germany, and France are:

Country	Unit	Value in U. S. Money	Approximate Value
England	Pound sterling (£)	$4.8665	$5.00
Germany	Mark (M.)	.238	.25
France	Franc (fr.)	.193	.20

193. **English Money**

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)

(£ is written *before* the number, *e.g.* £ 10.)

194. **German Money**

100 pfennigs (pf.) = 1 M.

195. **French Money**

100 centimes (c.) = 1 franc (fr.)

EXERCISES

196. Find the approximate value in U.S. money of:

1. £$2\frac{1}{2}$	7. £10 10*s.*	13. 9*s.* 6*d.*
2. 60 M.	8. 250 fr.	14. 5 M. 50 pf.
3. 50 fr.	9. 400 M.	15. 10 fr. 25 c.
4. 50 pf.	10. 20*s.*	16. £300
5. 50 c.	11. 40 pf.	17. 150 M.
6. 50*s.*	12. 40 c.	18. 150 fr.

EXERCISES

197. Find approximate values for ?

		English Money	German Money	French Money
1.	$20	£?	? M.	? fr.
2.	$100	£?	? M.	? fr.
3.	$2000	£?	? M.	? fr.
4.	$4	£?	? M.	? fr.
5.	$25	£?	? M.	? fr.
6.	$150	£?	? M.	? fr.
7.	$60	£?	? M.	? fr.
8.	$15	£?	? M.	? fr.
9.	$175	£?	? M.	? fr.
10.	$36	£?	? M.	? fr.

EXERCISES

198. Find the exact values of the following in U. S. money:

1. £14	3. 200 fr.	5. 16 M.
2. 200 M.	4. 16*s.*	6. 16 fr.

7. 50 pf.	9. 25 M. 50 pf.	11. £2 10*s.*
8. 50 c.	10. 18 fr. 50 c.	12. £5 10*s.*

EXERCISES

199. 1. Change 15 fr. 35 c. to U. S. money. (Exact value.)

Process

15 fr. 35 c. = 15.35 fr.

15.35 fr. = 15.35 × \$.193 = \$2.96255 = \$2.96.

2. Change 19 M. 75 pf. to United States money. (Exact value.)

Process

19 M. 75 pf. = 19.75 M.

19.75 M. = 19.75 × \$.238 = \$4.70050 = \$4.70.

3. Change £5 6*s.* to U. S. money. (Exact value.)

Process

£5 6*s* = £$5\frac{6}{20}$ = £5.3.

£5.3 = 5.3 × \$4.8665* = \$25.79245 = \$25.79.

Change the lower denominations to a fractional part of the standard unit; then change to U. S. money.

Give the exact equivalent of the following in U. S. money:

4. 26 M. 20 pf.	8. £4 15*s.*	12. £3 12*s.* 2*d.*
5. 18 fr. 15 c.	9. £6 10*s.*	13. 29 M. 5 pf.
6. 27 M. 12 pf.	10. £3 5*s.*	14. 16 M. 2 pf.
7. 30 fr. 24 c.	11. £5 14*s.*	15. 36 fr. 6 c.

* NOTE. \$4.87 = £1 is sufficiently accurate.

PROBLEMS

(Use approximate values.)

200. 1. A boy who is learning the trade of bricklaying in England receives 6*s*. per week. How much is that in U.S. money?

2. Four years later the boy receives an increase of 8*s*. Find his wages in U.S. money.

3. An English carpenter receives $10\frac{1}{2}d.$ per hour. He works 50 hr. per week. Find his wages in U.S. money.

4. An overcoat was bought in London for £2 3*s*. How much is that in U.S. money?

5. The price of a fan in Paris is 20 fr., the price in New York is $6. How much more do I pay for the fan in New York? (Give answer in U.S. money.)

6. I paid 22 M. for a pair of shoes in Berlin. The same kind of shoes cost $4 in New York. How much more did the shoes cost in New York? (Give answer in U.S. money.)

7. My bill at a German hotel was 6 M. 50 pf. per day for 6 da. State, in U.S. money, the amount paid in 6 da.

8. The cost of a railroad ticket was 10 M. 20 pf. State the equivalent amount in U.S. money.

9. A trip on a boat in France cost 3 fr. 25 c. Find the equivalent amount in U.S. money.

10. A traveler going from France to Germany had 476 fr. He changed his money to German money. How many Marks should he receive?

201. To find the exact value of U. S. money in foreign moneys.

1. Change $ 50 to French money.

Process

$50 \div .193 = 259.067$, or 259 fr. 7 c.

This problem is similar to Type Problem number 5 on p. 27.

There will be as many francs as $50 \div .193$.

Note that .06 of a franc equals 6 c., because 100 c. = 1 fr.

2. Change $ 50 to German money.

Process

$50 \div .238 = 210.084$, or 210 M. 8 pf.

This problem is similar to number 1 except that the divisor is .238.

Note that .08 of a mark equals 8 pf., because 100 pf. = 1 M.

3. Change $ 25 to English money.

Process

$25 \div 4.87 = 5.13$, or £5.13

This problem is similar to number 1 except that .13 of a pound must be changed to shillings and pence as follows:

Number of shillings = .13 of 20 = 2.6*s.*
Number of pence = .6 of 12 = 7.2*d.*
£5 2*s.* 7*d.** *Ans.*

* **Note.** Farthings are not carried.

EXERCISES

202. Change the following to exact equivalents:

	French Money	German Money	English Money
1. $10	?	?	?
2. $25	?	?	?
3. $100	?	?	?
4. $250	?	?	?
5. $350	?	?	?
6. $225	?	?	?
7. $2000	?	?	?

203. PROBLEMS

State the cost of the following articles (*a*) in French money, (*b*) in German money:

1. A table worth $60.

2. A piano worth $500.

3. An automobile worth $1800.

4. A set of books containing 12 vols. at $2.50 per volume.

5. 80 yd. of cloth at $2.25 per yard.

6. Find the value of the table (number 1) in English money.

7. Find the value of the piano (number 2) in English money.

8. Find the value of the automobile (number 3) in English money.

9. Find the value of the set of books (number 4) in English money.

10. Find the value of the 80 yd. of cloth (number 5) in English money.

The wages of various trades in London are stated below. Find the equivalent of the *weekly* wage in U. S. money.

11. Bricklayer, $10\frac{1}{2}d.$ per hour, 50 hr. a week.

12. Masons, $11\frac{1}{2}d.$ per hour, 50 hr. a week.

13. Painters, $9d.$ per hour, 50 hr. a week.

14. Plumber, $11d.$ per hour, 50 hr. a week.

15. Tailor, $7d.$ per hour, 55 hr. a week.

The wages of various trades in New York are given below. Find the *approximate equivalent* of the *weekly* wage in English money.

16. Bricklayer, 70 ct. an hour, 44 hr. a week.

17. Carpenter, $62\frac{1}{2}$ ct. an hour, 44 hr. a week.

18. Laborer, $37\frac{1}{2}$ ct. an hour, 44 hr. a week.

19. Painters, \$ 4 a day, $5\frac{1}{2}$ da. a week.

20. Plumber, \$ 5 a day, $5\frac{1}{2}$ da. a week.

21. Railroad engineers \$ 4.44 a day.

22. Railroad machinists \$ 3.25 a day.

BUSINESS FORMS

For Reading and Discussion

204. Goods for family use or consumption are usually bought from retail merchants. The form of the **bill** commonly sent by retail dry goods merchants to their customers is as follows:

Bills rendered monthly. All claims for errors must be made within ten days after receipt of Goods.

New York, January 2, 1913.
147 Madison Ave.

Mrs. R. T. Fuller.

Bought of ROBBINS & SMITH,

IMPORTERS AND RETAILERS OF DRY GOODS,

West 23d Street.

Salesperson	Date							
	Dec.		*Dr.*					
214	6	1 Coat			32	00		
		7½ yd. Suiting	1.25		9	38		
		2 yd. Elastic	.11			22	41	60
336	7	6 yd. Serge	.95		5	70		
		2 pc. Binding	.23			46	6	16
38	9	2 Suits	8.90		17	80	17	80
781	15	¼ yd. Veiling	.60			15		
		2 pr. Gloves	1.50		3	00	3	15
459	19	1 bx. Hdkfs.			2	50	2	50
							71	21
			Cr.					
	22	1 bx. Hdkfs.					2	50
							68	71

Verify each of the items on this bill.

WRITTEN EXERCISES

1. Prepare a bill for goods sold by Robbins and Smith to you as follows: May 2, 1913, $12\frac{1}{2}$ yd. Cloth @ $ 1.85, # 28 *; 1 pr. Gloves $ 2.80, # 16; 1 Jabot $ .85. May 7, 1913, 15 yd. Ribbon @ $ .48, # 47; 6 Hdkfs. @ $ .35. May 13, 1913, 23 yd. Sheeting @ $ .49, # 154; 6 Pillow Cases @ $ .75. May 20, 1913, $8\frac{1}{2}$ yd. Silk @ $ 2.25, # 890, 4 yd. Tulle @ $ .34, # 273.

2. Prepare a bill for the following goods sold to you by E. M. McLoughlin: June 1, 19—, 1 bbl. Pillsbury Flour $ 5.75; 25 lb. Sugar @ $ $.04\frac{1}{4}$; 30 lb. Raisins @ $ .09; 1 gal. Porto Rico Molasses $ .60; 1 gal. Olive Oil, $ 2.75; 2 doz. cans Tomatoes @ $ 1.38; 8 jars Jam @ $ .22; 15 bot. C. & B. Gherkins @ $ .28; 3 gross Matches @ $ .62; 18 lb. Rice @ $ $.07\frac{1}{2}$.

3. Make out in proper form a bill of goods sold by a local dealer, find the footing and receipt the bill.

For Reading and Discussion

205. Retail merchants do not usually manufacture the goods they sell nor do they purchase their merchandise directly from the manufacturer. They buy their goods from commission merchants or from jobbing merchants.

* This number identifies the salesperson. The symbol # stands for number when placed *before* a numeral.

The following letter is an **order** for goods from The Empire Hardware Company, retailers, to the H. C. Ackerson Company, jobbing merchants.

NEWBURG, N. Y., Apr. 11, 1913.

THE H. C. ACKERSON CO.
345 West Broadway, New York.

GENTLEMEN:

Please ship us at once by fast freight:

$\frac{1}{2}$ doz. Ladders, # 87
3 doz. Ladder Chairs, # 593
15 doz. Glass Lamps, # 623
$7\frac{1}{4}$ doz. Washbowls, # 183
5 gross Glasses, # 96

Yours truly,
EMPIRE HARDWARE CO.
By F. T. LEWIS.

206. Write the letter of The H. C. Ackerson Co., acknowledging the receipt of the order.

207. Observe these points in writing letters ordering goods:

1. The items are arranged one on a line.

2. The quantity and the catalogue number of each article are definitely stated.

3. Shipping directions are given.

208. Upon shipping the goods ordered, The H. C. Ackerson Co. sent the following bill or **invoice** to The Empire Hardware Co.

All claims must be made within five days after receipt of goods.

New York, April 15, 1913.

The Empire Hardware Co.,
Newburg, N. Y.

Bought of THE H. C. ACKERSON CO.,
HARDWARE, CROCKERY, AND GLASSWARE.

Terms, n/30.
Via Freight.
345 West Broadway.

87	$\frac{1}{2}$	doz. Ladders	@ 9.00	4	50
593	3	doz. Ladder Chairs	@ $6.12\frac{1}{2}$	18	38
623	15	doz. Glass Lamps	@ 4.17	62	55
183	5	doz. Washbowls	@ 10.50	52	50
96	5	gro. Glasses	@ 9.50	47	50
				185	43

1. Compare this invoice with the order in art. 205.

2. Upon receipt of the goods indicated in this invoice, The Empire Hardware Co. found that they were 3 doz. Lamps short. Write a letter asking credit for the shortage.

Notes. Term n/30 means that payment is to be made without discount in 30 da.

Terms 3/10, 2/30, 1/90, etc., mean that payment is to be made in 10, or 30, or 90 da. with a discount of 3 %, 2 %, or 1 %, respectively.

Terms 3/10, n/30 mean that 3 % will be allowed if payment is made in 10 da. or that the whole amount is due in 30 da.

209. A **receipted invoice** is shown below:

BOSTON, MASS., Oct. 14, 1913.

MR. EDWARD E. FOSTER,
123 Copley Square, City.

To ARMSTRONG BROTHERS, *Dr.*

Terms: 3/10, 1/30, net 60 da.

#15	136 Mahogany Dining Chairs	12.50	1700			
#314	120 Oak Dining Chairs	9.75	1170			
#81	6 Oak Chiffoniers	15.25	91	50		
#23 B	12 Brass Bedsteads	16.90	202	80		
#294	5 Mahogany Parlor Tables	17.75	88	75		
#709	28 Woven Wire Mattresses	3.65	102	20	3355	25
	Less 3%				100	65
					3254	60

Received Payment, Oct. 24, 1913.

ARMSTRONG BROS.,
per R. T. A.

210. The bill was paid by a **check** on The Merchants Bank in which E. E. Foster keeps his account.

No. Boston, Mass., *Oct. 23*, *1913.*

The Merchants Bank PAYABLE THROUGH THE CLEARING HOUSE.

Pay to *Armstrong Brothers* or order

Thirty-two Hundred Fifty-four $\frac{60}{100}$ Dollars

$3254 $\frac{60}{100}$ *Edward E. Foster.*

211. Acknowledgment of the payment of an invoice is generally made on a separate form of **receipt**, like the following:

> BOSTON, MASS., Oct. 24, 1913.
> Received of Edward E. Foster
> Thirty-two Hundred Fifty-four $\frac{60}{100}$ dollars
> in settlement of invoice of October 14, 1913.
> ARMSTRONG BROTHERS,
> 3254\frac{60}{100}$. *per* R. T. A.

EXERCISES

212. Write the letter ordering goods, and the other letters mentioned, the invoice, the check, and the receipt used in each of the following sets of transactions:

1. May 12, 19—. Jordan and Martin, retailers of hardware, Pittsburg, Pa., ordered of T. E. Fisk, Philadelphia, Pa., jobbing merchant, the following: 16 doz. carpenter's chisels, No. 16, @ $ 3.85; 15 doz. hammers, No. 59, @ $ 6.15; 43 doz. doorknobs, No. 77, @ 84 ct.; 8 grindstones, No. 9, @ $ 4.12; 22 doz. saws, No. 68, @ $ 19.50; 18 kegs wire nails, 8*d*, @ $ 2.85; 35 sets knives and forks, No. 101, @ 1.38; 6 Star ranges, No. 85, @ $ 10.50.

The goods were shipped May 20, by fast freight via Pennsylvania R.R.; the terms of sale were 8/10, 3/30, *n*/60. The bill was paid May 30.

2. Aug. 14, 19—. R. M. White & Co., Mobile, Ala., ordered of the J. M. Galton Manufacturing Company, Grand Rapids, Mich., the following:

10 Cheval Glasses, #29 A, @ $ 23.56.
124 Cane-seat Dining Chairs, # 71, @ $ 2.15.
18 Oak China Closets, # 35, @ $ 27.90.
9 Mahogany China Closets, # 43, @ $ 48.75.
16 Empire Tables, # 115, @ $ 42.18.
4 Library Tables, # 193 C, @ $ 22.35.
12 Oak Bookcases, # 12 B, @ $ 14.28.
27 Turkish Rockers, # 51, @ $ 14.89.
36 White Iron Bedsteads, # 72 B, @ $ 4.75.
48 Chenille Curtains, # 243 B, @ $ 4.68.

The goods were shipped Sept. 8, by freight via Michigan Southern R.R.; the terms of sale were 10/10, 5/30, n/90.

ACCOUNTS

For Reading and Discussion

213. Every individual or firm that engages in business must keep a record of goods bought and sold, of money received and paid out, of what he owes to others and what others owe him, and of all other transactions that have a money value. The keeping of this record in a systematic way is the art of bookkeeping. The record is kept in a number of books. The most important book is the **ledger**, which consists of a number of **a counts.**

214. An **account** is a formal * record of the business transactions relating to the same person or thing.

215. A **personal account** records the transactions with the person named in the title.

216. Every account has two sides, the left-hand, or **debit**, side and the right-hand, or **credit**, side.

217. The debit side of the personal account shows the items **against** the person, *i.e.* what he owes us or what we have paid to him.

218. The credit side of the personal account shows the items **in favor of** the person, *i.e.* what we owe him or what he has paid to us.

219. A personal account.

Dr. EMPIRE HARDWARE CO., *Cr.*

NEWBURG, N. Y.

1918			*			1918			†		
Apr.	15	Mdse.	110	185	43	Apr.	24	Cash	56	100	
	20	Mdse.	115	216	89		28	Cash	58	250	80
	25	Mdse.	121	57	92		29	Cash	59	65	35
	30	Mdse.	124	614	28	Apr.	31	Balance		658	37
				1074	52					1074	52
May	1	Balance		658	37						

This "personal account" records our dealings with the Empire Hardware Co. for April, 1913, as follows:

April 15, the Empire Hardware Co. bought from us merchandise amounting to $185.43; April 20, they bought a

* Formal here means according to regular and established forms and methods.

bill of $ 216.89; April 24, they paid us $ 100; April 25, they bought a bill of $ 57.92; April 28, they paid us $250.80; April 29, they paid $ 65.35; April 30, they bought a bill of $ 614.28. Their debits for the month amounted to $1074.52 and their credits to $416.15. They owe us the balance, $658.37.

Observe that the "balance" is entered on the smaller side of the account, so that when both sides are added or "footed," they will be equal; *i.e.* the two sides *balance* as the scales balance when the sugar on one side weighs as much as the pound weight on the other side. This entry is usually made in red ink to show that it does not record a cash payment or other real transaction.

After the account is ruled or "closed" the balance is brought down to the other side. It is entered in black ink because it records the actual state of affairs, to wit: the Empire Hardware Co. owes us $658.37.

The column marked * shows the pages of the "sales book" from which these items were taken; and the column marked † shows the pages of the "cash book" from which these items were taken.

EXERCISES

220. 1. Make the following entries in the account of The Keystone Manufacturing Company:

Jan. 1. They owe us a balance of $215.85.

NOTE. Enter on debit side as "Balance $215.85."

Jan. 3. Received from them cash on account, $150.

Jan. 4. Sold them merchandise on account, $108.92.

Jan. 6. They returned for credit goods ordered by mistake in shipment of Jan. 4, $28.94.

Jan. 7. Sold them merchandise on account, $ 76.54.

Jan. 8. * Received their note at 30 da. in settlement of balance unpaid on Jan. 3, $ 65.85.

Jan. 9. Sold them merchandise on account, $200.

Jan. 10. Received from them T. J. Smith's note in their favor which they have indorsed to us, $ 50.

Jan. 20. We have drawn on them a draft at sight, in favor of E. F. Locke, for $ 150.

Jan. 21. Sold them merchandise on account, $ 318.17.

Jan. 24. Received from them their draft at sight on Marshall, Field & Co. Chicago, Ill., in our favor for $ 115.85.

Jan. 30. We discount the balance of their account at $2\frac{1}{2}$% and they remit a check in settlement, which we accept.

Jan. 30. Sold them merchandise for $ 892.67.

Jan. 31. Received from them a receipted freight bill for $ 2.80 which should have been paid by us on shipment of Jan. 30.

What is the balance Jan. 31?

Write the notes mentioned on Jan. 8 and Jan. 10.

Write the check mentioned on Jan. 30.

Write a letter to the Keystone Manufacturing Company acknowledging the receipt of T. J. Smith's note.

* Notes and drafts are to be entered at their face value unless otherwise indicated.

2. Make the following entries in the account of Rogers and Marshall:

July 1. We owe them a balance of $ 116.75.

July 2. Bought of them merchandise on account, $ 62.72.

July 3. Paid them cash, $ 100.

July 5. We returned to them for credit goods purchased on July 2, amounting to $ 9.75.

July 9. We gave them our note at 60 da. in settlement in full of account.

July 13. Bought of them merchandise on account, $ 175.

July 15. We indorsed to them R. E. Swift's note at 30 da. in our favor for $ 68.50.

July 17. We accepted their draft drawn on us at 10 da. sight, in favor of The Reynolds Manufacturing Co., $ 106.50.

July 23. Bought of them merchandise on account, $ 95.70.

July 25. They allow us a claim for shortage in their shipment to us of Jan. 23, $ 8.35.

July 30. They allow us a discount of 2% on balance now due, and we remit to them a draft in their favor drawn on Hiram King & Co., Buffalo, N. Y., for $ 85.61.

What is the balance July 31?

221. A **cash account** records on its debit side the money received and on its credit side the money paid out. (See the Cash Account on pp. 126 and 127.)

EXERCISES

222. 1. Make the following entries in the Cash Account:

Sept. 1, 19—. We invested cash in the business, $ 2500.
Sept. 1, 19—. Paid rent, $ 50.
Sept. 1, 19—. Paid for furniture and fixtures, $ 350.
Sept. 1, 19—. Paid for books and stationery, $ 16.50.
Sept. 1, 19—. Received from E. R. Jones, $ 250.
Sept. 2, 19—. Purchased merchandise for Cash, $ 175.
Sept. 2, 19—. Cash sales, $ 24.50.
Sept. 3, 19—. Paid freight bill, $ 12.15.
Sept. 4, 19—. Received from E. O. Wilson, $ 316.80.
Sept. 5, 19—. Cash sales, $ 31.87.
Sept. 5, 19—. Paid S. E. Field, $ 118.09.
Sept. 8, 19—. Paid my note in favor of L. I. Boyle, $ 75 with interest $ 10.25.
Sept. 15, 19—. Cash sales, $ 428.91.
Sept. 29, 19—. Cash sales, $ 609.12.
Sept. 29, 19—. Received payment of E. M. Stafford's note, $ 87.50.
Sept. 30, 19—. Paid salaries of clerks, $ 54.75.
Sept. 30, 19—. Withdrew for personal use, $ 35.

CASH *Dr.*

Mo.	Day	Explanation	Page	$	Ct.
19–					
Feb.	1	Balance		735	84
		J. Duke & Co.		248	61
		G. Pall		75	94
	2	Sales		57	62
		F. Baldwin		24	73
		S. Reich		38	07
	6	Sales		87	56
		E. Short & Co.		15	45
				1287	82
				1287	82
Feb.	6	Balance		237	65

NOTES. 1. The names of persons are those from whom money was received (Dr.) or to whom money was paid (Cr.). We have personal accounts with all the persons named.

2. "Sales" means "sales for cash" for which no one owes us any money.

3. "Expense" means rent, light, telephone, stationery, and, perhaps, wages and other items.

4. "Purchases" means "purchases for cash" for which we owe no one.

5. "Personal" means money taken by the proprietor for his personal use.

6. The "Balance" shows the cash on hand. This item must be verified by counting the actual cash and by having the bank book "balanced" at the bank.

7. The entry "Balance 237.65" is first written in red ink and is then brought down to the other side of the account in black ink. Lines are also ruled in red ink.

CASH *Cr.*

Mo.	Day	Explanation	Page	$	Ct.
19–					
Feb.	1.	Expense		62	50
	2	S. Miller & Son		79	60
		Purchases		218	77
	3	G. Smith		25	00
		Personal		50	00
	5	Expense		3	75
		J. Weil & Co.		35	05
	6	M. Dyer		17	50
				1050	17
	6	Balance		237	65
				1287	82

What should be the balance of cash on hand Sept. 30?

2. Enter the following transactions in a Cash Account:

Jan. 1, 19—. Cash on hand, $1287.54.
Jan. 1, 19—. Paid for stationery, $21.16.
Jan. 1, 19—. Cash sales, $187.28.
Jan. 2, 19—. Paid rent, $125.
Jan. 2, 19—. Purchased horse and wagon, $275.
Jan. 3, 19—. Paid office salaries, $78.50.
Jan. 3, 19—. Withdrew for personal use, $50.
Jan. 4, 19—. Cash sales, $328.96.
Jan. 5, 19—. Paid R. S. MacAllister for his invoice of Dec. 28, $894.35.

Jan. 5, 19—. Cash purchases, $287.92.

Jan. 6, 19—. M. A. McCall paid on account, $350.

Jan. 8, 19—. The bank lends me $500 on my note.

Jan. 9, 19—. Paid E. R. Lawson $265 on account.

Jan. 10, 19—. Received from Isaac Stern his sight draft on E. J. Hollister, $125.50.

Close the account and find the amount of cash on hand.

NOTE. Sight drafts are treated the same as cash by bookkeepers.

PRACTICAL MEASUREMENTS

CARPETING

223. Carpet is sold in **strips,** usually $\frac{3}{4}$ **yd.** wide. In ascertaining the cost of carpet, it is necessary to know:

First: Whether the carpet is to be laid along the length or along the width of the room, because the number of strips may vary according to the way the carpet is laid.

Second: The total number of yards; this is found by multiplying the number of yards in each strip by the number of strips.

An additional strip may have to be purchased when the dimensions of a floor are not exact multiples of the width of a strip. For example, if a

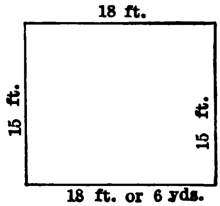
room is 18 ft. wide, 8 strips of carpet, $\frac{3}{4}$ yd. wide, are sufficient; but if the room is 19 ft. wide, the number of strips needed is $8\frac{4}{9}$; therefore, 9 strips must be purchased.

Some carpet may be wasted in matching patterns. The purchaser must pay for portions of a strip not used and for pieces wasted in matching.

Carpet is sold only in yards, half yards, quarter yards, and eighth yards.

To find the Number of Yards and Cost of Carpet

224. 1. How many yards of carpet, $\frac{3}{4}$ yd. wide, are required for a floor 18 ft. long and 15 ft. wide, if the carpet is laid along the length of the room?



Process

Number of strips $= 5 \div \frac{3}{4} = 5 \times \frac{4}{3} = 6\frac{2}{3}$.
Therefore, 7 strips must be purchased.
Total amount of carpet $= 7 \times 6$ yd. $= 42$ yd. *Ans.*

2. How many yards would be needed if the carpet were laid along the width of the room?

Process

Number of strips $= 6 \div \frac{3}{4} = 8$ (no remainder).
Total amount of carpet $= 8 \times 5$ yd. $= 40$ yd. *Ans.*
Therefore, 2 yd. less would be needed.

3. How many yards of carpet $\frac{3}{4}$ yd. wide are required for a floor 18 ft. long and 21 ft. wide, if

the carpet is laid along the length of the room? Allow for a loss of 6 in. for matching.

Process

Number of strips $= 6 \div \frac{3}{4} = 8$.
Each strip is 7 yd. + 3 in. long* $= 7\frac{1}{12}$ yd.
Total amount of carpet $= 8 \times 7\frac{1}{12} = 56\frac{2}{3}$, or $56\frac{3}{4}$ yd. *Ans.*

PROBLEMS

225. Find the cost of carpet for each of the following. Pupils are to decide whether the carpet should be laid along the length or the width of the room.

	Dimensions of Floor		Width of Carpet	Loss per Strip in Matching	Cost per Yard
	Length	Width			
1.	18 ft.	21 ft.	1 yd.	——	$1.25
2.	15 ft.	18 ft.	$\frac{3}{4}$ yd.	——	$1.25
3.	21 ft.	27 ft.	$\frac{3}{4}$ yd.	3 in.	$2.25
4.	30 ft.	30 ft.	$\frac{3}{4}$ yd.	——	$2.75
5.	27 ft.	24 ft.	$\frac{3}{4}$ yd.	6 in.	$3.00
6.	100 ft.	9 ft.	$\frac{3}{4}$ yd.	——	$1.50
7.	60 ft.	9 ft.	$\frac{3}{4}$ yd.	——	$1.75

8. How much is saved by covering a floor which is 24 ft. by 27 ft. with oilcloth at 50 ct. per square yard than with carpet $\frac{3}{4}$ yd. wide at $1.50 per yard?

* Note. Allowance is made here for loss in matching on each strip; but there is no loss on the first strip.

PAPERING

226. Wall paper is sold in **rolls 18 in. wide**; a **single roll** usually contains 8 yd., a **double roll** 16 yd.

An entire roll must be paid for, if any portion of it is used.

Since wall paper is 18 in. wide ($\frac{1}{2}$ yd. wide), the number of strips equals twice the perimeter of the floor of the room expressed in yards.

Allowance may be made for doors, windows, borders, bases, loss in matching, etc.

227. To find the Cost of Papering a Room

1. A room is 18 ft. long, 15 ft. wide, 15 ft. high. How many double rolls of paper 18 in. wide are required? (Make no allowance for doors, windows, etc.)

Process

Perimeter of floor $= (2 \times 18 \text{ ft.}) + (2 \times 15 \text{ ft.}) = 66 \text{ ft.} = 22$ yd.
Number of strips $= 2 \times 22 = 44$.
A double roll is 48 ft. long.
Number of strips from a double roll $= 48 \div 15 = 3\frac{1}{5}$.
Number of double rolls $= 44 \div 3\frac{1}{5} = 13\frac{3}{4}$, or 14 rolls. *Ans.*

2. How many double rolls of paper are required for the walls of a room 21 ft. long, 18 ft. wide, 15 ft. high, allowing for a door $3\frac{1}{2}$ ft. wide, 2 windows, each $3\frac{1}{2}$ ft. wide, a border 1 ft. wide, and a base 1 ft. high?

Process

Perimeter of floor $= (2 \times 21 \text{ ft.}) + (2 \times 18 \text{ ft.}) = 78$ ft.
Allowance, doors and windows $10\frac{1}{2}$ ft.
Remainder $67\frac{1}{2}$ ft., or $22\frac{1}{2}$ yd.
Number of strips $= 2 \times 22\frac{1}{2} = 45$.
15 ft. $-$ 1 ft. $-$ 1 ft. $=$ 13 ft. $=$ length of each strip.
Number of strips from a double roll $= 48 \div 13 = 3\frac{9}{13}$.
Number of double rolls $= 45 \div 3\frac{9}{13} = 12\frac{3}{16} = 13$ double rolls. *Ans.*

(Or 12 double rolls and 1 single roll if this paper can be bought in single rolls.)

PROBLEMS

228. Find the cost of papering the walls of each of the following:

1. A room 24 ft. long, 18 ft. wide, 15 ft. high. Allow for a base 1 ft. high, and a border 1 ft. wide. Use double rolls at \$1 per roll.

2. Use the dimensions in problem 1. Allow for the same base and border, and also for 2 doors, each $3\frac{1}{2}$ ft. wide, and 2 windows, each $3\frac{1}{2}$ ft. wide. Use double rolls at \$1.25 per roll.

3. Use the dimensions in problem 1. Allow for a base 1 ft. high; the paper is to be laid 8 ft. high. Make no other allowances. Use double rolls at \$1.50 per roll.

4. A hall 60 ft. long, 8 ft. wide, 20 ft. high. The double doors at the front of the hall extend along the entire width from the floor to the ceiling.

Allow also for a border 1 ft. wide. Use double rolls at $ 1.50 per roll.

5. A bathroom 12 ft. long, 8 ft. wide, and 15 ft. high. The door at the front of the room extends along half the width; a window at the other end is half the width of the room. The door and window extend from the floor to the ceiling. The lower half of the room is tiled; the upper half is to be papered. Use single rolls at 90 ct. a roll.

PLASTERING AND PAINTING

229. The cost of plastering and painting is based on the cost of a square yard. Allowances are sometimes made for doors, windows, etc., but there is no uniform custom.

To find the Cost of Plastering and Painting

PROBLEMS

1. Find the cost of plastering the walls and ceiling of a room 24 ft. long, 18 ft. wide, 15 ft. high. Allow for a baseboard 1 ft. high. Cost, $ 1.25 per square yard.

2. Find the cost of painting the walls of the room, including the baseboard, at 22 ct. per square yard.

3. Find the cost of kalsomining the ceiling of the room at 11¢ per square yard.

4. The walls and ceiling of a room are $20\frac{1}{2}$ by 18 by $16\frac{1}{2}$ ft. Allow for a door $3\frac{1}{2}$ ft. wide and $16\frac{1}{2}$ ft. high and 2 windows, each $3\frac{1}{2}$ ft. wide and $16\frac{1}{2}$ ft. high. Find the cost of plastering at \$ 1.25 per square yard.

5. Find the cost of plastering both sides and the ceiling of a hall 60 ft. long, 8 ft. wide, and 22 ft. high, at \$ 1.10 per square yard. (Doors at both ends entend all the way across the width of the hall from the floor to the ceiling.)

6. The walls and ceiling of a bathroom are 12 ft. long, 8 ft. wide, and 15 ft. high. Allow for a door at one end, and a window at the other, each extending halfway across the width from the floor to the ceiling. The lower half of the room is tiled. Find the cost of plastering the ceiling and the upper half of the walls, at \$ 1.15 per square yard.

7. Find the cost of varnishing both sides of the door in number 4, at 85 ct. per square yard.

8. Find the cost of painting the walls of the hall in number 5, at 22 ct. per square yard, and of varnishing both sides of the doors, at 85 ct. per square yard.

9. Find the cost of papering the ceiling of the room in number 6, with single rolls $\frac{3}{4}$ yd. wide, at 75 ct. per roll.

10. Find the cost of tiling the lower half of the walls in a bathroom 12 ft. long, 8 ft. wide, and 16 ft. high, allowing for a door 4 ft. by 16 ft., and a window 4 ft. by 16 ft., at 60 ct. per square foot.

LUMBER MEASURE OR BOARD MEASURE

The unit of measurement of lumber is the **board foot**; it is the amount of lumber in a board 1 ft. long, 1 ft. wide, and 1 in. thick.

A board 18 ft. long, 1 ft. wide, 1 in. thick contains 18 board feet; a board 12 ft. long, 1 ft. wide, $2\frac{1}{2}$ in. thick contains 30 board feet ($12 \times 1 \times 2\frac{1}{2} = 30$).

To find the number of board feet.

RULE. **Find the product of the length in feet, the width in feet, and the thickness in inches; the result is the number of board feet.**

In writing, the sign ′ is used for feet, and ″ for inches. In finding the number of board feet, the width is reckoned to the nearest half inch; a board $8\frac{1}{4}$ in. wide is reckoned 8 in. wide, a board $8\frac{5}{8}$ in. wide is reckoned $8\frac{1}{2}$ in. wide.

EXERCISES

230. Find the number of board feet in:

1. A board 1″ thick, 9″ wide, 13′ long.
2. A board 1″ thick, $10\frac{1}{4}$″ wide, 18″ long.
3. A board 2″ thick, 14″ wide, 10′ long.
4. A board $1\frac{1}{2}$″ thick, $8\frac{1}{2}$″ wide, 15′ long.
5. A board $2\frac{1}{4}$″ thick, $11\frac{1}{2}$″ wide, $12\frac{1}{2}$′ long.

6. 60 pieces, each $1'' \times 8'' \times 13'$.

7. 50 pieces, each $1\frac{1}{4}'' \times 9\frac{5}{8}'' \times 12'$.

8. 60 pieces, each $2'' \times 7\frac{5}{8}'' \times 14'$.

9. 35 pieces, each $3'' \times 10\frac{1}{4}'' \times 15'$.

10. 90 pieces, each $\frac{3}{4}'' \times 9\frac{1}{4}'' \times 15'$.

11. 70 pieces, each $1'' \times 15\frac{5}{8}'' \times 19\frac{1}{2}'$.

12. 75 pieces, each $1\frac{1}{2}'' \times 10\frac{5}{8}'' \times 14\frac{1}{2}'$.

13. 120 pieces, each $1\frac{1}{4}'' \times 12\frac{1}{4}'' \times 10'$.

Find the cost of:

14. 120 pieces, each $1\frac{1}{2}''$ thick, $11\frac{1}{2}''$ wide, $15'$ long, at 7 ct.

15. 90 pieces, each $2\frac{1}{4}'' \times 9\frac{5}{8}'' \times 14\frac{1}{2}'$, at $7\frac{1}{2}$ ct.

16. 325 pieces, each $1'' \times 14\frac{3}{4}'' \times 17'$, at $14\frac{1}{4}$ ct.

17. 250 pieces, each $1\frac{1}{4}' \times 13\frac{1}{4}'' \times 19\frac{1}{2}''$, at $8\frac{3}{8}$ ct.

ROOFING AND FLOORING

231. The cost of roofing and flooring is usually based on the cost of a **square** 10 ft. long, or **100 sq. ft.**

Shingles are usually 16 in. or 18 in. long and 4 in. wide. They are laid 4 in. or $4\frac{1}{2}$ in. "to the weather," each shingle covering about 18 sq. in. of roof.

Therefore, about **800 shingles** are required to cover a **square** (100 ft.), but in order to allow for waste, etc., from 900 to 1000 shingles are allowed for each 100 sq. ft. of roof surface.

Shingles are sold in **bundles** containing **250**. Dealers do not sell less than a bundle.

In laying floors, the "tongue" of one board fits into the "groove" of the next board. There is, therefore, a loss in the surface covered by the board. There is always some waste in cutting and fitting lumber.

To find the Cost of Roofing and Flooring

PROBLEMS

1. Find the cost of laying the floor of a room 20 ft. long and 18 ft. wide, at $3.50 per thousand square feet.

2. How many bundles of shingles are needed to cover a gable roof 30 ft. long and 20 ft. wide, allowing 1000 shingles to the square?

3. Find the cost of the shingles for No. 2 at $3.85 per thousand.

4. Find the cost of flooring a room 24 ft. by 20 ft., at $3.20 per thousand square feet, allowing for a loss in cutting and overlapping $\frac{1}{5}$ of the floor area.

5. Find the cost of shingling a gable roof 30 ft. by 20 ft. with slate shingles, at $12.50 per square.

6. The flat roof of a barn is 60 ft. by 35 ft. How many bundles of shingles must be purchased to cover the roof, allowing 900 shingles to the square?

7. How much will the shingles cost at $4 per M?

8. How long will it take a man to lay the shingles, if he can lay 2000 shingles a day?

9. Find the cost of laying a wooden floor and wooden ceiling of a room 24 ft. long and 22 ft. wide, at $4 per thousand square feet.

10. A hall is 100 ft. long and 8 ft. wide. Find the cost of laying a floor, at $2.90 per thousand square feet, allowing for a loss of $\frac{1}{6}$ of the floor area for waste.

11. How many square feet are covered by a board 22 ft. long and 10 in. wide?

12. If a board 18 ft. long and 9 in. wide and 3 in. thick is cut into boards 1 in. in thickness, how many square feet do they cover?

13. How many board feet in 570 cu. ft.?

14. How many feet of wood will be required for a fence 1 mi. long, 5 boards high, each board being 4 in. wide and 1 in. thick?

15. A banquet hall 60 ft. long and 35 ft. 8 in. wide is to be covered with maple flooring costing $36 per 1000 ft. A man getting $5 per day can lay 200 sq. ft. Find the cost of flooring the hall.

16. An apartment of 5 rooms is to be covered with yellow pine flooring costing $20 per 1000 ft. The dimensions of the rooms are:

8 ft. 10 in. × 7 ft. 6 in.
7 ft. 10 in. × 7 ft.
12 ft. × 10 ft. 9 in.
6 ft. 6 in. × 5 ft. 6 in.
6 ft. 10 in. × 5 ft. 4 in.

Find total cost of lumber.

17. How many board feet of lumber are in a pile 8 ft. high, 9 ft. wide, and 20 ft. long? Find the value of the lumber at $15 per 1000 ft.

18. How many feet, board measure, in 22 boards, each board being 19 ft. long, 10 in. wide, and 1 in. thick?

19. How many board feet in 32 joists, each joist being 20 ft. long and 5 in. square?

20. How many feet of lumber will be required for a box 8 ft. high, 4 ft. square?

VOLUME OR SOLID CONTENTS

232. A **rectangular solid** is a solid bounded by six rectangular surfaces.

When the six surfaces are squares, the solid is called a **cube**.

Solids have three **dimensions, length, breadth,** and **thickness**, or **height**.

To find the Volume of a Rectangular Solid

Find the volume of a rectangular solid 8 in. long, 4 in. wide, and 2 in. high.

Process

$$8 \times 4 \times 2 = 64 \text{ cu. in.}$$

233. RULE. **The volume of a rectangular solid is found by multiplying together the length, breadth, and thickness, or height, the three dimensions being expressed in like units.**

NOTE. For development of rule, see page 418.

ORAL EXERCISES

234. Find the solid contents of the following rectangular bins or boxes:

	Length	Width	Height		Length	Width	Height
1.	10 in.	6 in.	4 in.	4.	6 ft.	4 ft.	4 ft.
2.	15 ft.	5 ft.	2 ft. 6 in.	5.	10 ft.	3 ft.	12 in.
3.	20 ft.	3 ft.	9 in.	6.	13 yd.	5 ft.	3 ft.

235. Find the solid contents of the following bins or cisterns:

7. 10 ft. × 6 ft. × 3 ft.
8. 8 ft. × $7\frac{1}{2}$ ft. × 8 ft.
9. 5 ft. × 4 ft. × 2 ft. 6 in.
10. 2 ft. 6 in. × 3 ft. 6 in. × 2 ft.
11. 4 ft. × 3 ft. 3 in. × 2 ft.

To find the Number of Gallons in a Cistern

236. What is the capacity in gallons of a cistern 8 ft. × 3 ft. × 11 ft.?

Process

231 cu. in. = 1 gal.

$$\frac{1}{\cancel{231}} \text{ of } (8 \times \cancel{3} \times \cancel{11}) \times 1728 = \frac{13824}{7}$$
$$\cancel{21}$$
$$7 \qquad\qquad = 1974\tfrac{6}{7} \text{ gal.}$$

Explanation. One liquid gallon contains 231 cu. in. Find the capacity (volume or solid contents) of the cistern. The number of gallons the cistern holds is $\frac{1}{231}$ of the number of **cubic inches** in the cistern.

The contents are $8 \times 3 \times 11 = 264$ cu. ft. Change to cubic inches. 264×1728 cu. in. $= 456{,}192$ cu. in. $\frac{1}{231}$ of $456192 = 1974\ \frac{198}{231}$ gal.

Short Process

$(8 \times 3 \times 11) \times 7\frac{1}{2} = 1980$ gallons.

Explanation. The number of gallons is approximately $7\frac{1}{2}$ times the number of **cubic feet.**

To find the Number of Bushels in a Bin

237. Find the number of bushels in a bin 5 ft. long, 4 ft. wide, and 3 ft. high or deep.

Process

2150 cu. in.* = 1 bu.

$$\frac{1}{\cancel{2150}} \text{ of } \cancel{5} \times \cancel{4} \times 3 \times 1728 = 48\tfrac{48}{215} \text{ bu. } \textit{Ans.}$$
$$\cancel{430}$$
$$215$$

*Note. In a bushel there are 2150.42 cu. in. 2150 cu. in. is an approximation.

Explanation. One bushel contains 2150.42 cu. in. Therefore, the number of bushels is approximately $\frac{1}{2150}$ of the number of **cubic inches.**

Short Process

$$\tfrac{5}{4}\text{ cu. ft.} = 1\text{ bu.}$$

$$\frac{4}{\cancel{5}} \times \cancel{5} \times 4 \times 3 = 48\text{ bu.} \quad \textit{Ans.}$$

Explanation. 2150 cu. in. is approximately $\frac{5}{4}$ cu. ft. Each bushel occupies about $\frac{5}{4}$ cu. ft. Therefore, the number of bushels is $\frac{4}{5}$ of the number of **cubic feet.**

WRITTEN EXERCISES

Find the number of gallons that can be stored in the following cisterns:

1. 15 ft. × 5 ft. × 2 ft. 6 in.
2. 6 ft. × 4 ft. × 4 ft.
3. 3 ft. × 5 ft. × 3 ft.
4. 2 ft. 6 in. × 3 ft. 6 in. × 2 ft.

Find the number of bushels that can be stored in the following bins:

5. 2 ft. × 3 ft. × 9 in.
6. 10 ft. × 3 ft. × 12 in.
7. 8 ft. × $7\frac{1}{2}$ ft. × 8 ft.
8. 4 ft. × 3 ft. 3 in. × 2 ft.

9. How many gallons will a vat hold that is 4 ft. square and 12 in. high?

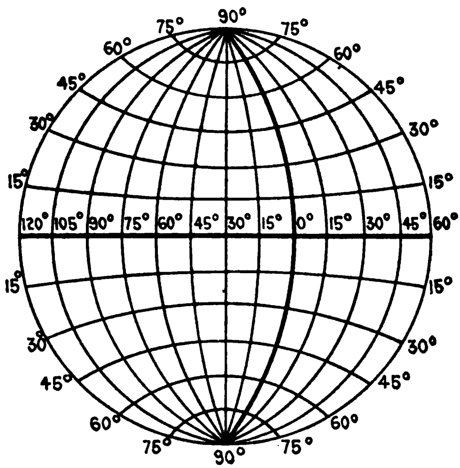
10. Find the cost of digging a cellar 25 ft. by 10 ft., and 4 ft. deep, at 75 ct. a cubic yard.

11. Find the cost of carting away the excavated earth at 25 ct. a cartload, each cart holding $1\frac{1}{4}$ cu. yd.

12. The weight of ice is .92 of the weight of water. What is the weight of a cake of ice 3 ft. long, 2 ft. wide, and 1 ft. high? (1 cu. ft. of water weighs $62\frac{1}{2}$ lb.)

LATITUDE AND LONGITUDE

238. To locate places on the earth's surface, geographers and navigators use two sets of im-



aginary circles drawn at right angles to each other. One set of circles is called parallels of latitude. The other set is called meridians of longitude.

239. The **equator** is a great circle around the earth midway between the poles.

240. **Parallels of latitude** are circles around the earth parallel to the equator.

241. **Latitude** is the distance north or south of the equator.

Latitude is expressed in terms of circular measure, degrees, minutes, and seconds.

Thus, the latitude of the equator is 0°; the latitude of the north pole is 90° N., and of the south pole, 90° S.; the latitude of New York City is 40° 45′ N.; of Washington, D. C., 39° N.; and of Melbourne, Australia, 38° S.

One degree of latitude measures about 69.16 statute miles; one sixtieth of a degree is called a geographical mile.

242. **Meridians of longitude** are semicircles drawn from the north pole to the south pole at right angles to the equator.

243. The **prime meridian** is the meridian which passes through the Royal Observatory at Greenwich, in London, England.

244. **Longitude** is distance east or west of the prime meridian.

Longitude is expressed in degrees, minutes, and seconds.

Since the meridians all meet at the poles, the distance in miles between two meridians one degree apart varies from 69.16 statute miles at the equator to nothing at the poles.

Since the entire distance around the earth measures 360°, there are 180° of east longitude and 180° of west longitude; that is, the 180th meridian is half way around the earth, either east or west from Greenwich.

EXERCISES

245. Find from the proper map, as accurately as you can, the latitude and longitude of the following places:

1. London.	2. Panama.	3. Quito.
4. Auckland.	5. Rio de Janeiro.	6. Para.
7. Manila.	8. Washington, D. C.	9. Rome.

Identify the islands or cities having the following locations:

	Latitude	Longitude		Latitude	Longitude
10.	40° N.	and 75° W.	11.	16° S.	and 6° W.
12.	30° N.	and 90° W.	13.	30° N.	and 31° E.
14.	23° N.	and 82° 30′ W.	15.	6° S.	and 39° E.

16. The *Titanic* sank in latitude 41° 27′ 36″ N., longitude 50° 08′ 02″ W. Find this spot on the map as closely as you can.

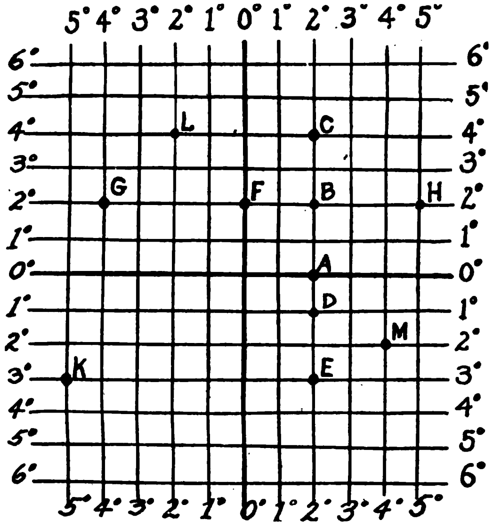
17. Find on the map the point on the earth's surface that has neither latitude nor longitude.

18. What point on the surface is directly opposite this point?

246. 1. To find the difference in latitude between two places whose latitudes are given.

247. 2. To find the difference in longitude between places whose longitudes are given.

The chart represents the equator and the other parallels of latitude from 4° S. to 4° N.; and also the prime meridian and the other meridians of longitude from 5° E. to 5° W.



SIGHT EXERCISES

Looking at the chart, tell the difference in latitude between the two places mentioned.

1. A and B.
2. B and C.
3. A and E.
4. C and E.

Tell the difference in longitude between:

5. F and B.
6. B and H.
7. G and B.
8. G and H.

Tell the difference (first) of latitude, and (second) of longitude between:

9. G and K.
10. C and E.
11. M and F.
12. D and L.

248. RULE. **To find the difference in latitude between two places whose latitudes are given: 1. Subtract the given latitudes if they are both north or both south. 2. Add the given latitudes if one is north and the other is south.**

RULE. **To find the difference in longitude between two places whose longitudes are given: 1. Subtract the given longitudes if they are both east or both west. 2. Add the given longitudes if one is east and the other is west.**

EXERCISES

249. 1. Find the difference in latitude between a place 40° 23′ 18″ N., and a place 84° 15′ 35″ N.

2. Find the difference in latitude between a place 18° 10′ 50″ S., and a place 54° 48′ 12″ N.

3. Find the difference in longitude between a place 14° 28′ 16″ W., and a place 156° 14′ 58″ W.

4. Find the difference in longitude between a place 98° 6′ E., and 78° 39′ 10″ W.

5. London is in latitude 51° 30′ N., longitude 0°, and Singapore is in latitude 1° 17′ N., longitude 103° 50′ 47″ E.

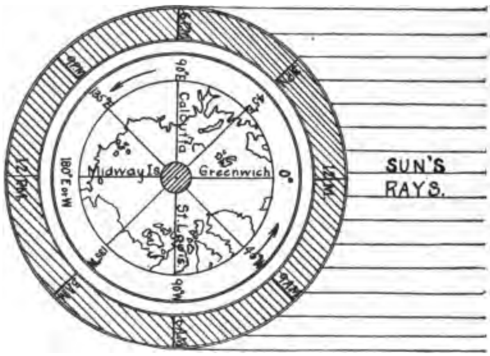
Find the difference in latitude, and the difference in longitude.

LONGITUDE AND TIME

250. The earth rotates on its axis from west to east, bringing each meridian in turn into sunlight and darkness. When the meridian on which you

live is directly under the sun, it is midday, or noon (M.), in all places on that meridian; it is forenoon, or ante-meridian (A.M.), in all places west of the meridian, and it is afternoon, or post meridian (P.M.), in all places east of the meridian. It takes twenty-four hours for a meridian to make a complete circuit and to arrive again under the sun. Hence, one hour of time corresponds to $\frac{1}{24}$ of 360°, or 15°.

"The Earth whirls through the hours."



" 'Tis always morning somewhere, and above
The awakening continents from shore to shore
Somewhere the birds are singing evermore."
—LONGFELLOW.

360° of longitude corresponds to 24 hours of time.
15° of longitude corresponds to 1 hour of time.
15′ of longitude corresponds to 1 minute of time.
15″ of longitude corresponds to 1 second of time.

When it is noonday where you are, it is one o'clock at places 15° east of you, and only eleven o'clock at places 15° west of you. Every place has its own time, called the **local time** or **true time** or **solar time**. The following exercises are in local time; but nowadays local time is not used very much Instead, people use "standard time," which will be explained in the next chapter.

ORAL EXERCISES

251. 1. How many hours of time correspond to 30° of longitude? To 45°? To 90°? To 180°?

2. How many degrees of longitude correspond to 2 hr. of time? To 3 hr.? To 5 hr.? To $\frac{1}{2}$ hr.?

3. How much difference of time is there between two meridians 15° apart? 90° apart? 120° apart?

4. Philadelphia, Pa., is in longitude 75° W.* Is Philadelphia time earlier or later than Greenwich time? How much?

5. St. Louis, Mo., is in longitude 90° W. Compare the time of St. Louis with that of Greenwich. Of Philadelphia.

6. Denver, Col., is in longitude 105° W. Compare the time of Denver with that of Greenwich. Of Philadelphia. Of St. Louis. Of Sydney, Cape Breton Island, longitude 60° W.

7. Compare the time of Sacramento, Cal. (longitude 120° W.), with the time of Greenwich. Of St. Louis.

* The longitudes given in this exercise are only approximately correct.

8. What is the difference in longitude between a point 30° E. and a point 30° W.? What is the difference in time?

Find (first) the difference in longitude, (second) the difference in local time, between places on the following meridians:

9. 50° W. and 40° E. 10. 20° E. and 110° E.

11. 90° W. and 180° W. 12. 150° W. and 90° E.

13. 60° W. and 90° E. 14. 10° E. and 55° E.

15. 15° W. and 30° E. 16. 180° W. and 180° E.*

* QUERY. What is the "international date line"?

Table of Longitudes

West Longitudes		East Longitudes	
Washington	73° 03′ 06″	Tokio	139° 44′ 30″
Boston	71° 03′ 50″	Paris	2° 20′ 13″
New York	53° 58′ 24″	St. Petersburg	30° 18′ 23″
Chicago	87° 36′ 41″	Rome	12° 28′ 54″
Galveston	94° 47′ 25″	Berlin	13° 23′ 44″
Seattle	122° 19′ 59″	Constantinople	29° 00′ 55″
San Francisco	122° 25′ 42″	Melbourne	145° 58′ 32″
Denver	104° 56′ 54″	Manila	120° 57′ 30″
Honolulu	157° 52′ 00″	Calcutta	88° 20′ 10″

252. Given the longitude to find the difference in local time.

1. What is the difference in time between New York and Berlin?

73° 58′ 24″	The difference in longitude is 87° 22′ 8″.
13° 23′ 44″	Since 15° corresponds to 1 hr., 15′ to 1 min.,
87° 22′ 8″	and 15″ to 1 sec., we have:

$$
\begin{array}{lrr}
87^\circ \text{ corresponds to } \frac{87}{15} \text{ hr.} & = 5 \text{ hr. } 48 \text{ min.} & \\
22' \text{ corresponds to } \frac{22}{15} \text{ min.} = & & 1 \text{ min. } 28 \text{ sec.} \\
8'' \text{ corresponds to } \frac{8}{15} \text{ sec.} = & & \frac{8}{15} \text{ sec.} \\
\hline
\left.\begin{array}{l}\text{Berlin is east of New York; there-} \\ \text{fore, Berlin time is later by}\end{array}\right\} & \multicolumn{2}{r}{5 \text{ hr. } 49 \text{ min. } 28\frac{8}{15} \text{ sec.}}
\end{array}
$$

SHORT METHOD

$$
\begin{array}{rrrr}
 & 73^\circ & 58' & 24'' \\
 & 13^\circ & 23' & 44'' \\
\hline
15) & 87^\circ & 22' & 8'' \\
\hline
 & \multicolumn{3}{r}{5 \text{ hr. } 49 \text{ min. } 28\frac{8}{15} \text{ sec.}}
\end{array}
$$

Since the *number* of hours is $\frac{1}{15}$ of the *number* of degrees, and since hours are subdivided into minutes (min.) and seconds (sec.) in the same manner (by 60ths) as degrees (°) are subdivided into minutes (′) and seconds (″), we may divide the number of degrees (°), minutes (′), and seconds (″) by 15 to obtain the number of hours, minutes, and seconds of time. Therefore, Berlin time is later than New York time by 5 hr. 49 min. $28\frac{4}{15}$ sec.

EXERCISES

Find the local time in New York when it is noon in each of the following cities:

2. Boston	3. Seattle	4. Denver
5. Galveston	6. Constantinople	7. Paris
8. Manila	9. Honolulu	10. Melbourne

253. Given the difference in local time, to find the difference in longitude.

1. St. Louis time is earlier than Vienna time by 7 hr. 6 min. 10.6 sec. What is the longitude of Vienna, if that of St. Louis is 90° 12′ 16.5″ W.?

Solution

7 hr. time corresponds to $7 \times 15° = 105°$	
6 min. time corresponds to $6 \times 15' =$	$1°\ 30'$
10.6 sec. time corresponds to $10.6 \times 15'' =$	$2'\ 39''$
St. Louis time is earlier than Vienna; therefore, Vienna is east of St. Louis by	$106°\ 32'\ 39''$
But St. Louis is west of Greenwich by	$90°\ 12'\ 16.5''$
Therefore, Vienna is east of Greenwich by	$10°\ 20'\ 22.5''$

Short Method

7 hr.	6 min.	10.6 sec.
		15
106°	32′	39″
90°	12′	16.5″
10°	20′	22.5″ E.

Since the *number* of degrees, minutes, and seconds of longitude is 15 times the *number* of hours, minutes, and seconds of time, the difference of longitude must be 106° 32′ 39″. The question states that St. Louis time is earlier than Vienna time, hence St. Louis is 106° 31′ 39″ west of Vienna. Therefore, Vienna is 10° 20′ 22.5″ east of the prime meridian.

Note. Pupils should be trained to represent the problem by a simple diagram as follows:

0°

90° 12′ 16.5″ W. ←——————————→ | ←——— ? ———→

St. L. ←——————— 7 hr. 6 min. 10.6 sec. ———————→ V

2. Baltimore time is 5 hr. 6 min. 26 sec. earlier than Greenwich time. What is the longitude of Baltimore?

Find the longitude of places having the following times when it is 12:00 M. in Washington.

3. 12:30 P.M.	4. 9:30 A.M.	5. 8:00 A.M.
6. 1:45 P.M.	7. 6:15 A.M.	8. 7:35 P.M.
9. 3:18 P.M.	10. 11:50 A.M.	11. 10:12 A.M.

12. (*a*) My watch was set right by New York time. But after traveling several days, I find that my watch is 2 hr. 10 min. slow compared with the local time. What is the longitude of the place at which I have arrived, assuming that my watch has kept perfect time?

(*b*) Suppose that my watch was 2 hr. 10 min. fast, what would the longitude of the place be?

13. A sea captain has his chronometer set by Greenwich time before he starts on a voyage across the Atlantic. Every day he observes the sun as it crosses the ship's meridian, and he notes the exact time by his chronometer, as follows:

First day, 12:15 P.M.; second day, 12:55:20 P.M.; third day, 1:16 P.M. Find the ship's longitude at the time of each observation.

14. A ship's chronometer carrying New York time indicated 11:02 A.M. as the sun crossed the ship's meridian. What was the longitude of the ship?

15. What is the time at Calcutta when it is 10 P.M., June 30, at San Francisco?

HINT. To avoid the difficulty of the international date line, always reckon the difference of time and of longitude *through* the prime meridian.

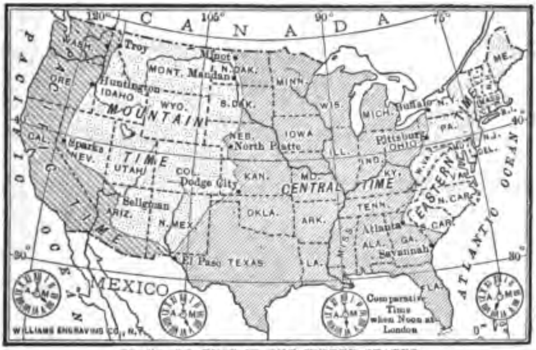
16. When it is noon, Oct. 1, at Seattle, what is the time at Manila? At Tokio? At Melbourne? At Honolulu?

STANDARD TIME

254. You have learned that only places on the same meridian have the same time and that as you travel east or west you must reset your watch frequently if you wish it to show the local or true time. In the days when people traveled by stage-coach or on horseback, the difference between the local time of to-day's stopping place and that of yesterday's was but slight. But after the steam-engine and railroad made it possible to travel a thousand miles or more a day, the local differences of time became confusing to the traveler and sometimes occasioned loss and delay to the merchant. It sometimes happened that in a city touched by two railroads, each road used a widely different local time. To overcome these difficulties, there has been established in all civilized countries, "standard time," whereby the local time of every fifteenth meridian serves as the time for the wide belt of country through which the meridian passes.

In North America there are five of these time belts, as shown on the accompanying map: **Atlantic,** or Colonial, time is the local time of the 60th meridian; **Eastern**, or Philadelphia, time is that of the 75th meridian; **Central** time is that of the 90th meridian; **Mountain**, or Denver, time is that of the 105th meridian; and **Pacific** time is that of the 120th meridian.

Since 15 degrees of longitude correspond to one hour of time, there is a change of one hour as we pass from one belt to the next. Thus, we ignore all the small changes of minutes and seconds which made the old system of local or true time so troublesome.



STANDARD TIME IN THE UNITED STATES

The boundaries of these belts take their irregularity from the location of the cities on the different railroads at which the time is conveniently changed from one standard to the next.

Every day the United States Naval Observatory at Washington telegraphs the correct time to all parts of the United States.

EXERCISES IN STANDARD TIME

For these exercises consult the map on this page to ascertain the time meridian of cities in the United States. The time meridian of foreign cities is given wherever needed.

255. When it is noon at Greenwich, what is the standard time at the following cities?

1. New York
2. Chicago
3. Halifax
4. Los Angeles
5. St. Louis
6. Boston
7. St. Paul
8. New Orleans
9. Salt Lake City
10. Portland, Ore.
11. Portland, Me.
12. Austin, Tex.

When it is noon at Washington, what is the standard time at the following cities?

13. Berlin, Germany (15° E.)
14. Cairo, Egypt (30° E.)
15. Yokohama, Japan (135° E.)
16. Constantinople, Turkey (30° E.)
17. Vienna, Austria (15° E.)
18. Madrid, Spain (0°)
19. Auckland, New Zealand (165° E.)
20. Cape Town (22° 30′)

21. A dispatch from London was received in Detroit at 10 P.M. Central time. Allowing no time for transmission, at what time was it sent?

22. When the Western Union "time ball" drops at noon in New York City, what is the time in Rome, Italy (15° E.)?

23. The Stock Exchange in New York opens at 10 A.M. What is the time in Berlin (15° E.)? In Gibraltar (0°)?

24. The Bourse in Berlin (15° E.) opens at 10 A.M. What is the time in Chicago?

25. France uses the time meridian of Paris, which is 2° 20′ 13″ east of Greenwich. If the Paris Bourse opens at 10 A.M., what is the time in San Francisco?

METRIC SYSTEM

For Reading and Discussion

256. The Metric System of weights and measures is the system used in all European countries except Great Britain and Russia. It has also been adopted by Mexico, Brazil, Chile, and other nations in South America, and it has been made a legal system in many other countries, including the United States.

In this country the Metric System is not in general use. It is used, however, for scientific purposes.

257. The **unit** on which the Metric System is based is the **meter**; a meter equals **39.37 inches**, a little more than 1 yd.

All measurements less than a meter are formed by taking **decimal parts** of a meter.

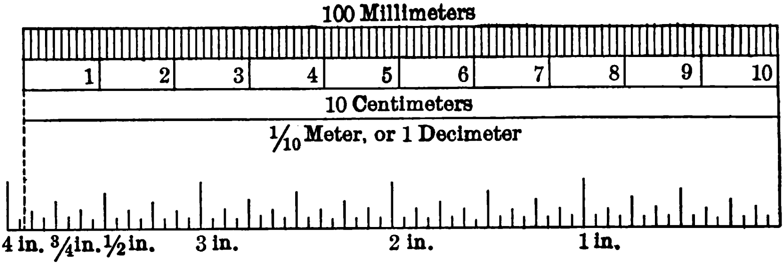
$\frac{1}{10}$ of a meter is called a **decimeter.**

$\frac{1}{100}$ of a meter is called a **centimeter.**

$\frac{1}{1000}$ of a meter is called a **millimeter.**

All measurements greater than a meter are formed by **multiplying** a meter by **10** or some **multiple** of **10**.

A distance of 10 meters is called a **dekameter.**
A distance of 100 meters is called a **hektameter.**
A distance of 1000 meters is called a **kilometer.**



Comparison of 1 decimeter, 10 centimeters, 100 millimeters with an inch. (1 decimeter equals about 3$\frac{15}{16}$ in.)

258. Metric System. Table of Length

10 **millimeters** (mm.)	= 1 **centimeter** (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 **meter** (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 **kilometer** (Km.)

Note that abbreviations for decimal parts of a meter begin with small letters, and that abbreviations for multiples of a meter begin with capital letters.

NOTE. All the other tables of the Metric System are printed in the Appendix.

EXERCISES

259. Change the following to meters or to a decimal of a meter:

1. 3 Dm.	3. 5 Hm.	5. 5 Km.
2. 16 dm.	4. 125 cm.	6. 125 mm.

EQUIVALENTS

To change from Our System to the Metric System

260. 1. From yards to meters; multiply .914 m. by the number of yards (or take $\frac{9}{10}$ of the number of yards).

2. From miles to kilometers; multiply 1.6 Km. by the number of miles.

3. From square yards to square meters; multiply .83 sq. m. by the number of square yards.

4. From acres to hectares; multiply .404 Ha. by the number of acres (or take $\frac{2}{5}$ of the number of acres).

5. From cubic yards to cubic meters; multiply .765 cu. m. by the number of cubic yards (or take $\frac{3}{4}$ of the number of cubic yards).

6. From quarts (dry) to liters; multiply 1.101 l. by the number of dry quarts (or take $\frac{11}{10}$ of the number of quarts).

7. From quarts (liquid) to liters; multiply .946 l. by the number of liquid quarts.

8. From bushels to hektoliters; multiply .35 Hl. by the number of bushels.

9. From pounds (avoir.) to kilograms; multiply .45 Kg. by the number of pounds.

10. From tons to metric tons; multiply .907 M.T. by the number of tons (or take $\frac{9}{10}$ of the number of tons).

EXERCISES

261. Change the following from our system to the metric system. (Approximate equivalents may be used.)

1. 1 in. to m.; to cm.
2. 1 ft. to m.; to dm.
3. 1 yd. to m.; to Dm.
4. 6 in. to m.; to dm.
5. 16 ft. to m.; to dm.
6. 30 yd. to m.; to Dm.
7. 130 yd. to m.; to Dm.
8. 4 mi. to Km.
9. $15\frac{1}{2}$ mi. to Km.
10. 2000 yd. to Km.
11. 18 sq. yd. to sq. m.
12. 150 sq. in. to sq. dm.
13. $54\frac{1}{2}$ sq. yd. to sq. m.
14. 2 sq. mi. to sq. Km.
15. 15 A. to Ha.
16. $3\frac{1}{2}$ A. to Ha.

262. To change from the Metric System to Our System

1. From meters to inches; multiply 39.37 in. by the number of meters.

2. From kilometers to miles; multiply .62 mi. by the number of kilometers (or take $\frac{5}{8}$ of the number of kilometers).

3. From square meters to square yards; multiply 1.19 sq. yd. by the number of square meters.

4. From hectares to acres; multiply 2.47 acres by the number of hectares (or take $2\frac{1}{2}$ times the number of hectares).

5. From cubic meters to cubic yards; multiply 1.3 cu. yd. by the number of cubic meters (or take $1\frac{1}{3}$ times the number of cubic meters).

6. From liters to dry quarts; multiply .908 quarts (dry) by the number of liters (or take $\frac{9}{10}$ of the number of liters).

7. From liters to liquid quarts; multiply 1.05 liquid quarts by the number of liters.

8. From hektoliters to bushels; multiply 2.83 bu. by the number of liters.

9. From kilograms to pounds (avoir.); multiply 2.204 lb. by the number of kilograms (or multiply the number of kilograms by $2\frac{1}{5}$).

10. From metric tons to tons; multiply 1.1 T. by the number of metric tons (or take $\frac{11}{10}$ of the number of metric tons).

EXERCISES

263. Change the following from the metric system to our system. (Approximate equivalents may be used.)

1. 8 m. to in.; to ft.
2. 9 dm. to yd.; to ft.
3. 15 m. to in.; to yd.
4. 7 dm. to in.; to ft.
5. 10 Km. to mi.; to yd.
6. 8 cm. to in.
7. 3.7 m. to in.; to ft.
8. $6\frac{1}{2}$ m. to yd.; to ft.
9. 100 m. to in.; to yd.
10. 16 Km. to mi.; to yd.
11. 12 sq. m. to square yards; to square feet.
12. $15\frac{1}{2}$ sq. m. to square yards; to square feet.
13. 4 sq. Km. to square miles.
14. 50 sq. Km. to square miles.

PROBLEMS

(Use approximate equivalents whenever possible.)

264. 1. A boy ran 100 yd. How many meters?

2. It is necessary to ride 8 times around a track to go 1 mi. What part of a kilometer has a boy ridden who has finished 3 laps? How many meters?

3. How many meters has the boy ridden when he has finished 5 laps?

4. A building in New York is 750 ft. high. How would the height be stated if the building were in Paris?

5. A train travels 20 mi. an hour for 5 hr. How many kilometers has it gone?

Express the following distances in kilometers:

	Distances from New York to	Miles	Km.		Distances from New York to	Miles	Km.
6.	Philadelphia	69.4	?	9.	Cleveland	578.9	?
7.	Harrisburg	123.3	?	10.	do	700.5	?
8.	Pittsburg	440.5	?	11.	Detroit	760.1	?

12. A wall contains 360 sq. ft. Express this in square meters.

13. A plot of ground is 50 ft. by 100 ft. How many square meters in the plot?

14. A bin is 8 ft. long, 4 ft. wide, and 3 ft. high. Express the contents of the bin in cubic meters.

15. Express the contents of the bin in bu.

Tell how the following would be expressed in a French geography:

16. Area of Texas equals 265,780 sq. mi.

17. Length of the Hudson River equals 280 mi.

18. A large auto truck carries 8 T. of coal. How many metric tons?

19. How many cubic meters are there in a bin 12 ft. long, 4 ft. high, and 4 ft. wide?

20. A barrel of flour weighs 196 lb. Express the weight in kilograms.

21. A can of milk contains 40 qt. Express the capacity in liters.

22. 46 qt. of apples are obtained from 1 bbl. How many liters may be obtained from the barrel?

23. The distance from Paris to Rouen is 136 Km. Express this in miles.

24. The height of one of the Dolomite Mountains is 2250 m. Express this in feet.

25. A foreign time-table gives the following distances between stations. Find values for ? .

	Km.	Mi.
26. From 1st to 2d station	4	?
27. From 2d to 3d station	13	?
28. From 3d to 4th station	18	?
29. From 4th to 5th station	22	?

30. The Danube is 2600 Km. in length. Express this in miles.

31. Zeppelin's dirigible balloon flew 480 Km. in 9 hr. How many miles per hour did it fly?

32. On Nov. 20, 1909, Louis Paulhan flew in a biplane from Mourmelon to Chalons, France, and return, in 55 min. The total distance was about 40 Km. Find his speed per hour in miles.

33. A French boy ran a race of 100 m. An American boy ran a race of 100 yd.

(*a*) Which is the longer race?

(*b*) How many more feet in one race than the other?

34. I spent 625 fr. for silk at $12\frac{1}{2}$ fr. per meter. How many yards of silk did I buy?

35. 100 k. of flour are what part of the weight of a barrel of flour in our system (196 lb. = 1 bbl. of flour)?

36. A jar containing 40 l. of cologne will fill how many bottles if each bottle holds $\frac{1}{2}$ pt.?

37. At 40 ct. a square meter, find the cost of painting a ceiling 15.4 m. long by 12.5 m. wide.

38. At $ 2.25 a yard, find the cost of 400 m. of silk.

39. How many bottles each containing 40 cl. may be filled from 300 l.?

40. At 25 ct. a deciliter, how much will 12.75 l. cost?

PERCENTAGE

MEANING OF PER CENT RELATIONS

For Reading and Discussion

265. Per cent, written %, means **hundredths.** It is an abbreviation of the Latin *per centum.*

20 per cent is written 20 %; it means 20 out of every 100.

$33\frac{1}{3}$ per cent is written $33\frac{1}{3}$ %; it means $33\frac{1}{3}$ out of every 100.

$\frac{1}{2}$ per cent is written $\frac{1}{2}$ %; it means $\frac{1}{200}$ of every 100.

Changing Decimal Fractions and Common Fractions to Per Cents.

EXERCISES

266. Change the following decimal fractions to per cents:

Process. $.09 = 9\,\%$. $.035 = .03\frac{1}{2} = 3\frac{1}{2}\,\%$.

First read as hundredths; then state as per cent:

1. .07	2. $.02\frac{1}{2}$	3. 10.00
4. .15	5. .125	6. 5.50
7. .29	8. .005	9. .625
10. .18	11. .0025	12. .0325
13. .06	14. .001	15. $.05\frac{1}{2}$

EXERCISES

267. Change the following common fractions to per cents:

1. $\frac{1}{2}$. **Process.** $\frac{1}{2} = \frac{50}{100} = 50\,\%$.

First change to hundredths; then state as per cent:

2. $\frac{1}{4}$	**3.** $\frac{1}{5}$	**4.** $\frac{7}{10}$
5. $\frac{1}{3}$	**6.** $\frac{4}{5}$	**7.** $\frac{3}{20}$
8. $\frac{2}{3}$	**9.** $\frac{1}{8}$	**10.** $\frac{3}{5}$
11. $\frac{3}{4}$	**12.** $\frac{5}{8}$	**13.** $\frac{9}{10}$
14. $\frac{1}{6}$	**15.** $\frac{1}{10}$	**16.** $\frac{5}{6}$

PER CENT AND COMMON FRACTIONS

268. Memorize this table; it contains the most important per cents and their equivalents in common fractions.

$$\left\{\begin{array}{l} 25\,\% = \frac{1}{4} \\ 50\,\% = \frac{1}{2} \\ 75\,\% = \frac{3}{4} \end{array}\right. \qquad \left\{\begin{array}{l} 12\frac{1}{2}\,\% = \frac{1}{8} \\ 37\frac{1}{2}\,\% = \frac{3}{8} \\ 62\frac{1}{2}\,\% = \frac{5}{8} \\ 87\frac{1}{2}\,\% = \frac{7}{8} \end{array}\right. \qquad \left\{\begin{array}{l} 14\frac{2}{7}\,\% = \frac{1}{7} \\ 71\frac{3}{7}\,\% = \frac{5}{7} \end{array}\right. \quad \left\{\begin{array}{l} 1\,\% = \frac{1}{100} \\ 5\,\% = \frac{1}{20} \end{array}\right.$$

$$\left\{\begin{array}{l} 10\,\% = \frac{1}{10} \\ 20\,\% = \frac{1}{5} \\ 40\,\% = \frac{2}{5} \\ 60\,\% = \frac{3}{5} \\ 80\,\% = \frac{4}{5} \end{array}\right. \qquad \left\{\begin{array}{l} 33\frac{1}{3}\,\% = \frac{1}{3} \\ 66\frac{2}{3}\,\% = \frac{2}{3} \end{array}\right. \quad \left\{\begin{array}{l} 16\frac{2}{3}\,\% = \frac{1}{6} \\ 83\frac{1}{3}\,\% = \frac{5}{6} \end{array}\right. \qquad \left\{\begin{array}{l} 100\,\% = 1 \\ 200\,\% = 2 \\ 120\,\% = \frac{6}{5} \\ 125\,\% = \frac{5}{4} \\ 133\frac{1}{3}\,\% = \frac{4}{3} \end{array}\right.$$

First Type Problem

269. To find a per cent of a number.

By Common Fractions

1. How much is $\frac{3}{4}$ of \$160?

Process. $\frac{3}{\cancel{4}}$ of \$$\overset{40}{\cancel{160}}$ = \$120 *Ans.*

By Percentage

2. How much is 60% of \$250?

Process. 60% of a number = $\frac{3}{5}$ of the number

$\frac{3}{\cancel{5}}$ of \$$\overset{50}{\cancel{250}}$ = \$150 *Ans.*

By Decimal Fractions

3. How much is .37 of 240 mi.?

Process.

```
 240 mi.
  .37
 1680
 720
88.80 mi.  Ans.
```

By Percentage

4. How much is 53% of \$865?

Process.

```
 $865
   .53
  2595
 4325
$458.45  Ans.
```

270. From these problems we see that to find **a per cent of a number,** we proceed as in finding **a fractional part of a number.**

Whenever convenient, reduce the per cent to a common fraction in its lowest terms; then proceed as in common fractions.

In other cases, reduce the per cent to a decimal fraction and proceed as in decimal fractions.

271. In the question, How much is 60% of $250?

60% is called the **Rate** or **Rate per cent**

$250 is called the **Base**

$150 is called the **Percentage.**

The **Base** is the number on which the percentage is calculated ($250).

The **Rate** or **Rate per cent** is the number of hundredths of the base to be taken (60%).

The **Percentage** is the number obtained by taking the required per cent of the base ($150).

The percentage is always the same denomination as the base. Why?

272. To find the percentage when the base and rate are given.

RULE. **Multiply the base by the rate expressed as a common fraction or a decimal fraction.**

EQUATION AND FORMULA.

$$\textbf{Percentage} = \textbf{Rate} \times \textbf{Base}$$

$$P = R \times B.$$

273. ORAL EXERCISES

1. Find 16% of 300.	2. Find 1% of 350.
3. Find 8% of 50.	4. Find 100% of 425.
5. Find 4% of 30.	6. Find 300% of 125.
7. Find 12% of 80.	8. Find $1\frac{1}{2}$% of 200.
9. Find 6% of 90.	10. Find $2\frac{1}{2}$% of 100.
11. Find 5% of 40.	12. Find 2% of 400.
13. Find 3% of 120.	14. Find 18% of 50.
15. Find 9% of 100.	16. Find 200% of 75.
17. Find 11% of 150.	18. Find $\frac{1}{2}$% of 800.
19. Find 15% of 60.	20. Find $4\frac{1}{2}$% of 50.
21. Find 2% of 150.	22. Find $\frac{1}{8}$% of 160.

274. ORAL EXERCISES

1. Find 25% of 40; of 80; of 100; of 1000.
2. Find 50% of 30 bu.; of 800 ft.; of \$420.
3. Find $33\frac{1}{3}$% of \$960; of 240 mi.; of 330 yd.
4. Find 60% of 120; of 200; of 10; of 75.
5. Find $66\frac{2}{3}$% of 99; of 180; of 300; of 1500.
6. Find 75% of 160 bbl.; of \$1600; of 240.
7. Find 150% of 200; of 320; of 40; of 100.
8. Find $37\frac{1}{2}$% of \$1.60; of \$4; of \$24.40.
9. Find 10% of \$17.30; of \$63.20; of \$89.20.

NOTE. To find 1%, point off two decimal places from the right (*e.g.* 1% of 185 = 1.85); if the base ends in two zeros, cancel the zeros (*e.g.* 1% of 500 = 5).

10. Find 20% of \$15.30; of \$39.20; of \$12.45.
11. Find $12\frac{1}{2}$% of \$80; of \$240; of \$400.
12. Find $\frac{1}{8}$% of \$80; of \$240; of \$400.
13. Find 25% of \$64; of \$200; of \$800.
14. Find $\frac{1}{4}$% of \$64; \$200; of \$800.
15. Find 50% of \$80; of \$120; of \$1000.
16. Find $\frac{1}{2}$% of 80; of 120; of 1000.

DRILLS

For Limited Time

275. *First:* Begin at the top of a column and work down. Find values for all (*e.g.* 10% of 12 = ?; 10% of 24 = ?, etc.).

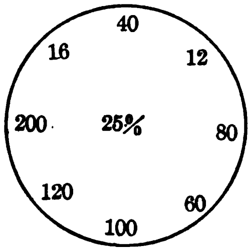
Second: Begin at the left and work to the right. Find values for all (*e.g.* 10% of 12 = ?; 25% of 12 = ?, etc.).

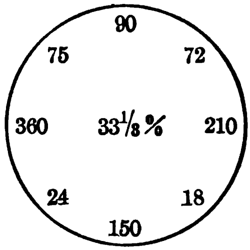
	Base	10%	25%	$33\frac{1}{3}$%	50%	75%	100%
1.	12	?	?	?	?	?	?
2.	24	?	?	?	?	?	?
3.	72	?	?	?	?	?	?
4.	60	?	?	?	?	?	?
5.	144	?	?	?	?	?	?
6.	1	?	?	?	?	?	?
7.	36	?	?	?	?	?	?
8.	120	?	?	?	?	?	?
9.	48	?	?	?	?	?	?
10.	84	?	?	?	?	?	?

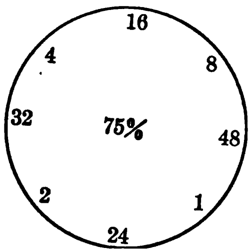
DRILLS

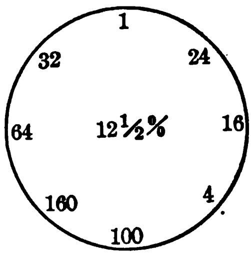
For Limited Time

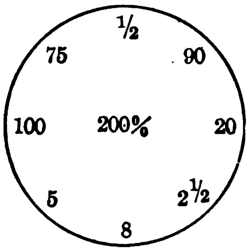
276. The number in the center is the rate; each number in the ring is the base; find the percentage.

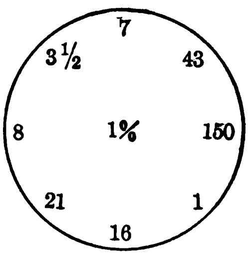












Note. Other devices suitable for drills in percentage may be found in The Pupils' Arithmetic, Book IV.

DRILLS

For Limited Time

277. Proceed as in art. 275.

	Base	1%	10%	20%	40%	50%	200%
1.	1	?	?	?	?	?	?
2.	5	?	?	?	?	?	?
3.	10	?	?	?	?	?	?
4.	20	?	?	?	?	?	?
5.	40	?	?	?	?	?	?
6.	50	?	?	?	?	?	?
7.	100	?	?	?	?	?	?
8.	150	?	?	?	?	?	?
9.	200	?	?	?	?	?	?
10.	1000	?	?	?	?	?	?

DRILLS

For Limited Time

278. Proceed as in art. 275.

	Base	2%	3%	4%	6%	8%	12%
1.	200	?	?	?	?	?	?
2.	300	?	?	?	?	?	?
3.	500	?	?	?	?	?	?
4.	600	?	?	?	?	?	?
5.	1000	?	?	?	?	?	?
6.	1200	?	?	?	?	?	?
7.	1500	?	?	?	?	?	?
8.	1750	?	?	?	?	?	?
9.	2250	?	?	?	?	?	?
10.	2500	?	?	?	?	?	?

DRILLS

For Limited Time

279. Proceed as in art. 275.

	Base	$\frac{1}{2}$%	$\frac{1}{4}$%	$\frac{1}{8}$%	1%	$2\frac{1}{2}$%	$3\frac{1}{2}$%
1.	100	?	?	?	?	?	?
2.	200	?	?	?	?	?	?
3.	400	?	?	?	?	?	?
4.	800	?	?	?	?	?	?
5.	1200	?	?	?	?	?	?
6.	1600	?	?	?	?	?	?
7.	3200	?	?	?	?	?	?
8.	4000	?	?	?	?	?	?
9.	1000	?	?	?	?	?	?
10.	10000	?	?	?	?	?	?

WRITTEN EXERCISES

280. 1. Find 26% of 950.
2. Find 130% of $1300.
3. Find 18% of 260.
4. Find 16% of 120.
5. Find 33% of 175.
6. Find 38% of 210.
7. Find 51% of 18.
8. Find 66% of 160.
9. Find 24% of 150.
10. Find 79% of 1500.
11. Find 37% of 320.
12. Find 55% of 325.
13. Find 69% of 165.
14. Find 73% of 700.
15. Find 85% of 320.
16. Find 92% of 125.
17. Find 43% of 275.
18. Find 15% of 640.
19. Find 97% of 1800.
20. Find 82% of 230.
21. Find 71% of 500.
22. Find 49% of 390.

ORAL PROBLEMS

281. 1. Minneapolis has a population of 300,000 people. Oakland has a population 50% as large. Find the population of Oakland.

2. A local train takes 15 hr. to go 240 mi. An express train takes only 40% as long. How long does the express train take?

3. George has 120 marbles. Harry has 125% as many. How many marbles has Harry?

4. The railroad fare between two cities was \$2.70. The company increased the fare $33\frac{1}{3}\%$. Find the increase in fare.

5. Meat which cost 16 ct. a pound has increased $37\frac{1}{2}\%$ in price. How much more does a pound cost now?

6. A company that employed 220 men discharged 20% of them. How many men were discharged?

7. A plot of ground is 440 ft. long. The width is $12\frac{1}{2}\%$ of the length. Find the width.

8. A book cost \$1.20. It was sold for 80% of its cost. For how much was it sold?

9. The trunk of a tree was 350 ft. long. It was cut into two pieces, one being $14\frac{2}{7}\%$ as long as the trunk. How long was each piece?

10. 150 glass globes were shipped by express. 6 % of them were broken. How many were broken?

11. A farmer had 250 cows. He sold 12 % of them. How many did he sell?

12. There were 500 children in a school. 8 % were not promoted. How many were not promoted?

13. A baseball team played 50 games. It won 90 % of them. How many games were won?

WRITTEN PROBLEMS

1. A school contains 3000 children. 54 % of them are boys. How many boys are there?

2. 40 words were given to each boy in an examination in spelling. George spelled 95 % of them correctly; Harry spelled 85 % of them correctly; Frank spelled 70 % correctly. How many words did George spell correctly? Harry? Frank?

3. A house is worth $ 16,000. A second house is worth 92 % as much. Find the value of the second house.

4. During January, a store sold $ 1500 worth of goods; during February, it sold 110 % as much. Find the value of the goods sold in February.

5. The cost of sending freight by railroad is \$ 850. The cost of sending it by boat is 76 % as much. How much is saved by sending the freight by boat?

6. How many days are there in 78 % of a year? (365 da. = 1 yr.)

7. One piano cost \$ 550. A second piano cost 130 % as much. Find the cost of the second piano.

8. The distance between A and B is 420 mi. The distance between A and C is 54 % of the distance between A and B. (Draw diagram.)

(*a*) Find the distance from A to C.
(*b*) Find the distance from C to B.

9. 20,000 copies of a book were sold during the first year. During the second year the sales increased 22 %. How many copies were sold during the second year?

10. Desks which were marked \$ 65 each were reduced 12 %.

(*a*) Find the reduction.
(*b*) How much is saved by buying 6 desks at the lower price?

11. A man who receives \$ 2000 a year saves 32 % of it. How much will he save in five years at that rate?

12. A man who receives \$ 3000 a year saves

35 % of it. How many years will it take him to save $ 4725 at that rate?

13. A number of books were packed in a box. The total weight of the books and the box was 65 lb. The box weighed 6 % of the total weight.

(a) Find the weight of the box.

(b) Find the weight of the books.

14. An orchard yielded 2500 bbl. of apples. 65 % of them were sold at $ 2.40 per barrel. The remainder were sold at $ 2.70 per barrel. How much was received for all the apples?

15. A telephone company received 700 calls at one station. 95 % of them were 5-ct. calls, the remainder were 10-ct. calls. How much money did the company receive for all the calls?

282. To find the amount when the base and the rate are given.

WHEN THE RATE IS EASILY REDUCIBLE TO A COMMON FRACTION

1. A boy who had 160 marbles won 25 % as many. How many marbles has he now?

Process

Represent the original number of marbles by 100 %.
The number of marbles gained = 25 % of the original number.
The number of marbles the boy has now = 125 %, or $\frac{5}{4}$ of the original number.
Therefore, $\frac{5}{4}$ of 160 = 200 marbles. *Ans.*

N

When the Rate is not easily reducible to a Common Fraction

2. Five years ago the population of a city was 200,000. To-day the population is 23 % greater. Find the population to-day.

Process

Represent the population 5 yr. ago by 100 %.
The increase in population = 23 % of the population 5 yr. ago.
The present population = 123 % of the population 5 yr. ago.

$1.23 \times 200{,}000 = 246{,}000.$ *Ans.*

283. Second Method

Problem 1. $\frac{1}{4}$ of $160 = 40$; $160 + 40 = 200$.
Problem 2. $.23$ of $200{,}000 = 46{,}000$;
$200{,}000 + 46{,}000 = 246{,}000$.

284. The answers are the **Amount.**

285. Rule. **To find the amount: Add the percentage to the base, or multiply the base by (1 plus the rate).**

Equation and Formula.

Amount = Base + Percentage ; $A = (B + P)$;
or
Amount = Base × (1 plus Rate); $A = B \times (1 + R)$.

286. Proof. Check the answers by Type 2 (p. 183), or by Type 3, Case 2 (p. 197).

287. To find the difference when the base and the rate are given.

WHEN THE RATE IS EASILY REDUCIBLE TO A COMMON FRACTION

1. A boy who had 120 marbles lost 20 % of them. How many marbles were left?

Process

Represent the original number of marbles by 100 %.

The number of marbles lost = 20 % of original number.

The number of marbles left = 80 %, or $\frac{4}{5}$ of the number he had at first.

$$\frac{4}{\cancel{5}} \text{ of } \overset{24}{\cancel{120}} = 96 \text{ marbles. } \textit{Ans.}$$

WHEN THE RATE IS NOT EASILY REDUCIBLE TO A COMMON FRACTION

2. A merchant had 750 boxes of oranges. He sold 24 % of them. How many boxes were left?

Process

Represent the total number of boxes by 100 %.

The number of boxes sold = 24 % of total number.

Number of boxes not sold = 76 % of total number.

$.76 \times 750$ boxes $= 570$ boxes. *Ans.*

288. SECOND METHOD

Problem 1. $\frac{1}{5}$ of $120 = 24$; $120 - 24 = 96$.

Problem 2. $.24$ of $750 = 180$; $750 - 180 = 570$.

289. The answers are the **Difference.**

290. RULE. **To find the difference: Subtract the percentage from the base; or multiply the base by (1 – rate).**

EQUATION AND FORMULA.

Difference = Base − Percentage ; $D = (B - P)$,

or

Difference = Base × (1 − Percentage) ;

$$D = B \times (1 - P).$$

291. Proof. Check the answers by Type 2 (p. 183), or by Type 3, Case 3 (p. 198).

ORAL PROBLEMS

292. 1. William weighed 60 lb. in June; in September his weight was 10% more. Find his weight in September.

2. One kind of tea costs 50 ct. a pound. A better kind costs 20% more. Find the cost of 3 lb. of the better kind.

3. A wagon cost $80. A horse cost 300% as much as the wagon. Find the cost of the horse and the wagon.

4. Nellie had $4. The first day, she spent 50% of it; the second day, she spent 50% of the remainder. How much did she spend the second day?

5. A has $200; B has 50% as much as A; C has 25% as much as B. How much money has C?

6. The number of pupils in one school is 400; the number in a second school is 25% greater; the number in a third school is 50% greater than the number in the second school. Find the number of pupils in the third school.

WRITTEN PROBLEMS

293. 1. The cost of a railroad ticket was $65. The railroad company increased the fare $3\frac{1}{2}\%$. Find the price of a ticket now.

2. The population of Albany in 1900 was about 94,000. In 1910 the population was 9% greater. Find the population in 1910.

3. A horse that cost $350 was sold at a gain of 18%. How much was received for it?

4. 12,500 pr. of shoes were made in a factory in one year. The next year 12% more shoes were made. Find the number of pairs of shoes made during the second year.

5. A man who formerly paid $45 a month for rent now pays 15% more. Find the amount paid for rent now.

6. The wages paid in a factory were as follows:

Skilled Workmen	$25 per week.
Unskilled Workmen . . .	$15 per week.
Boys	$8 per week.

The skilled workmen received an increase of 25%, the unskilled workmen an increase of 15%, and the boys an increase of 7%. How much does each class receive per week after the increase?

7. The receipts of a store during the first week were $350; the amount received during the sec-

ond week was 25% greater than the amount received during the first week; the amount received during the third week was 35% greater than the amount received during the second week. Find the total amount received during the three weeks.

8. There are three grades of lead pencils. Grade A costs 5 ct. each; grade B costs 20% more than grade A; grade C costs 50% less than grade B. Find the cost of 6 doz. of grade C.

9. A tailor has 90 overcoats. He sells $33\frac{1}{3}\%$ of them at \$15 each; 50% of the remainder at \$18 each, and those still on hand at \$20 each. How much is received for all the overcoats?

10. The annual salary of a clerk is \$1200. He spends 30% for rent, 40% for other expenses, and saves the remainder. How much does he save?

11. A firm that employed 250 men discharged 12% of that number.

(*a*) How many men are still employed?

(*b*) How much money is paid to them if each man receives \$22 a week?

12. One grade of butter costs 35 cents a pound; another grade costs 40% more. Find the cost of 5 lb. of the better quality.

13. 1500 people bought tickets for a theater. 48% of them paid \$2 each; 26% paid \$1 each; the remainder paid 75 ct. each. Find the total amount of money paid for tickets.

14. The amount received in a store during the first week was \$400; the amount received during the second week was 40% greater than the amount received during the first week; the amount received during the third week was 18% greater than the amount received during the second week.

(*a*) Find the receipts for the third week.

(*b*) Find the total amount received during the three weeks.

15. When a train started from Chicago it carried 200 passengers. At the first station, 25% more passengers got aboard; at the second station, 40% of those in the train left; at the third station, 50% of those still aboard left. How many were still on the train?

Second Type Problem

294. To find what per cent one number is of another.

1. A class contained 50 children. 10 of them were not promoted. What per cent of the class was not promoted?

The number not promoted equals 10 out of 50, or $\frac{10}{50}$ of the class.

$\frac{10}{50} = \frac{1}{5} = 20\%$. *Ans.*

2. A storekeeper had 65 baseballs. He sold 51 balls. What per cent of his stock did he sell?

The number sold equals 51 out of 65, or $\frac{51}{65}$ of the entire stock.

$\frac{51}{65} = 78\frac{6}{13}\%$. *Ans.*

295. RULE. **To find the rate: Divide the percentage by the base.**

EQUATION AND FORMULA.

$$\textbf{Rate} = \frac{\textbf{Percentage}}{\textbf{Base}}; \quad R = \frac{P}{B}.$$

NOTE. Be careful to calculate the rate per cent on the base.

296. Proof. Check the answers by Type 1 (p. 167), or by Type 3, Case 1 (p. 191).

ORAL EXERCISES

297. What per cent is

1. 5 of 25?	2. 10 of 100?	3. 2 of 8?
4. 10 of 50?	5. 6 of 12?	6. 12 of 48?
7. 9 of 9?	8. 6 of 9?	9. 8 of 32?
10. 3 of 8?	11. 3 of 9?	12. 3 of 4?
13. 4 of 4?	14. 3 of 5?	15. 4 of 8?
16. 3 of 10?	17. 4 of 16?	18. 6 of 100?
19. 4 of 20?	20. 10 of 5?	21. 8 of 1?
22. 9 of 3?	23. 1 of 8?	24. 9 of 6?
25. 6 of 2?	26. 20 of 5?	27. 25 of 5?
28. 100 of 100?	29. 100 of 400?	30. 150 of 600?
31. 250 of 500?	32. 250 of 1000?	33. 200 of 800?
34. 300 of 1000?	35. 740 of 1000?	36. 250 of 1000?
37. 600 of 1000?	38. 300 of 1000?	39. 300 of 2000?

ORAL EXERCISES

298. 1. $4\frac{1}{2}$ is what per cent of 18?

Process. $\frac{4\frac{1}{2}}{18}=\frac{\frac{9}{2}}{18}=\frac{9}{2}\times\frac{1}{18}=\frac{1}{4}=25\%$.

What per cent is

2. \$3 of \$2.50?
3. $2\frac{1}{2}$ of 15?
4. \$200 of \$140?
5. $\frac{1}{2}$ of 8?
6. \$650 of \$500?
7. $3\frac{1}{2}$ of 10?
8. 90 da. of $\frac{1}{2}$ da.?
9. $5\frac{1}{2}$ of 25?
10. 175 da. of 365 da.?
11. 50 of 45?

ORAL EXERCISES

299. Change both quantities to the same denomination before trying to find the rate.

What per cent is

1. 1 ft. of a yard?
2. 2 ft. of 2 yd.?
3. 2 ft. of a yard?
4. 6 in. of 2 ft.?
5. 4 hr. of a day?
6. 3 hr. of 1 da.?
7. 12 hr. of a day?
8. 3 hr. of 1 hr.?
9. 1 da. of a week?
10. 3 ft. of 8 ft.?
11. 5 da. of a week?
12. 2 yd. of 8 ft.?
13. 3 in. of a foot?
14. 40 cents of \$1?
15. 9 in. of a foot?
16. 70 cents of \$1?
17. 20 min. of an hour?
18. 1500 lb. of 1 T.?
19. 40 min. of an hour?
20. 3000 lb. of 1 T.?

DRILLS

For Limited Time

300. *First:* Begin at the top of a column and work down (*e.g.* 5 is what per cent of 5? 5 is what per cent of 10? 5 is what per cent of 20? etc.).

Second: Begin at the left and work to the right (*e.g.* 5 is what per cent of 5? 10 is what per cent of 5? 20 is what per cent of 5? etc.).

	Base	5	10	20	40	50	100
1.	5	? %	? %	? %	? %	? %	? %
2.	10	? %	? %	? %	? %	? %	? %
3.	20	? %	? %	? %	? %	? %	? %
4.	30	? %	? %	? %	? %	? %	? %
5.	40	? %	? %	? %	? %	? %	? %
6.	50	? %	? %	? %	? %	? %	? %
7.	60	? %	? %	? %	? %	? %	? %
8.	80	? %	? %	? %	? %	? %	? %
9.	100	? %	? %	? %	? %	? %	? %
10.	1000	? %	? %	? %	? %	? %	? %

DRILLS

For Limited Time

301. Proceed as in the preceding drill.

First: 1 is what per cent of 1? of 2? of 3? etc.

Second: 1 is what per cent of 1? 2 is what per cent of 1? 3 is what per cent of 1? etc.

	BASE	1	2	3	4	6	10
1.	1	? %	? %	? %	? %	? %	? %
2.	2	? %	? %	? %	? %	? %	? %
3.	3	? %	? %	? %	? %	? %	? %
4.	4	? %	? %	? %	? %	? %	? %
5.	5	? %	? %	? %	? %	? %	? %
6.	6	? %	? %	? %	? %	? %	? %
7.	8	? %	? %	? %	? %	? %	? %
8.	10	? %	? %	? %	? %	? %	? %
9.	20	? %	? %	? %	? %	? %	? %
10.	25	? %	? %	? %	? %	? %	? %

ORAL PROBLEMS

302. 1. Butter cost 35 ct. a pound. The price was increased 5 ct. Find the per cent of increase.

2. Meat which cost 20 ct. a pound now costs 25 ct. a pound. Find the per cent of increase.

3. A book was marked 80 ct. The price was reduced 10 ct. Find the per cent of reduction.

4. A book which was marked 90 ct. was sold for 60 ct. Find the per cent of reduction.

5. One package weighed 10 lb. Another package weighed 15 lb.

(*a*) The weight of the first package was what per cent of the weight of the second package?

(*b*) The weight of the second package was what per cent of the weight of the first package?

6. A boy who sold newspapers began with $ 2.

At the end of the day he had $ 4. What per cent of his original amount had he gained?

7. There were 60 books on a shelf. 10 were arithmetics; the rest were readers.

(*a*) What per cent of the books were arithmetics?

(*b*) What per cent of the books were readers?

8. A book that cost 80 ct. is sold for 90 ct. What per cent of the cost is gained?

9. George receives $ 8 a week; Harry receives $ 4 a week. George's salary is what per cent of Harry's salary?

10. One store charges 25 ct. a pound for meat. A second store charges 20 ct. a pound for the same quality of meat. The first price is what per cent of the second price?

11. Lead pencils are bought at 10 ct. a dozen. What per cent of the cost is gained if they are sold at 1 ct. each?

12. 3 toys cost 20 ct. each. They are sold for 25 ct. each.

(*a*) What per cent of the cost is gained on 1 toy?

(*b*) What per cent of the cost is gained on the 3 toys?

13. A baseball club won 8 games and lost 2 games.

(*a*) What per cent of the games played were won?

(*b*) What per cent of the games played were lost?

14. 35 pupils of a class are present. 5 are absent. What per cent of the class is present? What per cent is absent?

15. Handkerchiefs that cost $4 a dozen are sold at 50 ct. each. Find the per cent profit.

WRITTEN PROBLEMS

303. 1. In a class of 43 pupils, 39 are promoted.

(*a*) What per cent are promoted?

(*b*) What per cent are not promoted?

2. A plot of ground is 75 ft. long and 175 ft. wide.

(*a*) The length is what per cent of the width?

(*b*) The width is what per cent of the length?

3. Mr. Jones, Mr. Frank, and Mr. Williams contribute to a fund. Mr. Jones contributes $150, Mr. Frank, $175, and Mr. Williams, $225.

(*a*) What per cent of the total amount is contributed by Mr. Jones? (*b*) By Mr. Williams?

4. A street is 400 ft. long and 20 yd.* wide. The length is what per cent of the width?

5. 1500 ft. is what per cent of 1 mi.?

6. The population of a town increased from 2500 to 3400. Find the per cent of increase.

7. Three years ago the population of a town was 3000. To-day the population is 750 greater. Find the per cent of increase.

* Note. Always reduce all terms to the same denomination before finding the rate per cent.

8. A man who is paid at the rate of $30 a week received $17.50. What per cent of the weekly wage did he receive?

9. $1000 is divided equally among 15 children. What per cent of the sum is given to each child?

10. I buy 2 doz. picture post cards at 1 ct. each and sell them at 2 ct. each. What per cent of the cost is gained?

11. Shoes that were marked at $4 a pair were sold for $2.50 a pair.

(*a*) Find the per cent of reduction on 1 pr.

(*b*) Find the per cent of reduction on 3 pr.

12. A baseball club won 104 games and lost 16 games. What per cent of the games did the club win?

13. Mr. Daly receives a salary of $3000. He spends 25% of it for rent, $600 for other expenses, and saves the rest. What per cent of his salary does he save?

14. Mr. Wood had 250 baseballs. He sold 20% of them the first day and 40% of the remainder the second day. What per cent of his original stock was left?

15. A school contains 750 boys and 650 girls.

(*a*) The boys form what per cent of the total register?

(*b*) The girls form what per cent of the total register?

Third Type Problem (Case 1)

304. To find the base when the percentage and the rate are given.

BY COMMON FRACTIONS

1. $\frac{3}{8}$ of the number of boys in a class is 15. How many boys are in the entire class?

Process

$\frac{3}{8}$ of the no. in the class = 15,
$\frac{1}{8}$ of the no. in the class = 5,
$\frac{8}{8}$ or the no. in the class = 40,

or (shorter method),

$15 \div \frac{3}{8} = \overset{5}{\cancel{15}} \times \frac{8}{\cancel{3}} = 40$. *Ans.*

BY PERCENTAGE

2. $37\frac{1}{2}\%$ of the number of boys in a class is 15. How many boys in the entire class?

Process

$37\frac{1}{2}\%$ of the number in class = 15.

Therefore $\frac{3}{8}$ of the number in the class equals 15.

The solution is the same as in problem 1.

The number in class = $15 \div \frac{3}{8}$, or $\overset{5}{\cancel{15}} \times \frac{8}{\cancel{3}} = 40$. *Ans.*

BY DECIMAL FRACTIONS

3. .28 of the distance traveled by a boat is 560 mi. Find the entire distance traveled by the boat.

Process

.28 of the distance equals 560 mi. Therefore the total distance equals

$560 \div .28 = 2000$. *Ans.*

BY PERCENTAGE

4. 28% of the distance traveled by a boat is 560 mi. Find the entire distance traveled by the boat.

Process

28% = .28.

Therefore the solution is the same as in problem 3.

$560 \div .28 = 2000$ mi. *Ans.*

305. Rule. **To find the base: Divide the percentage by the rate.**

Equation and Formula.

$$\textbf{Base} = \frac{\textbf{Percentage}}{\textbf{Rate}};\ B = P \div R.$$

Note. If the rate can be reduced easily to a common fraction, reduce the rate to a common fraction, invert the fraction and multiply, as in example 2, page 191. If the rate cannot be reduced easily to a common fraction, express the rate as a decimal fraction and proceed as in division of decimal fractions, as in example 4, page 191.

306. Proof. Check the answers by Type 1 (p. 167), or by Type 2 (p. 183).

ORAL EXERCISES

307. Find the number if

1. 50% of it $= 5$.
2. $16\frac{2}{3}\%$ of it $= 10$.
3. $33\frac{1}{3}\%$ of it $= 9$.
4. $37\frac{1}{2}\%$ of it $= 6$.
5. 10% of it $= 15$.
6. $87\frac{1}{2}\%$ of it $= 14$.
7. 20% of it $= 8$.
8. 50% of it $= 4\frac{1}{2}$.
9. $12\frac{1}{2}\%$ of it $= 6$.
10. 75% of it $= 9$.

ORAL EXERCISES

1. 5 is 20% of what number?
2. 9 is 30% of what number?
3. 6 is 75% of what number?
4. $3\frac{1}{2}$ is 25% of what number?
5. $\frac{1}{4}$ is $12\frac{1}{2}\%$ of what number?
6. $\frac{3}{4}$ is $33\frac{1}{3}\%$ of what number?
7. $\frac{2}{3}$ is 50% of what number?

8. 60 is $16\frac{2}{3}\%$ of what number?

9. 48 is 150% of what number?

10. 20% of a number = 30; find 40% of the number.

11. $12\frac{1}{2}\%$ of a number = 5; find $62\frac{1}{2}\%$ of the number.

12. 10% of a number = 9; find 35% of the number.

DRILLS

For Limited Time

308. *First:* Begin at the top of a column and work down (*e.g.* 3 is 10% of ? 4 is 10% of ? 5 is 10% of ? etc.).

Second: Begin at the left and work to the right (*e.g.* 3 is 10% of ? 3 is 20% of ? 3 is 25% of ? etc.).

	PERCENTAGE	10% of	20% of	25% of	50% of	$33\frac{1}{3}\%$ of	$16\frac{2}{3}\%$ of
1.	3 is	?	?	?	?	?	?
2.	4 is	?	?	?	?	?	?
3.	5 is	?	?	?	?	?	?
4.	6 is	?	?	?	?	?	?
5.	8 is	?	?	?	?	?	?
6.	10 is	?	?	?	?	?	?
7.	12 is	?	?	?	?	?	?
8.	20 is	?	?	?	?	?	?
9.	25 is	?	?	?	?	?	?
10.	30 is	?	?	?	?	?	?

ORAL DRILLS

For Limited Time

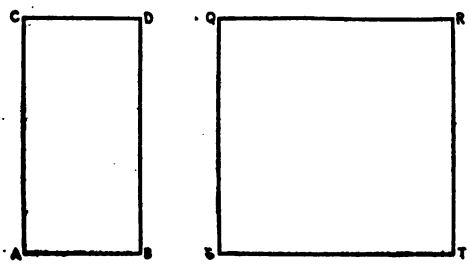
309. Proceed as in art. 308.

	Percentage	25% of	50% of	75% of	$33\frac{1}{3}$% of	$66\frac{2}{3}$% of	$12\frac{1}{2}$% of
1.	12 is	?	?	?	?	?	?
2.	24 is	?	?	?	?	?	?
3.	36 is	?	?	?	?	?	?
4.	60 is	?	?	?	?	?	?
5.	48 is	?	?	?	?	?	?
6.	72 is	?	?	?	?	?	?
7.	144 is	?	?	?	?	?	?
8.	120 is	?	?	?	?	?	?
9.	180 is	?	?	?	?	?	?
10.	240 is	?	?	?	?	?	?

ORAL PROBLEMS

310. 1. The line *AB*, which is 25% of *AC*, is 4 in. long. How long is *AC*?

2. The area of *ABCD* is 50% of the area of *QRST*. If *ABCD* contains 125 sq. in., find the area of *QRST*.



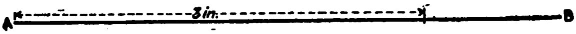
Hint. Indicate as nearly as possible on *QRST* the area of *ABCD*.

3. A boy can walk 4 blocks in $12\frac{1}{2}\%$ of an hour. How many blocks can he walk in an hour?

4. 20% of the water in this jar is 2 qt. How many quarts does the jar hold?

HINT. Indicate as nearly as possible the height of water in the jar.

5. 75% of the distance from A to B is 3 mi. Find the total distance from A to B.



6. Harry has 20 ct. This is 25% of the amount George has. How much has George?

7. William has 50 ct. This is 200% of the amount John has. How much has John?

8. $12\frac{1}{2}\%$ of the number of boys in a class is 5. How many boys are in the class?

9. Find the height of a flagpole if $16\frac{2}{3}\%$ of the height equals 6 ft.

10. The width of a room is $66\frac{2}{3}\%$ of the length. The width is 20 ft. Find the length.

WRITTEN PROBLEMS

311. 1. A train can travel 10.8 mi. in 18% of an hour. How far can it travel in 1 hr.?

2. The height of a flagpole is 78% of the height of a building. If the pole is 27.3 ft. high, how high is the building?

3. There are 1000 pupils in one school. This is 250 % of the number in a second school. How many pupils are there in the second school?

4. The population of a city is 288,000. This is 72 % of the population of a second city. Find the population of the second city.

5. 92 % of the number of pupils in a school equals 2024. How many pupils are there in the school?

6. Two men, A and B, were candidates for an office. A received 7500 votes. This was 150 % of the number B received.

(*a*) Who was elected, A or B? Why?

(*b*) How many votes did B receive?

7. The population of Buffalo is 190 % of the population of Rochester. The population of Buffalo is about 425,600.

(*a*) Which city has the larger population?

(*b*) Find the population of Rochester?

8. A printer has an order to print 1750 cards. If it takes 5 hr. to print 65 % of them, how long will it take to print all the cards?

9. 12,000 writing pencils were sold. This is 120 % of the number of drawing pencils sold. How many drawing pencils were sold?

10. If it requires $14\frac{1}{2}$ da. to build 85 % of a wall, how many days will be required to build the entire wall?

11. By going at the rate of $38\frac{1}{4}$ mi. per hour for 12 hr., a train traveled 85% of the distance between two cities. Find the total distance between the cities.

Third Type Problem (Case 2)

312. To find the base when the amount and the rate of increase are given.

WHEN THE RATE IS EASILY REDUCIBLE TO A COMMON FRACTION

1. A boy's salary was increased to \$14. This is $16\frac{2}{3}$% more than he received last week. How much did he receive last week?

Process

Represent original salary by 100%.
The increase in salary = $\underline{16\frac{2}{3}\%}$ of the original salary.
The new salary = $116\frac{2}{3}\%$ of the original salary.
Therefore, $116\frac{2}{3}\%$, or $\frac{7}{6}$ of the original salary = \$14.

$$\text{Original salary} = \$14 \div \frac{7}{6} = \$\overset{2}{\cancel{14}} \times \frac{6}{\cancel{7}} = \$12. \quad \textit{Ans.}$$

Or

Represent original salary by $\frac{6}{6}$.
Increase in salary = $\frac{1}{6}$ of the original salary.
New salary = $\frac{7}{6}$ of the original salary.
Original salary = $\$14 \div \frac{7}{6} = \$12$. *Ans.*

WHEN THE RATE IS NOT EASILY REDUCIBLE TO A COMMON FRACTION

2. In December, a store employed 15% more clerks than in November. If it employed 345 clerks during December, how many did it employ during November?

Process

Represent the number employed by	100 %
The increase during November	= 15 %
The number employed during December	= 115 %

of the number employed during November.

Therefore, 115 % of the original number = 345.

Number employed during November = 345 ÷ 1.15 = 300 clerks. *Ans.*

313. Rule. **To find the base: Divide the amount by 1 plus the rate.**

Equation and Formula.

Base = Amount ÷ (1 + Rate), $B = A \div (1 + R)$.

314. Proof. Check the answers by Type 1 (p. 177, to find the amount), or by Type 2 (p. 183).

Third Type Problem (Case 3)

315. To find the base when the difference and the rate of decrease are given.

When the Rate is easily reducible to a Common Fraction

1. After selling 20% of his stock, a storekeeper has 48 books left. How many books had he at first?

Process

Represent the original number of books by	100 %
Number books sold	= 20 % of original number.
Number books left	= 80 % of original number.

Therefore,

80 % or $\frac{4}{5}$ of the original number = 48.

$\frac{1}{5}$ of original number = $\frac{1}{4}$ of 48 = 12.

$\frac{5}{5}$ of original number = 5 × 12 = 60,

or

Shorter Form of Explanation

$\frac{4}{5}$ of original number = 48.

The original number $= 48 \div \frac{4}{5} = \overset{12}{\cancel{48}} \times \frac{5}{\cancel{4}} = 60$ books. *Ans.*

When the Rate is not easily reducible to a Common Fraction

2. A farmer sent a number of barrels of apples to the city. 18% of them were destroyed. The remainder, 410 bbl., were in good condition. How many barrels were sent by the farmer?

Process

Represent the total number of barrels by	100 %	
The number destroyed	= 18 %	of the total number.
The number in good condition	= 82 %	of the total number.
Therefore, 410 bbl.	= 82 %	of the total number.
Therefore, total number	= 410 ÷ .82 = 500 bbl. *Ans.*	

Rule. **To find the base: Divide the difference by 1 minus the rate.**

Equation and Formula.

$$\textbf{Base} = \textbf{Difference} \div (\textbf{1} - \textbf{Rate}),$$

$$B = D \div (1 - R).$$

316. Proof. Check the answers by Type 1 (p. 179, to find the difference), or by Type 2 (p. 183).

ORAL PROBLEMS

317. 1. On promotion day, 10 % of the pupils were not promoted. If 45 pupils were promoted, what was the total number before promotion?

2. A boy sold $12\frac{1}{2}$ % more papers to-day than yesterday. He sold 45 papers to-day. How many did he sell yesterday?

3. The price of butter was increased 10 %. If butter costs 33 ct. per pound now, what did it cost before the increase in price?

4. A boy's salary was increased to $ 9 a week. This was an increase of 20 %. What was the salary before the increase?

5. On Monday, a storekeeper had a number of books. On Tuesday, he had only 34 books left. This was $66\frac{2}{3}$ % less than the number he had on Monday. How many books had he sold?

6. One quality of coffee costs 35 ct. a pound. This is $12\frac{1}{2}$ % cheaper than a better quality. Find the cost of a pound of the better quality.

7. A statue and its base weigh 80 lb. This is 25 % more than the weight of the statue. Find the weight of the statue.

8. A boy weighed 120 lb. in September. This was $14\frac{2}{7}$ % more than he weighed in June. How much did he weigh in June?

WRITTEN PROBLEMS

318. 1. The price of a suit of clothes this year is $ 27.25, which is 9 % greater than the price last year. Find the price last year.

2. The area of Mr. Howell's farm is 42 A. This is 16 % less than the area of Mr. Holt's farm. Find the area of Mr. Holt's farm.

3. 14 % of the length of a log is 3.36 ft. Find the length of the log.

4. The price of a pound of butter is 24 % greater than it was last month. Find the cost of 5 tubs, each containing 52 lb., at last month's price, if the present price of butter is 31 ct. per pound.

5. The population of Massachusetts increased 20 % during the last 10 yr. If the present population is 3,366,000, what was the population 10 yr. ago?

6. The population of Iowa decreased .3 % during the last 10 yr. If the present population is 2,134,000, find the decrease during the last 10 yr.

7. The ocean steamship *Olympic* is 882 ft. long. This is 11.7 % more than the length of the *Lusitania*. How much longer is the *Olympic* than the *Lusitania?*

8. In 1911 a company sold 20 % more lamps than in 1910. In 1912 the company sold 20 % more lamps than in 1911. If 720 lamps were sold in 1912,

(*a*) How many lamps were sold in 1911?

(*b*) How many lamps were sold in 1910?

9. A horse was sold for $ 595. This was a loss of 15 % on the cost. Find the cost.

10. After using 84 % of his flour, a baker had 40 bbl. left.

(*a*) How many barrels had he at first?

(*b*) How many barrels did he use?

11. A number of boys took part in a race. After a while 25 % of them dropped out. Later, 40 % of the remainder dropped out. There were 18 boys left. How many started on the race?

12. The number of houses sold in a city in 1912 was 20 % less than the number sold in 1911; but the number sold in 1911 was 10 % greater than the number sold in 1910. If the number sold in 1912 was 1056,

(*a*) How many were sold in 1911?

(*b*) How many were sold in 1910?

13. The number of boys in a school is 50 % greater than the number of girls. The number of boys is 1440. Find the number of girls.

MISCELLANEOUS ORAL PROBLEMS

1. A building is 600 ft. high. Another building is 40% as high. How high is the second building?

2. One piano costs $1200. Another piano costs $66\frac{2}{3}\%$ as much. Find the cost of the second piano.

3. A baseball team played 60 games. It won $66\frac{2}{3}\%$ of them. How many games were lost?

4. Harry had 60 words to spell in an examination. He spelled only 75% of them correctly. How many words did he spell incorrectly?

5. A class contains 40 pupils. 30 of them are boys.

(*a*) What per cent of the class are boys?

(*b*) What per cent of the class are girls?

6. A workman who was sick $33\frac{1}{3}\%$ of a week received $20 for the days he worked. How much would he have received if he had worked the whole week? (6 da. = 1 wk.)

7. A boy received $8 a week. The employer gave him an increase of salary of $37\frac{1}{2}\%$. How much does the boy receive per week now?

8. A chair which was marked at $12 was sold at a reduction of $33\frac{1}{3}\%$. Find the selling price.

9. 40% of the weight of an iron bar equals 10 lb. Find the weight of the bar.

10. A man spends $37\frac{1}{2}$% of his weekly salary. If he spends $9, how much is his weekly salary?

11. Mary has 40 ct. This is $62\frac{1}{2}$% of the amount Jane has. How much has Jane?

12. George earned $10 a week. His employer increased his salary $2. What was the per cent of increase?

13. A boy's salary was increased from $10 to $14 per week. Find the per cent of increase.

14. A merchant had 150 pr. of shoes. He sold 60% of them. How many pair were not sold?

15. A class contained 40 pupils. 30% of them were boys. Find the number of girls in the class.

16. How many gallons of water does a tank hold if, after 75% of the water has been used, there are still 22 gal. in the tank?

17. 10% of the pupils in a class were not promoted. How many pupils were in the class, if 4 pupils were not promoted?

18. The length of a room is 30 ft.; the width is 24 ft. The width is what per cent of the length?

19. Chairs were reduced $12\frac{1}{2}$% in price and were sold at $2.10 each. How much was saved by buying 6 chairs at the lower price?

20. In 1900 there were 100,000 people in Omaha; in 1910 the population was 20% greater. How many more people lived in Omaha in 1910 than in 1900?

21. The Ohio River is 950 mi. long. The Hudson River is 30% as long. How long is the Hudson River?

22. A man receives $12 a week. He loses $4 by sickness. What per cent of his salary does he lose?

23. The height of a church is 270 ft. This is 10% less than the height of an adjoining building. How high is the building?

24. 20 words were given in a spelling lesson. George had 19 correct; William had 15 correct. Find the per cent for each boy.

25. A book that cost 80 ct. is sold at a profit of 20 ct. What per cent of the cost is gained?

26. The expenses of a storekeeper are $66\frac{2}{3}$% of his receipts. The expenses are $100. Find the receipts.

27. An express train goes at the rate of 60 mi. per hour. This is 20% greater than the rate of a local train. Find the rate of the local train.

28. Mr. Jones raised 500 bbl. of apples and 800 baskets of peaches. Mr. Smith raised 70% as many barrels of apples and 200% as many baskets of peaches.

(*a*) How many barrels of apples did Mr. Smith raise?

(*b*) How many baskets of peaches?

29. 240 T. of coal were burned in a furnace in one month. During the next month, $87\frac{1}{2}\%$ as much coal was burned. How much coal was burned during the second month?

30. The distance between two villages is 150 mi. What per cent of the distance is left, after 90 mi. have been traveled?

31. A company employed 300 men. During the busy season it increased the force by $16\frac{2}{3}\%$. How many additional men were employed?

32. A salesman sold \$300 worth of goods on Monday; 40% as much on Tuesday; and 15% as much on Wednesday. How much did he sell on Tuesday? On Wednesday?

33. The width of a rectangular plot of ground is 200 ft., which is $33\frac{1}{3}\%$ less than the length. Find the length.

34. George had \$1.20. He spent 25% of his money. How much money was left?

35. A newsboy buys 10 papers for 5 ct. He sells them at 1 ct. each. Find his per cent profit.

36. A newsboy had 50 papers. He sold all except 5. What per cent of his papers did he sell?

37. A man who was sick for 25% of a week received \$21 for the time he worked. How much money did he lose by illness?

38. A grocer sold $83\frac{1}{3}$% of his oranges He still has 30 oranges. How many did he sell?

39. If a workman can build 40% of a wall in one day, how long will it take him to build the entire wall?

40. If a workman can build 30% of a wall in 3 da., how long will it take him to build the entire wall?

41. How many minutes are there in 75% of an hour? In 120% of an hour? In 100% of an hour?

42. How many more minutes are there in 75% of an hour than in 50% of an hour?

43. The length of a plot of ground is 300% of its width. The width is 120 ft. Find the length.

44. The cost of a piano is 150% of the cost of a bookcase. The piano costs \$600. Find the cost of the bookcase.

45. Books were marked \$1 each. Later the price was reduced 30%. Find the cost of 5 books at the reduced price.

46. A statue was made of copper and zinc. 95 lb. of the statue were copper; 5 lb. were zinc.

(*a*) What per cent of the weight of the statue was copper?

(*b*) What per cent of the weight of the statue was zinc?

47. After completing $87\frac{1}{2}\%$ of a journey, a boy had 1 mi. to travel. How far had he traveled?

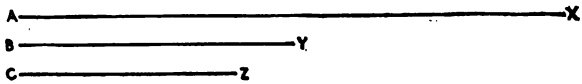
48. William lost 50% of his marbles in one game, and 50% of all he had left in a second game. He then had 40 marbles left. How many had he lost?

49. One village contains 800 people. Another village contains 75% as many people. How many more people are there in the first village than in the second?

50. James had 120 marbles. He lost 25% of them the first day and 50% of the remainder the second day. How many marbles are left?

51. The length of AX is 200% of the length of BY. The length of BY is 120% of the length of CZ. If AX is 24 ft. long, how long is CZ?

52. If CZ is 15 ft. long, how long is BY? How long is AX?



MISCELLANEOUS PROBLEMS

(Use pencil and paper only when necessary.)

1. One building is 700 ft. high. A second building is 82 % as high. How high is the second building?

2. In a school containing 850 pupils, 824 are present.

(*a*) What per cent are present?

(*b*) What per cent are absent?

3. If a train travels 38 % of the distance between two cities in an hour, how long will be required to travel the entire distance?

	City	Approx. Pop. 1900	Increase in 10 yr.	Pop. in 1910
4.	Cincinnati	325,500	11.6 %	?
5.	Cleveland	381,700	47 %	?
6.	New York	3,437,000	38.7 %	?
7.	Chicago	1,698,500	38.8 %	?
8.	Denver	133,500	59.4 %	?

9. Best quality tea costs 56 ct. per pound. This is 12 % more than the cost of the second quality. Find the cost of the second quality per pound.

10. Three men were candidates for an office. A received 2500 votes; B, 3500; C, 4000 votes.

(*a*) B's vote was what per cent of C's vote?

(*b*) What per cent of the total vote was cast for A?

11. After having traveled 47 % of the distance between two cities, a train still had 91 mi. to go. Find the distance between the cities.

12. Mr. Jordan spent \$1500. 32 % was spent for a horse, 15 % for a wagon, and the rest for tools. How much was spent for tools?

13. 58 desks were bought for \$25 each. They were sold at a gain of 16 %. How much was received for them?

14. 64% of the cost of a desk is \$115.20. Find the cost of the desk.

15. One river is 950 mi. long. Another river is 42 % as long. How long is the second river?

16. An ocean steamship is 800 ft. long. This is $114\frac{2}{7}$ % of the height of the Singer building in New York. How high is the Singer building?

17. 230 da. is what per cent of 1 yr. ($365\frac{1}{4}$ da.)?

18. \$15.50 is what per cent of \$75?

19. The population of Virginia increased 11.2 % during the last 10 yr. If the present population is 2,168,400, find the increase during the last 10 yr.

20. 82 % of the water in a tank weighs 205 lb. Find the weight of all the water in the tank.

21. How many feet are there in 62 % of a mile?

22. A manufacturer of shoes made 350 pr. of children's shoes, and 120 % as many pairs of ladies' shoes. How many pairs of ladies' shoes were made?

23. A company employed 220 men during the regular season. During the busy season it increased its force by 35 %. How many men were employed during the busy season?

24. A factory made 1189 gross of pencils during June; this was 18 % less than the number made during May. Find the total number of pencils made during May and June.

25. The expenses of a manufacturer are 92 % of his receipts. The expenses are $ 5704. Find the receipts.

26. 40 words were given in an examination in spelling. George spelled 40 correctly; Mary spelled 38 correctly; Harry, 35 correctly. Find the per cent of each pupil.

27.

	Register	No. Present	Per cent Present	Per cent Absent
Class 8 B	42	39	?	?
Class 8 A	41	40	?	?
Class 7 B	45	43	?	?
Class 7 A	44	44	?	?

28. In a school containing 1800 children, 3 children were late in the morning and 2 children were late in the afternoon.

(*a*) What per cent of the children were late in the morning?

(*b*) What per cent of the children were late in the afternoon?

29. A piece of cloth contained 30 yd. 75% of it was sold at $2 a yard; the remainder was sold at $3 a yard. How much was received for the entire piece of cloth?

30. During July the receipts of a store were $2750; during August, $2250. Find the per cent of decrease.

31. A school contains 800 boys. This is 125% of the number of girls.

(*a*) Are there more girls or boys in the school?

(*b*) Find the number of girls.

32. A manufacturer spent $12,000. 74% of this money was paid to his employees. The remainder was spent for materials.

(*a*) How much was paid to the employees?

(*b*) How much was spent for materials?

33. The workmen in a factory obtained an increase of 9% in wages. If the owners of the factory pay $2997.50 in wages after the increase, how much did they pay before the increase in wages?

34. The number of pupils present in school is 96% of the number on register. How many pupils are on register, if the number present is 720?

35. In one year a field produced 900 bu. of wheat. The next year it produced 175 bu. less. Find the per cent of decrease.

36. 12,160 persons or 8 % of the population of a city were born in a foreign country. Find the population of the city.

37. In 1900 the population of a city was 375,000. In 1910 the population was 22 % greater. Find the increase in population.

38. A man who receives $ 2500 a year spends 23 % of it for rent, 45 % for food and clothing, 12 % for other expenses, and saves the remainder.

(*a*) How much is spent for rent?

(*b*) How much is spent for food and clothing?

(*c*) How much is saved?

39. Pianos which were marked at $525 each were reduced 12 %. Find the reduced price.

40. A house was sold for $575. This was a gain of 15 % on the cost. Find the cost.

41. The profits of a business man in 1912 were $ 5200. This was $8\frac{1}{3}$ % greater than the profits in 1911, and 30 % greater than the profits in 1910.

(*a*) Find the profits in 1911.

(*b*) Find the profits in 1910.

42. A ton of coal is put into 35 bags, each containing an equal amount. What per cent of the ton is in each bag?

43. A post-office clerk sold the following:

1000 one-cent stamps.

30 % more two-cent stamps than one-cent stamps.

20 % less post cards (1 cent each) than two-cent stamps.

(*a*) How much was received for the post cards?

(*b*) How much was received in all?

44. I buy 4 doz. picture post cards at 1 ct. each and sell them at 2 ct. each. What per cent of the cost is gained?

45. The value of a building is $ 38,500. This is 23 % less than the value of an adjoining building. Find the value of the second building.

46. 500 people were employed in a factory. 72 % of them were men; 22 % of them were women; the rest were children. How many men were employed? How many women? How many children?

47. A merchant bought 150 overcoats. He paid $ 12 each for 40 % of them and $ 15 each for the remainder. How much did he pay for all the overcoats?

48. Apples that cost 2 for 1 ct. are sold at 1 ct. each. What per cent of the cost is gained?

49. Oranges that cost 1 ct. each are sold at the rate of 3 for 5 ct. What per cent of the cost is gained?

50. Lead pencils that cost 20 ct. a dozen are sold for 2 ct. each.

(*a*) What per cent is gained on 1 doz.?

(*b*) What per cent is gained on 3 doz.?

51. A man who desired to buy a house saved \$850 a year for 6 yr. He thus saved 68 % of the cost of the house. How much more money must he save to buy the house?

52. By sailing at the rate of 30 mi. an hour, a boat completed $56\frac{1}{4}$ % of its journey in $4\frac{1}{2}$ hr. How many more miles must the boat sail to complete the whole distance?

53. A baseball team lost 35 % of its games. If it won 78 games, how many games were lost?

54. 60 % of the population of one town equals 80 % of the population of a second town. If the population of the second town is 3600, find the population of the first town.

COMMISSION

For Reading and Discussion

319. Commission may be earned in various ways.

1. A firm that has a large number of customers may employ collectors to collect bills that are due. These men may be paid a commission on the amount they collect.

2. Agents are often employed to buy or sell goods; *e.g.* a contractor may employ an agent to buy 100 teams of horses; or a firm may employ agents to sell goods. These agents are paid a percentage of the value of the goods bought or sold.

3. Agents, called brokers, are employed to buy and sell real estate. They are paid a percentage of the value of the property bought or sold. The money they receive is usually called **brokerage**.

4. Salesmen are sometimes paid a commission on the amount of goods they sell. Sometimes they are paid a weekly salary and a commission.

5. Agents who buy or sell stocks or bonds, agents who draw up insurance policies, and agents who procure loans, charge a commission or brokerage. Commissions are paid also to architects, builders, contractors, auctioneers, and other persons who render valuable services to their principals.

To find the Commission or Brokerage

320. 1. Mr. Graham sells $400 worth of goods for Mr. Brown. His rate of commission is 6 %.

(*a*) How much is his commission?

(*b*) How much does he send to Mr. Brown?

Commission or Brokerage is always calculated on the value of the goods bought or sold. Mr. Graham's commission is 6 % of $400.

Process

$400		$400	Value of goods.
.06		24	Commission.
$24.00	Commission.	$376	Amount remitted to Mr. Jones.

Mr. Graham is called a **Commission Merchant** or **Commission Agent**.

2. A contractor orders his agent to buy 400 horses at \$75 each. The agent charges 4% commission.

(*a*) Find the commission.

(*b*) How much does the contractor pay for the horses?

Process

400 horses at \$75 each, cost \$30,000.

\$30,000	\$30,000 Cost of horses.
.04	1200 Commission
\$1200.00 Commission.	\$31200 Total Cost.

Therefore the method of finding the Commission is similar to the method of finding a per cent of a number (First Type Problem, p. 167).

321. The Value of the Goods corresponds to the **Base.**

The Rate of Commission / **The Rate of Brokerage** corresponds to the **Rate Per Cent.**

The **Commission** or **Brokerage** corresponds to the **Percentage.**

The Amount remitted (Net Proceeds) corresponds to the **Difference.**

The Total Cost (in problem 2) corresponds to the **Amount.**

RULE. **To find the commission or brokerage: Multiply the value of the goods bought or sold by the rate expressed as a decimal or a common fraction.**

ORAL EXERCISES

322. Find the commission in each of the following:

	Value of Goods	Rate of Commission		Value of Goods	Rate of Commission
1.	$800	10 %	11.	$1500	6 %
2.	$1000	10 %	12.	$700	3 %
3.	$500	5 %	13.	$1640	$12\frac{1}{2}$ %
4.	$1200	5 %	14.	$2000	2 %
5.	$250	4 %	15.	$5000	$1\frac{1}{2}$ %
6.	$400	4 %	16.	$10,000	$2\frac{1}{2}$ %
7.	$5000	2 %	17.	$2500	5 %
8.	$600	$1\frac{1}{2}$ %	18.	$1850	5 %
9.	$900	1 %	19.	$4000	$\frac{1}{8}$ %
10.	$1400	$\frac{1}{2}$ %	20.	$1600	$\frac{1}{8}$ %

WRITTEN PROBLEMS

323. 1. A firm employs Mr. A to collect bills. On Monday he collects $250; on Tuesday, $400; on Wednesday, $150; on Thursday, $500; on Friday, $100; and on Saturday, $50. Find Mr. A's commission, if he receives $1\frac{1}{2}$% of the amount he collects.

2. An agent sells $250 worth of apples for a farmer. He charges 5% commission.

(*a*) How much does he receive as commission?

(*b*) How much does he remit to the farmer?

3. An agent buys $500 worth of goods for Mr. B, and charges $3\frac{1}{2}\%$ commission.

(*a*) Find his commission.

(*b*) Find the gross cost.

4. An agent sold a house for $15,000. He charged $2\frac{1}{2}\%$ commission. Find his commission and the amount of money he sent to the owner.

5. An agent sold 60 lots at $500 a lot. Find his commission at $2\frac{1}{2}\%$.

6. Mr. Jones sent his agent 200 bbl. of apples, 300 bbl. of potatoes, and 150 bbl. of cabbages.

The agent sold the apples at $2.75 per barrel, the potatoes at $1.50 per barrel, and the cabbages at 85 ct. per barrel.

(*a*) Find his commission at 4%.

(*b*) How much did Mr. Jones receive?

7. Mr. Smith sent his agent 450 lb. of poultry, 75 crates of eggs, and 150 lb. of butter.

The agent sold the poultry at 15 ct. per pound, the eggs at $1.44 per crate, and the butter at 22 ct. per pound.

Find the agent's commission at 4%. Find the net proceeds.

8. Mr. Brown sent the following to his agent: 150 bbl. of apples, 225 bbl. of potatoes, 75 bbl. of turnips. The agent sold $\frac{1}{2}$ the apples at $2.50 per barrel and the remainder at $3 per barrel;

he sold the potatoes at $1.75 per barrel, and the turnips at $1.25 per barrel.

Find the agent's commission at 6%. Find the net proceeds.

9. Mr. Herman is a salesman for a sewing machine company. He receives a commission of 6% on all machines he sells. During the first week he sold 10 machines; during the second week, 14 machines; during the third week, 20 machines.

Find his total commission for three weeks if the machines are sold at $32 each.

10. Mr. Williams is employed by a book company. He receives a salary of $30 a week and a commission of 10% on all sales. During the first week he sold 20 books; during the second week, 35 books; during the third week, 50 books.

Find the total amount earned by Mr. Williams in three weeks, if the books were sold at $2.50 each.

To find the Rate of Commission or Brokerage

324. 1. An agent sells $500 worth of goods and retains $10 for his commission. Find the rate of commission.

The rate is always calculated on the value of the goods bought or sold.

Process. The rate per cent $= \frac{10}{500} = \frac{1}{50} = 2\%$. *Ans.*

2. An agent who buys $1250 worth of goods charges $37.50 for his services. Find the rate of commission.

Process

The rate per cent $= \frac{37.50}{1250} = 37.50 \div 1250 = 3\%$. *Ans.*

Therefore, the method of finding the rate of commission or brokerage is similar to the method of finding what per cent one number is of another (Second Type Problem, p. 183).

325. To find the rate of commission or brokerage.

RULE. **Divide the commission or brokerage by the value of the goods bought or sold.**

EXERCISES

326. Find the rate of commission in each of the following:

	Value of Goods	Commission		Value of Goods	Commission
1.	$ 200	$ 40	11.	$ 500	$ 10
2.	$ 400	$ 40	12.	$ 600	$ 3
3.	$ 50	$ 10	13.	$ 800	$ 20
4.	$ 250	$ 10	14.	$ 300	$ 10.50
5.	$ 1000	$ 20	15.	$ 640	$ 80
6.	$ 60	$ 3	16.	$ 1000	$ 25
7.	$ 320	$ 80	17.	$ 75	$ 17
8.	$ 120	$ 20	18.	$ 950	$ 9.50
9.	$ 400	$ 4	19.	$ 800	$ 2
10.	$ 600	$ 9	20.	$ 1600	$ 2

WRITTEN PROBLEMS

327. 1. An agent collected bills amounting to \$5000 and received \$125 for his commission. Find his rate of commission.

2. A commission merchant sold goods valued at \$750. Find his rate of commission, if he received \$37.50 for his services. Also find the amount remitted to the owner.

3. A real estate broker sold a house for \$20,000. His brokerage was \$300. Find the rate of brokerage or commission. How much did the owner receive?

4. An agent sold two houses, one for \$15,000 and one for \$17,500. What rate of commission did he charge, if he received \$1625?

5. An agent sold 40 sets of books, each set containing 6 vols., at \$3 a volume. Find the rate of commission, if the agent received \$43.20.

6. A salesman sold 20 tables at \$25 each and 120 chairs at \$1.50 each. Find the rate of commission, if the salesman received \$27.20.

7. A salesman is paid a weekly salary of \$25 and a commission on all the tables he sells. He sells 10 tables at \$35 each in one week. Find the rate of commission paid to him if his total earnings during the week were \$40.

To find the Value of Goods Bought or Sold when the Commission and the Rate are Given

328. 1. An agent receives $2\frac{1}{2}\%$ commission for selling a piece of land. Find the value of the land if the agent's commission was \$500.

Process

\$500 is $2\frac{1}{2}\%$ of the value of the land.
Therefore the value of the land = \$500 ÷ .02½ = \$20,000.

Therefore the method of finding the value of goods bought or sold is similar to the method of finding the base when a part of it is given (Third Type Problem, Case 1, p. 191).

To find the value of goods bought or sold

RULE. **Divide the commission or brokerage by the rate per cent expressed as a decimal or common fraction.**

EXERCISES

329. In each of the following, find the value of the goods bought or sold:

	Commission	Rate of Commission		Commission	Rate of Commission
1.	\$50	5%	7.	\$8	2%
2.	\$80	5%	8.	\$24	4%
3.	\$100	10%	9.	\$30	6%
4.	\$25	10%	10.	\$3	$1\frac{1}{2}\%$
5.	\$7.50	1%	11.	\$15	$2\frac{1}{2}\%$
6.	\$10	$\frac{1}{2}\%$	12.	\$10	$\frac{1}{8}\%$

WRITTEN PROBLEMS

330. 1. An agent received $200 for his commission for selling an automobile.

(*a*) Find the value of the automobile if the agent's commission was $12\frac{1}{2}\%$.

(*b*) How much did the firm receive for the automobile?

2. A real estate agent earned $3000 in commissions in a year by selling property. Find the value of the property sold, if the agent's commissions were $1\frac{1}{2}\%$ of the value of the property.

3. A commission merchant earned $28 by selling apples at $2 a barrel. His commission was 4% of the value of the apples sold. How many barrels of apples did he sell?

4. An agent earned $625 by selling pianos on commission, his rate of commission being 20%.

(*a*) Find the value of the pianos sold.

(*b*) How many pianos did he sell, if each piano cost $625?

5. A salesman is paid $20 a week and 5% commission for selling pictures. At the end of 2 wk. he receives $100. Find the value of the pictures sold.

331. To find the amount to be expended and the commission, when both are included in the amount sent by the principal to the agent

1. I send my agent $630 with instructions to deduct a commission of 5% and to buy books with

the balance. How much will be deducted as commission? How much will he invest in books?

Process

$630 includes money for books and money for commission at 5%.

Represent the money spent for books by 100%.

Represent the money spent for commission by 5% of money spent for books.

Then the amount sent to the agent equals 105% of the amount spent for books.

105% of amount spent for books = $630.

Amount spent for books = $630 ÷ 1.05 = $600.

$630 − $600 = $30 commission. *Ans.*

The method of solving this problem is similar to the method of finding the base when the base plus a part is given (Third Type Problem, Case 2, p. 197).

332. To find the amount to be expended for goods, when the amount sent to the agent includes both the commission and the money to be expended.

RULE. **Divide the total amount sent to the agent by 1 plus the rate of commission.**

PROBLEMS

333. 1. Mr. Jones sent $1947.50 to his agent with instructions to deduct his commission at $2\frac{1}{2}$%, and to expend the remainder for an automobile.

(*a*) How much money is spent for the automobile?

(*b*) How much does the agent keep for his commission?

2. An agent receives $20,600 to pay for the cost of horses and for his commission at 3 %.

(*a*) How much does he spend for the horses?

(*b*) How much does he deduct for his commission?

3. An agent receives $1001.25 with instructions to deduct his commission at $\frac{1}{8}$ %, and to expend the remainder for wheat.

(*a*) How much money does he spend for wheat?

(*b*) How much does he deduct for his commission?

(*c*) How many bushels of wheat does he buy at $1 per bushel?

4. An agent receives $6597.50 to include the cost of flour and his commission at $1\frac{1}{2}$ %. He buys 1000 bbl. of flour.

(*a*) How much money does he spend for flour?

(*b*) How much does he pay per barrel?

TRADE DISCOUNT OR COMMERCIAL DISCOUNT

For Reading and Discussion

334. Many manufacturers issue catalogues containing lists of the goods they sell and the prices at which the goods are sold. In these catalogues the goods are "listed" at a price greater than the firm expects to receive for them. The manufacturer then sends out discount lists, stating reductions or discounts from the catalogue or list prices.

For example, sewing machines may be listed at \$40 each, but the firm may allow a reduction or discount of 10% from the list price. The sewing machine, therefore, actually costs \$40 − \$4 or \$36.

\$40 is called the **List Price** or **Catalogue Price.**
4% is called the **Rate of Discount.**
\$4 is called the **Discount.**
\$36 is called the **Net Price** or **Actual Cost.**

There are many reasons why manufacturers follow this custom. It enables them to give larger discounts to those who buy in large quantities than to those who buy in small quantities; it also enables them to increase or decrease their rates of discount when prices change, without reprinting their catalogue.

Frequently two or more discounts are allowed. Automobiles may be listed at \$1500, with discounts of 20% and 5%. This does not mean a discount of 25%; it means a discount of 20% on \$1500, or \$300, making the price \$1200; and a further discount of 5% on \$1200 or \$60, making the net cost of the automobile \$1140.

Discounts are also given for payment before a bill is due. Usually a man who buys goods from one with whom he deals frequently is allowed 30 da. in which to make payment. If he pays

cash, he may be allowed a discount of 5% or 2%; if he pays the bill at the end of 10 da., he may be allowed a discount of 1%. Firms that buy large quantities of goods save considerable money each year by paying their bills promptly; they also obtain an excellent reputation among those with whom they deal.

335. To find the discount and the net price, when the list price and the rates of discount are given

ONE DISCOUNT

1. The list price of a piano is $650. The discount is 10%. Find the discount and the net cost.

Process

$\frac{1}{10}$ of \$650 = \$65 Discount. *Ans.*

\$650 – \$65 = \$585 Net Cost.

Or

$\frac{9}{10}$ of \$650 = \$585 Net Cost.

TWO OR MORE DISCOUNTS

2. The list price of a piano is $700. Discounts of 10% and 2% are allowed. Find the total discount and the net cost of the piano.

Process

$\frac{1}{10}$ of \$700 = \$70, Discount at 10%.

\$700 – \$70 = \$630 Price after deduction of first discount.

$$\begin{array}{r} \$630 \\ .02 \\ \hline \$12.60 \end{array}$$ Discount at 2%.

\$630 – \$12.60 = \$617.40 Net Cost.

\$700 – \$617.40 = \$82.60 Total Discount.

Or Shorter Method

$\frac{9}{10} \times \frac{98}{100} \times \$700 = \$617.40$. *Ans.*

There is another short method shown in the following:

3. Find the net cost of a phonograph listed at \$150, with discounts of 20% and 5%.

A discount of 20% leaves a net cost of 80%.
A discount of 5% leaves a net cost of 95%.
The net cost $= .80 \times .95 \times \$150 = \$114$. *Ans.*
Or $\frac{4}{5} \times \frac{19}{20} \times \$150 = 114$. *Ans.*
$\$150 - \$114 = \$36$ Discount. *Ans.*
Total discount $= \$150 - \$114 = \$36$. *Ans.*

Therefore, the method of finding the discount is similar to the method of finding a per cent of a number (First Type Problem, p. 167).

The method of finding the Net Price when one discount is allowed is similar to the method of finding the Difference (p. 179).

Rule. **To find the net price when more than one discount is allowed:**

Deduct the first discount from the list price and each subsequent discount from each successive remainder. The last remainder is the net price.

Note. The order in which discounts are subtracted does not affect the final result; *e.g.* Discounts of 10, 5, and 2 may be deducted in any order.

ORAL EXERCISES

336. In each of the following find (*a*) the discount, (*b*) the net cost.

ORAL EXERCISES

	List Price	Rates of Discount
1.	$500	20 and 10
2.	$1000	10 and 10
3.	$100	10 and 5
4.	$200	10 and 2
5.	$400	5 and 2

NOTE. 20 and 10 means 20% and 10%.

WRITTEN EXERCISES

337. Find the net cost and the total discount:

	List Price	Rate of Discount		List Price	Rate of Discount
1.	$1200	20 and 5	11.	$1500	25 and 2
2.	$500	20 and 10	12.	$2500	25 and 2
3.	$200	25 and 2	13.	$150	30 and 2
4.	$600	30 and 2	14.	$360.25	5 and 5
5.	$75	10 and 2	15.	$140.75	10 and 1
6.	$40	10 and 5	16.	$240	$12\frac{1}{2}$ and 2
7.	$85	5 and 2	17.	$975	$33\frac{1}{3}$ and 5
8.	$115	40 and 10	18.	$2160	$16\frac{2}{3}$ and 5
9.	$235	50 and 10	19.	$1580	10, 5, and 2
10.	$740	25 and 10	20.	$4260	10, 5, and 2

ORAL PROBLEMS

338. 1. A storekeeper buys $5000 worth of goods in a month. He is allowed 2% off for paying cash. How much is saved by paying cash?

2. A storekeeper buys \$ 2500 worth of furniture. He is allowed 2 % off for cash or 1 % off if he pays in 10 days. How much is saved by paying cash?

3. A mahogany table is listed at \$ 125, with a discount of 20 %. Find the net cost.

4. A set of books containing 12 vols. is listed at \$ 5 a volume. Find the net cost of the set, if a discount of $33\frac{1}{3}$ % is allowed.

5. Mr. Gray buys 30 doz. knives and 30 doz. forks at \$ 9 a dozen. Find the net cost if he receives a discount of 10 %.

WRITTEN PROBLEMS

339. Find the net cost of the following articles:

1. 6 chairs at \$ 2.50 each; discount of 25 %.

2. 30 books at \$ 3.50 each; discount of 40 %.

3. 40 doz. fountain pens at \$ 1.25 each; discounts of 30 % and 10 %.

4. 120 bbl. of flour at \$ 5.75 each; discounts of 20 % and 5 %.

5. 72 crates of eggs, 6 doz. in a crate, at 25 ct. a dozen; discount, 10 %.

Find the net cost and discount in each of the following:

6. William Furman bought of George Burns:

6 doz. books, Style A, @ \$ 6.60 per dozen.
12 doz. books, Style B, @ \$ 8.40 per dozen.
18 doz. books, Style C, @ \$ 9.60 per dozen.
Discounts 40 % and 10 %.

7. John Davis bought of William Chandler:

5 M envelopes, No. 5, @ $1.30 per M.
10 M envelopes, No. 6, @ $2.20 per M.
50 M envelopes, No. 7, @ $2.60 per M.
Discounts 5% and 2%.

8. Henry Henderson bought of George Wharton:

24 doz. lead pencils, American, @ $4.38 a dozen.
36 doz. lead pencils, Eagle, @ $4.37 a dozen.
60 doz. lead pencils, Dixon, @ $4.69 a dozen.
Discounts $1\frac{1}{2}$% and 1%.

9. Henry Roth bought of Harry Tower:

10 gross pens, No. 10, @ $ .60 per gross.
20 gross pens, No. 20, @ $ .75 per gross.
25 gross pens, No. 30, @ $1.10 per gross.
Discounts $2\frac{1}{2}$% and 2%.

10. James Murphy bought of James Dunn:

10 doz. rubber erasers, Circular, @ $5.40 per gross.
18 doz. rubber erasers, Triangle, @ $6 per gross.
18 doz. rubber erasers, Typewriter, @ $9 per gross.
Discounts 6%, 5%, and 1%.

11. Make out the bill, in proper form, for problem number 6. Find the net cost, allowing an additional discount of 2% for cash.

12. (*a*) Make out the bill, in proper form, for problem number 7. Find the cost, allowing an additional discount of 1 % for cash.

(*b*) Make out a check in payment of the bill.

13. (*a*) Make out the bill, in proper form, for problem number 8, allowing an additional discount of 2 %.

(*b*) Make out a check in payment of this bill.

14. Make out the bill, in proper form, for problem number 9, allowing an additional discount of 5 %.

15. Make out the bill, in proper form, for problem number 10, allowing an additional discount of 2 %.

16. Discounts of 20 % and 5 % were allowed from the list price of a canoe. If the canoe was listed at $ 250, find the net cost.

17. How much do I pay for 6 rugs listed at $ 150 each, if I am allowed discounts of 10 % and 5 %?

18. I can buy goods listed at $ 2000 at a discount of 25 %, or at a discount of 20 % and 5 %.

(*a*) Which rate gives me the greater discount?

(*b*) How much greater?

19. How much greater is a discount of 10 % and 5 % than a discount of 12 %?

20. (*a*) How much less is a discount of 20 % and 10 % than a discount of 30 %?

(*b*) How much greater is a 30 % discount on $ 2700 than a discount of 20 % and 10 %?

21. A large store buys $10,000 worth of goods each month. If it pays cash, it receives 2% discount. How much is saved in a year by paying cash?

22. A store buys $5000 bill of goods each week. A regular discount of 10% is allowed on the bill and another discount of $1\frac{1}{2}$% for cash. How much is saved in a year by taking advantage of these discounts?

To find a Single Discount equivalent to a Number of Discounts.

340. 1. What single rate of discount is equivalent to discounts of 20% and 10%?

Process

Base	=	1.00
First Discount	=	.20
First Remainder	=	.80
10% of 80	=	.08
Net Price	=	.72.

100% − 72% = 28%. Discount of 28% = equals discounts of 20% and 10%.

To find a single discount equivalent to a series of discounts.

RULE. **Consider 100% as the list price; deduct each successive discount until the net price is obtained; subtract the net price from 100%; the remainder is the single discount equivalent to the given series of discounts.**

EXERCISES

Find a single discount equivalent to the given series of discounts.

1. 25 % and 10 %
2. 10 % and 2 %
3. 10 % and 5 %
4. $33\frac{1}{3}$ % and 10 %
5. 20 % and 5 %
6. 10 % and 5 % and 2 %
7. 20 % and 5 % and 2 %
8. 25 % and 5 % and 2 %
9. 20 % and 10 % and 5 %
10. 20 % and 10 % and 2%

PROFIT AND LOSS

For Reading and Discussion

341. Mr. Clark keeps a grocery store. He buys fruits, vegetables, cereals, etc., from the wholesale dealers and sells them to his customers. To carry on his business, he has to pay \$ 60 a month to a clerk, \$ 50 a month for rent, and \$ 5 for light. If Mr. Clark sold his goods at the prices he paid for them, he would not be able to pay these expenses nor make a profit for himself. Therefore, it is necessary to sell the goods at prices greater than the cost.

By taking into account his monthly expenses and the profit he desires for himself, and by making an allowance for goods that may be lost or become decayed, Mr. Clark decides that he must sell his goods at a gain of 20 %. Apples that cost \$2 a barrel must be sold at a profit of — ct. per

barrel; eggs that cost 15 ct. a dozen must be sold at a gain of — ct. per dozen.

The price that Mr. Clark pays for his goods is the **Cost**; the price at which he sells them is the **Selling Price**. A **gain** or **profit** is made when the Selling Price is GREATER than the Cost; a **loss** is sustained when the Selling Price is LESS than the Cost. The per cent of the cost gained or lost is called the **per cent gained** or the **per cent lost**; it is also called the **rate of gain** or the **rate of loss**.

To find the Gain or Loss and the Selling Price when the Cost and the Rate of Gain or Loss are given

342. 1. A book that cost $2 is sold at a gain of 25 %. Find the gain. Find the selling price.

Process

Gain = 25 %, or $\frac{1}{4}$ of the Cost.
Gain = $\frac{1}{4}$ of $2 = $.50 Gain. *Ans.*
Selling Price = Cost + Gain.
Selling Price = $2 + $.50 = $2.50. *Ans.*

ANOTHER METHOD OF FINDING THE SELLING PRICE

$$\text{Selling Price} = \frac{4}{4} + \frac{1}{4} = \frac{5}{4} \text{ of Cost.}$$

$$\text{Selling Price} = \frac{5}{\cancel{4}} \text{ of } \$\overset{50}{\cancel{200}} = \$2.50. \textit{ Ans.}$$

2. A chair cost $80. As it was damaged, it was sold at a loss of 20 %. Find the loss. Find the selling price.

The problem corresponds to the First Type Problem in percentage (see p. 167).

Process

Loss $= 20\,\%$ or $\frac{1}{5}$ of the Cost.

Loss $= \frac{1}{\cancel{5}}$ of $\$\overset{16}{\cancel{80}} = \$16$ Loss. *Ans.*

Selling Price $=$ Cost $-$ Loss $= \$80 - \$16 = 64$. *Ans.*

Another Method of finding the Selling Price

Selling Price $= \frac{5}{5} - \frac{1}{5} = \frac{4}{5}$ of Cost.
Selling Price $= \frac{4}{5}$ of $\$80 = \$64$. *Ans.*

The method of finding the **Gain** or **Loss** is similar to the method of finding the **Percentage** (First Type Problem, on p. 167).

The method of finding the **Selling Price** (when there is a gain) is similar to the method of finding the **Amount** (p. 177).

243. The method of finding the **Selling Price** (when there is a loss) is similar to the method of finding the **Difference** (p. 179).

The **Cost** corresponds to the **Base.**

The **Gain Per Cent** or **Loss Per Cent** corresponds to the **Rate.**

The **Gain** or **Loss** in money corresponds to the **Percentage.**

The **Selling Price** corresponds to the **Amount** or to the **Difference.**

344. To find the gain or loss.

RULE. **Multiply the cost by the rate of gain or loss expressed as a common fraction or a decimal.**

FORMULA AND EQUATION. **Gain or Loss = Rate × Cost.**

$$\mathbf{G \text{ or } L = R \times C.}$$

To find the selling price.

RULE. **Add the gain to the cost or subtract the loss from the cost.**

EQUATION AND FORMULA.

Selling Price = Cost + Gain, or Selling Price = Cost − Loss.

$$\mathbf{SP = C + G \text{ or } C - L.}$$

ORAL PROBLEMS

345. 1. Pencils cost 2 ct. each. They are sold at a gain of 100%. Find the gain. Find the selling price.

2. A blank book costs 4 ct. It is sold at a gain of 50%. Find the gain.

3. Apples cost 3 ct. They are sold at a gain of $33\frac{1}{3}$%. Find the gain on 1 doz. apples.

4. Butter which cost 30 ct. a pound is sold at a gain of $16\frac{2}{3}$%.

(*a*) Find the selling price of 1 lb.

(*b*) Find the selling price of 6 lb.

5. Lead pencils that cost 5 ct. each are sold at a gain of 20%. Find the selling price of 6 lead pencils.

6. Mr. Brown bought picture post cards at 3 ct. each and sold them at a gain of $66\frac{2}{3}\%$. How much did he receive for each card?

7. Oranges that cost 8 ct. each are sold at a loss of 25%. Find the loss. Find the selling price.

8. Books that cost $1.50 each are sold at $33\frac{1}{3}\%$ less than the cost. Find the selling price.

9. I bought 8 yd. of lace at 50 ct. a yard and sold it at a gain of 10%. How much did I receive for the lace?

10. A box of fruit that cost $4 was sold at a loss of $12\frac{1}{2}\%$. Find the selling price.

11. Shoes that cost $3 per pair are sold at a gain of 50%. Find the selling price of 6 pr.

12. It costs a manufacturer $1.50 to manufacture a cap. For how much must the cap be sold to gain 20%?

13. Bottles of ink cost 4 ct. each. For how much must each bottle be sold to gain 25%?

14. Baseballs that cost 50 ct. each are to be sold so as to gain 30%. For how much must each ball be sold?

15. One dozen neckties cost $6. Find the selling price of each tie if I gain 20%.

EXERCISE

346. Find the value of ? in each of the following:

	Cost	Gain Per Cent	Loss Per Cent	Gain	Loss	Selling Price
1.	$ 3.00	50	——	?	——	?
2.	.15	100	——	?	——	?
3.	16.00	——	$12\frac{1}{2}$	——	?	?
4.	8.00	——	25	——	?	?
5.	200.00	20	——	?	——	?
6.	30.00	30	——	?	——	?
7.	250.00	——	50	——	?	?
8.	40.00	——	75	——	?	?
9.	90.00	$66\frac{2}{3}$	——	?	——	?
10.	17.00	10	——	?	——	?
11.	60.00	$16\frac{2}{3}$	——	?	——	?
12.	360.00	$33\frac{1}{3}$	——	?	——	?
13.	105.00	——	10	——	?	?
14.	80.00	40	——	?	——	?
15.	14.00	25	——	?	——	?

WRITTEN PROBLEMS

347. 1. A desk that cost $55 was sold at a gain of 14%.

(*a*) Find the gain.

(*b*) Find the selling price.

2. How much is gained by selling a house that cost $30,000 at a gain of 15%? How much is received for the house?

3. $1500 worth of goods were sold at a loss of 12%.

(*a*) Find the loss.

(*b*) Find the selling price.

Find the gain and the selling price in each of following:

4. A piano that cost $500 was sold at a gain of 12%.

5. An automobile that cost $1800 was sold at a gain of 21%.

6. A flag that cost $45 was sold at a gain of 16%.

Find the loss and the selling price in each of the following:

7. A crate of oranges that cost $12 was sold at a loss of 36%.

8. A boat that cost $70 was sold at a loss of 23%.

9. A picture that cost $240 was sold at a loss of 35%.

10. I bought two farms for $10,000 each. I sold one of them at a gain of 18%, and the other at a loss of 18%. Did I gain or lose in all?

11. A grocer bought 40 boxes of fruit at $2.50 per box. He sold it at a gain of 33%. How much did he receive for all the fruit?

12. 150 yd. of dress goods were bought at $1.75 a yard and sold at an average of 40%. How much was gained? How much was received for the goods?

13. A newsboy bought 160 papers at 2 for 1 ct. He sold them at a gain of 100%, and spent all the money for more papers at 2 for 1 ct. How many papers did he buy the second time?

14. A farmer sells turkeys to an agent at 16 ct. a pound. The agent sells them to a butcher at an advance of 25%. The butcher sells to a customer at a gain of 25%.

(*a*) How much does the customer pay per pound?

(*b*) How much more does the butcher receive per pound than the farmer?

15. A stationer buys post cards at 2 for 5 ct. and sells them at a gain of 20%. How much does he gain on 480 cards?

16. If the stationer buys cards at the rate of 3 for 5 ct. and sells them at a gain of 30%, how much does he receive for 50 doz. cards?

17. 100 doz. drinking glasses are bought by a dealer at $3 per doz. 10% of them are broken. The remainder are sold for 20% more than the cost. How much does the dealer gain or lose on the transaction?

18. 75 lamp chimneys are bought for $7.50. They are sold at a gain of 50%. How much is received for each chimney?

19. A contractor builds a fence. He employs 3 carpenters at \$4.50 per day. It takes 5 da. to build the fence. The lumber costs \$18. How much should the contractor charge in order to pay all his expenses and make a profit of 15% for himself?

20. 50 A. of land were bought at \$175 per acre. They were sold at a gain of 32%. Find the total selling price.

348. To find the Per Cent of Gain or Loss when the Selling Price and the Cost are Given

1. A bottle of ink that cost 4 ct. is sold at a gain of 1 ct. Find the per cent gain (the rate of gain).

Process

The gain is always calculated on the cost.
Therefore the per cent gain $= \frac{1}{4}$, or 25%. *Ans.*

2. A barrel of apples that cost \$2.50 is sold at a loss of 75 ct. Find the loss per cent (the rate of loss).

Process

The loss is always calculated on the cost.
Therefore the per cent loss $= \frac{75}{250} = \frac{3}{10} = 30\%$. *Ans.*

From these problems we see that the method of finding the **rate** of **gain** or **loss** is similar to the method of finding **what per cent one number is of another** (Second Type Problem, p. 183).

RULE. **The rate of gain or gain per cent** } **equals the gain divided by the cost.**

The rate of loss or loss per cent } **equals the loss divided by the cost.**

EQUATION AND FORMULA.

$$\textbf{Rate of Gain} = \frac{G}{C}. \quad \textbf{Rate of Loss} = \frac{L}{C}.$$

PROBLEMS

349. 1. Tea that cost 40 ct. a pound is sold at a gain of 10 ct. per pound. Find the per cent gain.

2. A baseball that cost 25 ct. is sold at a gain of 10 ct. Find the per cent gain.

3. What per cent is gained by selling hats that cost $ 2 at a gain of 50 ct.?

4. Turkeys that cost 25 ct. a pound are sold at a loss of 3 ct. a pound. Find the per cent loss.

5. What per cent is lost by selling a bicycle that cost $ 50 at a loss of $ 25?

* 6. Coffee that cost 20 ct. a pound is sold for 25 ct. a pound. Find the per cent gain.

* 7. A box of fruit that cost $ 6 is sold for $ 4. Find the per cent loss.

*** NOTE. When the selling price and cost are given, find the gain or loss; then proceed as in numbers 1 and 2, p. 243.**

*8. 5 books are bought at 50 ct. each and sold at an advance of 25 ct. Find the per cent gain.

*9. 2 books are bought at 50 ct. each and sold at a gain of 25 ct. each. Find the per cent gain.

*10. Flour that cost 6 ct. a pound is sold for 8 ct. a pound. Find the gain per cent on 1 lb.

*11. Flour that cost 5 ct. a pound is sold for 7 ct. a pound. Find the gain per cent on 10 lb.

12. 10 books are bought for $ 20. They are sold at $ 3 each. Find the per cent gain.

WRITTEN PROBLEMS

350. 1. What per cent is gained by manufacturing hats at $ 1.25 each and selling them at a gain of $ 3.00?

2. What per cent is gained by buying pencils at 55 ct. a dozen and selling them at 72 ct. a dozen?

3. Library cards that cost 80 ct. per M are sold at $ 1.25 per M. Find the gain per cent.

4. What per cent is lost by selling books that cost $ 2.75 each at a loss of 75 ct. each?

5. What per cent is lost by buying shirts at $ 1.50 each and selling them at $ 1.30 each?

6. Flowers that cost 90 ct. a hundred are sold at 78 ct. a hundred. Find the loss per cent.

7. Shoes are bought for $ 1.80 per pair and sold for $ 2.25 per pair. Find the gain per cent on 1 pr.

* Note. Compare 8 and 9; 10 and 11.

8. Shoes are bought for \$ 2.25 per pair and sold for \$ 2.60 per pair.

(*a*) Find the gain per cent on 1 pr.

(*b*) Find the gain per cent on 100 pr.

9. If a grocer makes a profit of 25 % on each pound of flour he sells, what per cent of the profit does he make by selling a barrel of flour?

10. Penholders are bought at 35 ct. a dozen and sold at 5 ct. each. Find the per cent gain.

11. Flour is bought at \$ 6 a barrel and sold at the rate of $5\frac{1}{2}$ ct. a pound. Find the per cent gain.

12. Post cards are bought at 50 ct. a hundred and sold at the rate of 3 for 5 ct. Find the gain per cent.

13. A can of milk containing 40 qt. costs \$ 1.60. What per cent is gained by selling the milk for 6 ct. a quart?

14. A florist bought 72 roses for 80 ct. He made bouquets of them, putting 12 roses into each bouquet. What per cent profit did he make if he sold the bouquets for 25 ct. each?

15. A retail coal dealer buys a ton of coal for \$ 5.50. He sells the coal by the pailful, charging 25 ct. for each pailful. What per cent profit does he make if he gets 40 pailfuls from each ton?

16. An iceman pays 30 ct. for each 100 lb. of ice. He sells it in 10-ct. pieces. What per cent profit does he make if he cuts 7 pieces out of each 100 lb.?

17. A department store sold 50 yd. of dress goods at \$1.40 a yard and 75 yd. at \$1.60 a yard. The goods cost \$1.50 a yard. What per cent did the store gain or lose?

18. Mr. Martin bought 200 umbrellas for \$1.25 each. He sold $\frac{1}{2}$ of them at a gain of 20%, and the remainder at a loss of 10%. Find his per cent of gain or loss on the entire lot.

19. Mr. Dunn buys 400 overcoats for \$15 each. He sells 75% of them at a gain of 30% and the remainder at a loss of 20%. Find his per cent of gain or loss on the whole transaction.

351. To find the Cost when the Gain and the Gain Per Cent are Given

Or

To find the Cost when the Loss and the Loss Per Cent are Given

When the Rate is reduced to a Common Fraction

352. 1. I sold a book at a gain of 60 ct., thereby gaining $37\frac{1}{2}$% of the cost. Find the cost.

Process

First Method

$37\frac{1}{2}$% or $\frac{3}{8}$ of the Cost = 60 ct.

$\frac{1}{8}$ of the Cost = $\frac{1}{3}$ of 60 ct. = 20 ct.

$\frac{8}{8}$ or the Cost = 8×20 ct. = \$1.60 Cost. *Ans.*

Or Shorter Method

$\frac{3}{8}$ of the Cost = 60 ct.

Cost = 60 ct. $\div \frac{3}{8}$.

Cost = 60 ct. $\times \frac{8}{3}$ = \$1.60 Cost. *Ans.*

When the Rate is reduced to a Decimal Fraction

2. I sold a horse at a loss of $ 80, thereby losing 16 %. Find the cost.

Process

16 % of the Cost = $ 80.
Cost = $80 ÷ .16 = $ 500. *Ans.*

From these problems, we see that the method of finding the **Cost** is similar to the method of finding the **Base** when the percentage and the rate per cent are given (Third Type Problem, Case 1, p. 191).

353. To find the cost when the gain and gain per cent or loss and loss per cent are given.

Rule. **Divide the gain or loss by the gain or loss per cent.**

Equation and Formula.

$$\textbf{Cost} = \textbf{Gain} \div \textbf{Gain Per Cent} \quad \left(\mathrm{C} = \frac{\mathrm{G}}{\mathrm{G}\ \%}\right).$$

Or

$$\textbf{Cost} = \textbf{Loss} \div \textbf{Loss Per Cent} \quad \left(\mathrm{C} = \frac{\mathrm{L}}{\mathrm{L}\ \%}\right).$$

ORAL PROBLEMS

354. 1. By selling meat at a gain of 4 ct. a pound a butcher makes a profit of 25 %. Find the cost of 1 lb.

2. Flour is sold at a gain of 50 %. If the actual gain is $2\frac{1}{2}$ ct. per pound, find the cost of 1 lb. of flour.

3. I gain $66\frac{2}{3}$ %, or 2 ct., on each lead pencil sold. How much do I pay for each pencil?

4. A bicycle was sold at a gain of $5. This was 10% of the cost. Find the cost.

5. Eggs were sold at a loss of $9. How much did they cost if the loss was $37\frac{1}{2}$% of the cost.

6. A real estate dealer made a profit of $500 on a plot of ground. The rate of profit was 20%. Find the cost of the plot.

7. Butter was sold at a loss of 9 ct., or 30%, on each pound. Find the cost of 1 lb.

8. A profit of 24 ct., or 40%, was made on a picture. Find the cost of the picture.

9. Find the cost of a boat, if a loss of $9 equals $12\frac{1}{2}$% of the cost.

10. Shirts were sold at a profit of $3 a dozen, thereby gaining 50%. How much did 6 doz. shirts cost?

WRITTEN PROBLEMS

1. A coal dealer makes a profit of 77 ct., or 14%, on each ton of coal he sells. Find how much he paid for each ton.

2. A barrel of potatoes was sold at a profit of 35 ct., or $23\frac{1}{3}$%. Find the cost.

3. If I gain 38%, or $199.50, by selling a piano, how much did I pay for it?

4. A house and lot were sold at a profit of $3410. The rate of profit was 22%.

(*a*) Find the cost of the house and lot.

(*b*) Find the selling price.

5. A profit of $ 1710, or 18 %, was made on an automobile. Find the cost and the selling price.

6. By selling a phonograph at a gain of $ 23.25, a dealer made a profit of 31 %. Find cost of the phonograph.

7. By selling a suit of clothes at a loss of 8 %, a tailor lost $ 1.42. Find the selling price.

8. Goods that had been damaged by fire were sold at a loss of 15 %. The actual loss was $ 187.50. Find the cost and the selling price.

9. A gain of 250 % was made by selling a diamond ring. The actual gain was $ 150. Find the cost.

10. A dry goods store sells its goods at a profit of 18 %. Find the cost of a hat on which a profit of $ 1.44 is made.

355. To find the Cost when the Selling Price and the Gain Per Cent are Given

When the Rate is reduced to a Common Fraction

356. 1. A desk was sold for $ 60. This was at a gain of 25 %. Find the cost.

Process

Represent the Cost by $\frac{4}{4}$.
Gain $= \frac{1}{4}$ of Cost.
Selling Price $= \frac{5}{4}$ of Cost.
Therefore $\frac{5}{4}$ of Cost $= \$60$.
Cost $= \$60 \div \frac{5}{4} = \$60 \times \frac{4}{5} = \$48$. *Ans.*

WHEN THE RATE IS REDUCED TO A DECIMAL FRACTION

2. By selling a picture for \$168, I gained 12%. How much did the picture cost?

Process

Represent the Cost by 100%.
Gain = 12% of Cost.
Selling Price = 112% of Cost.
Therefore 112% of the Cost = \$168.
Cost = \$168 ÷ 1.12 = \$150 Cost. *Ans.*

From problems 1 and 2 we see that the method of finding the **Cost** when the **Selling Price** and the **Gain Per Cent** are given is similar to the method of finding the **Base** when the **Amount** and **Rate of Increase** are given (Third Type Problem, Case 2, p. 197).

357. To find the cost when the selling price and the gain per cent are given.

RULE. **Divide the selling price by 1 plus the gain per cent.**

EQUATION AND FORMULA.

$$C = SP \div (1 + G.\%),$$

or

$$C = \frac{SP}{1 + G.\%}.$$

358. To find the Cost when the Selling Price and the Loss Per Cent are Given

WHEN THE RATE IS REDUCED TO A COMMON FRACTION

359. 1. A plot of ground was sold at a loss of $16\frac{2}{3}$%. The plot was sold for \$2000. Find the Cost.

Process

Represent the Cost by $\frac{6}{6}$.
Then the Loss $= \frac{1}{6}$ of the Cost,
and the S. P. $= \frac{5}{6}$ of Cost.
Therefore $\frac{5}{6}$ of the Cost $= \$2000$.
Cost $= \$2000 \div \frac{5}{6}$.

$$\text{Cost} = \$\overset{400}{\cancel{2000}} \times \frac{6}{\cancel{5}} = \$2400. \quad \textit{Ans.}$$

WHEN THE RATE IS REDUCED TO A DECIMAL FRACTION

2. A carriage was sold for $322, which was at a loss of 8%. How much did the carriage cost?

Process

Represent the Cost by 100%.
Then the Loss = 8% of the Cost,
and the S. P. = 92% of Cost.
Therefore 92% of the Cost = $322.
Cost = $322 ÷ .92 = $350. *Ans.*

From problems 1 and 2 we see that the method of finding the **Cost** when the **Selling Price** and the **Loss Per Cent** are given is similar to the method of finding the **Base** when the **Difference** and the **Rate of Decrease** are given (Third Type Problem, Case 3, p. 198).

360. To find the cost when the selling price and the loss per cent are given.

RULE. **Divide the selling price by 1 minus the loss per cent.**

EQUATION AND FORMULA.

$$C. = S.P. \div (1 - L.\%),$$

or

$$C. = \frac{S.P.}{1 - L.\%}.$$

ORAL EXERCISES

361. Find the cost of the following:

	Selling Price	Gain Per Cent		Selling Price	Gain Per Cent
1.	$100	20	2.	$200	25
3.	$90	50	4.	$36	$12\frac{1}{2}$
5.	$40	$33\frac{1}{3}$	6.	$100	100

ORAL EXERCISES

362. Find the cost in each of the following:

	Selling Price	Loss Per Cent		Selling Price	Loss Per Cent
1.	$90	25	2.	$60	$33\frac{1}{3}$
3.	$50	50	4.	$80	20
5.	$45	10	6.	$75	$66\frac{2}{3}$

WRITTEN EXERCISES

363. Find the cost in each of the following:

	Selling Price	Gain or Loss		Selling Price	Gain or Loss
1.	$ 232.50	L., 7%	2.	$ 226.00	G., 13%
3.	$ 220.40	L., 24%	4.	$ 378.00	L., 16%
5.	$ 940.00	L., 6%	6.	$ 682.50	L., 9%
7.	$1470.00	L., 2%	8.	$1300.00	G., 100%
9.	$ 262.60	G., 30%	10.	$ 500.00	L., 20%

WRITTEN EXERCISES

364. Find the cost in each of the following:

	Selling Price	Gain or Loss		Selling Price	Gain or Loss
1.	\$ 146.25	L., $2\frac{1}{2}\%$	2.	\$ 71.82	L., $5\frac{1}{2}\%$
3.	\$ 309.75	G., $3\frac{1}{4}\%$	4.	\$ 668.00	L., $16\frac{1}{2}\%$
5.	\$ 2610.00	G., $30\frac{1}{2}\%$	6.	\$ 259.80	G., $8\frac{1}{4}\%$
7.	\$ 626.50	L., $10\frac{1}{2}\%$	8.	\$ 935.75	L., $1\frac{1}{2}\%$
9.	\$ 1518.75	G., $1\frac{1}{4}\%$	10.	\$ 2412.50	L., $3\frac{1}{2}\%$

WRITTEN EXERCISES

Find the cost in each of the following:

365. 1. Selling Price, \$ 930.00 Gain, $3\frac{1}{3}\%$

Process

$1.03\frac{1}{3})\overline{\$\,930}$ Multiply both divisor and dividend by 3.
$3.10)\overline{\$\,2790} = \$\,900.$ *Ans.*

	Selling Price	Gain or Loss		Selling Price	Gain or Loss
2.	\$ 532.00	L., $6\frac{2}{3}\%$	3.	\$ 343.00	G., $16\frac{2}{3}\%$
4.	\$ 521.90	G., $2\frac{1}{3}\%$	5.	\$ 393.00	L., $12\frac{2}{3}\%$

ORAL PROBLEMS

366. 1. Oranges are sold at 24 ct. a dozen. If this is at a gain of 20%, find the cost of 1 doz.

2. Magazines are sold at 10 ct. each. This is at a gain of 25%. Find the cost.

3. By selling newspapers at 5 ct. each, a boy makes 100%. How much did each newspaper cost?

4. A baseball bat was sold at a loss of $33\frac{1}{3}\%$. Find the cost of the bat, if the selling price was 40 ct.

5. A barrel of apples was sold for $ 2.80, which was at a loss of $16\frac{2}{3}\%$. Find the cost.

6. Picture post cards are sold at a gain of 40%. If they are sold for 14 ct. a dozen, how much did they cost per dozen?

7. A pound of tea is sold for 36 ct. This is at a loss of $14\frac{2}{7}\%$. Find the cost.

8. Ribbon is sold for 18 ct. a yard, which is at a gain of $12\frac{1}{2}\%$. Find the cost.

9. A profit of 10% is made by selling a book for $ 2.20. Find the cost.

10. Mr. White marks a damaged chair at $37\frac{1}{2}\%$ less than it cost him. If the marked price is $ 25, find the cost.

11. How much did I pay for a horse, if I gain 25% by selling it for $ 300?

12. Oats were sold at a loss of 10%. How much did I pay for them per bushel, if I sold them for 36 ct. per bushel?

13. A profit of 25% is made by selling 100 ducks for $ 250. Find the cost of each duck.

14. 20 bbl. of apples were sold for $ 60. This was at a loss of $33\frac{1}{3}\%$. How much did the apples cost per barrel?

WRITTEN PROBLEMS

367. 1. A manufacturer of pianos sold them at 24% more than they cost. Find the cost, if the selling price was \$800.

2. Flour is sold at \$6.16 per barrel, which is 12% greater than the cost. Find the cost per barrel.

3. Eggs were sold at \$6.08 per box. This was a loss of $33\frac{1}{3}$%. Find the cost.

4. A farm was sold for \$4140, which was a gain of 15%. Find the cost.

5. By selling a farm of 20 A. for \$7000, Mr. Sheldon gained $14\frac{2}{7}$%. How much did he pay for the farm per acre?

6. Cranberries were sold at a loss of 20%. How much did the berries cost, if they were sold for \$560?

7. 200 bbl. of cranberries were sold for \$1080, which was at a gain of 8%. Find the cost of the berries per barrel.

8. A load of melons was sold at $8\frac{1}{3}$% above cost. If the selling price was \$1300, what was the cost?

9. 350 crates of melons were sold at a gain of $16\frac{2}{3}$%. If the selling price was \$525, what was the cost of the melons per crate?

10. 50 bbl. of cabbages were sold for \$ 32.20. This was 10% less than the cost. How much did the cabbages cost?

11. 125 bbl. of cabbages were sold for \$ 93.75 This was 25% greater than the cost.

(*a*) How much did the cabbages cost per barrel?

(*b*) How much money was gained on each barrel?

12. I sold two farms for \$ 25,000 each. On one I gained 20%; on the other I lost 20%. How much did I gain or lose in all?

13. I bought two farms for \$ 20,000 each. I sold one at a gain of 17%, and the other at a loss of 17%. How much did I gain or lose in all?

14. 12,500 bu. of wheat were purchased by Mr. Jones. He sold them for \$ 14,375, which was 15% more than he paid for them.

(*a*) How much did Mr. Jones pay for the wheat per bushel?

(*b*) How much did he receive for the wheat per bushel?

15. A profit of $33\frac{1}{3}$% was made by selling 80 cows for \$ 6000. How much did each cow cost?

EXERCISES

368. Find values for ? in each of the following. (Use paper and pencil only when necessary.)

	Cost	Selling Price	Gain in Money	Loss in Money	Gain Per Cent	Loss Per Cent
1.	$ 6	$ 9	?	——	?	——
2.	$ 8	$ 6	——	?	——	?
3.	?	$10	$5	——	?	——
4.	?	?	——	$ 5	——	20 %
5.	$20	?	$4	——	?	——
6.	$50	?	——	$10	——	?
7.	$40	?	?	——	$12\frac{1}{2}$ %	——
8.	$10	?	?	——	50 %	——
9.	$60	?	——	?	——	$33\frac{1}{3}$ %
10.	$32	?	——	?	——	$37\frac{1}{2}$ %
11.	?	?	$9	——	25 %	——
12.	?	?	——	$12	——	20 %
13.	$ 7.50	$15	?	——	?	——
14.	$12	$ 6	——	?	——	?
15.	?	?	$2.50	——	$33\frac{1}{3}$ %	——
16.	?	?	$4.50	——	——	25 %
17.	?	$50	?	——	25 %	——
18.	?	$20	——	?	——	20 %
19.	?	$16	?	——	$33\frac{1}{3}$ %	——
20.	?	$12	——	?	——	25 %
21.	?	?	$2.50	——	10 %	——
22.	?	?	——	$ 5	——	$12\frac{1}{2}$ %
23.	$ 4	$12	?	——	?	——
24.	?	$15	——	?	——	$16\frac{2}{3}$ %
25.	?	$30	?	——	$66\frac{2}{3}$ %	——

TAXES

For Reading and Discussion

369. All governments spend money for schools, for police departments and fire departments, for prisons, for salaries, for roads, etc. The government obtains the money it needs by taxing the people.

Some of the ways in which money is obtained are:

1. By levying a tax on the value of real estate. This tax is paid directly by each person who owns real estate; it is paid indirectly by each person who pays rent.

2. By levying a tax on the value of personal property, *e.g.* pictures, books, jewelry, securities, etc. This tax is paid directly by each person who owns personal property.

3. By levying a tax on incomes. This tax is called an **income tax**; it is paid by each person whose income exceeds a certain amount of money.

A tax rate of 2% on property means that

For every $1 of property a man owns, he must pay 2 ct. in taxes.

For every $ 100 of property a man owns, he must pay $ 2 in taxes.

For every $ 1000 of property a man owns, he must pay $ 20 in taxes.

To find the Amount of Money to be paid in Taxes

370. To find the amount of money to be paid in taxes, proceed as in the First Type Problem in Percentage (p. 167).

> The **value** of the **property** corresponds to the **base**.
> The **tax rate** corresponds to the **rate per cent**.
> The **amount** of **taxes** corresponds to the **percentage**.

ORAL EXERCISES

371. Find the amount of taxes paid by each of the following men who live in the town of Danville. The tax rate is $1\frac{1}{2}\%$.

Name	Value of Property	Name	Value of Property
1. Mr. Jordan	$10,000	4. Mr. Dolan	$24,000
2. Mr. Hurley	$ 6,000	5. Mr. Gruber	$30,000
3. Mr. Bates	$18,000	6. Mr. Walsh	$35,000

7. Find the taxes on Mr. Jordan's property, if it is taxed on $\frac{4}{5}$ of its value at $1\frac{3}{4}\%$ per annum.

8. Mr. Jones is taxed $3 on every $100 of property. How much does he pay on $15,000 worth of property?

9. Find the amount Mr. Hayes pays in taxes on $28,000 worth of property at $1\frac{7}{8}$%.

10. If the rate is $4 on $100, find the amount paid by Mr. Cameron, whose property is worth $40,000.

11. Find the taxes on $20,000 worth of property at 2.15%.

12. Find the taxes on $30,000 worth of property at 1.54%.

WRITTEN PROBLEMS

372. If the tax rate is 2.14%, find the amount paid by each of the following companies:

Name	Value of Property
1. Consolidated Gas Co.	$100,000
2. Main St. R. R.	$250,000
3. American Steel Co.	$ 75,000
4. Acme Insurance Co.	$ 50,000
5. The Harbor Coal Co.	$ 25,000
6. The Crystal Ice Co.	$ 37,500

7. Mr. Daly owns 2 houses valued at $25,000 each. How much does he pay in taxes, if the rate is $2.12 per $100?

8. Mr. Mayer owns 6 lots valued at $15,500 each and 2 houses valued at $3000 each. How

much does he pay in taxes, if the tax rate is 2.21 %?

9. Mr. Holly owns \$3500 worth of personal property and \$15,500 worth of real estate. How much does he pay in taxes, if the tax rate is 2.25 %?

10. Last year the tax rate was 1.35 %. This year it is 1.85 %. How much more must be paid this year than last year on \$120,000 worth of property?

11. I own a house worth \$20,000. I spend \$500 a year for repairs and pay taxes of $2\frac{1}{2}$ %. What is my net income from the property during the year, if my tenant pays me \$1500 a year in rent?

To find the Rate of Taxation

To find the rate of taxation, proceed as in the Second Type Problem in Percentage (p. 183).

ORAL EXERCISES

373. Find the tax rate in each of the following instances:

	Value of Property	Amount to be Raised		Value of Property	Amount to be Raised
1.	\$100,000	\$ 1,000	4.	\$300,000	\$90,000
2.	\$500,000	\$10,000	5.	\$150,000	\$ 1,500
3.	\$250,000	\$ 5,000	6.	\$600,000	\$12,000

WRITTEN PROBLEMS

374. Find the tax rate in each of the following cases:

	Value of Property	Amount to be Raised
1.	$ 750,000	$ 11,250
2.	$ 1,500,000	$ 37,500
3.	$ 2,500,000	$ 56,250
4.	$ 3,000,000	$ 82,500
5.	$10,000,000	$175,000

6. The total value of the real estate and personal property is $ 650,000. The amount to be raised by taxation is $ 14,950. Find the tax rate.

7. The value of the real estate in a country is $ 2,500,000. The value of the personal property is $ 400,000. What rate of taxation will yield $ 40,000?

8. The total value of the real and personal property is $ 3,000,000. Last year the expenses were $ 52,500. This year the expenses are $ 15,000 greater than last year.

(*a*) Find the tax rate for last year.

(*b*) Find the tax rate for this year.

9. A county government spent the following:

For schools	$ 75,000
For police department . . .	27,500
For fire department	27,500
For all other expenses . . .	50,000

(*a*) Find the tax rate, if the real estate is appraised at \$10,000,000 and the personal property is appraised at \$2,000,000.

(*b*) How much will a man have to pay in taxes who owns 10 lots worth \$1750 each, two houses worth \$15,000 each, and has personal property worth \$5000?

To find the Appraised Valuation of Property when the Amount of Taxes Paid and the Rate of Taxation are Given

1. The tax rate is $1\frac{1}{2}\%$. Mr. Jones pays \$300 in taxes, each year, on his property. Find the valuation of his property.

Process

$1\frac{1}{2}\%$ of the valuation of the property = \$300.
Valuation of property = \$300 ÷ .015 = \$20,000.

The method of solution is similar to the method in the Third Type Problem, Case 1 (p. 191).

PROBLEMS

375. Find the valuation of the property in each of the following:

Amount of Tax	Tax Rate	Amount of Tax	Tax Rate per \$100
1. \$270	$1\frac{1}{2}\%$	6. \$382.80	\$1.74
2. \$225	$1\frac{1}{2}\%$	7. \$419.25	\$2.15
3. \$210	$1\frac{3}{4}\%$	8. \$273.00	\$1.95
4. \$350	$1\frac{3}{4}\%$	9. \$261.00	\$2.175
5. \$273	1.56%	10. \$560.00	\$2.24

DUTIES OR CUSTOMS

For Reading and Discussion

376. The National Government or United States Government spends large sums of money each year. Some of the money is spent for the army and navy, some for pensions for disabled soldiers and their families, some for the large number of government employees, some for the United States Courts, etc. The amount spent is more than $1,000,000 a day. The United States Government does not levy direct taxes on real and personal property; it obtains money for its expenditures by indirect taxation and by an income tax.

One way by which the United States Government obtains money is by taxing goods imported into this country. These taxes are called **Duties** or **Customs**. They are collected by "customs officials" at the great ports of entry, as New York, San Francisco, Boston, etc.

Duties are of two kinds: **specific duties** and **ad valorem duties.**

377. A **specific duty** is a tax levied on **each bushel** or **each pound** or **each gallon**, etc., without regard to the value of the goods.

378. An **ad valorem duty** is, as the name indicates, a tax levied on the **value** of the goods imported.

379. Some goods are subject to both a specific duty and an ad valorem duty. For example, if oilcloth is taxed 12 ct. per square yard, and 15% ad valorem; the tax on 200 sq. yd. of oilcloth worth \$100 would be \$24, specific duty, plus \$15 ad valorem duty, or \$39, total duty.

Some goods are not taxed; they are admitted free of duty.

380. The law stating the different rates and kinds of duties is called the **tariff law**. Tariff laws are passed by Congress and may be changed at any session.

381. **Tare** is an allowance made for the weight of barrels, boxes, bottles, or other covering of goods. **Leakage** is an allowance made for loss of liquids by leakage or evaporation when imported in barrels, casks, etc.

Gross weight is the total weight of the goods and the receptacles in which they are shipped. The **net weight** is the weight after the allowances for tare and leakage have been made. For example, the **gross weight** of 10 kegs of nails is the total weight of the nails and the kegs; the **tare** is the weight of the kegs; the **net weight** is the gross weight minus the tare.

To find the Specific Duty

382. Suppose the tariff law imposes the following specific duties:

Fruit	25 ct. per bu.	Potatoes	25 ct. per bu.
Chocolate	$2\frac{1}{2}$ ct. per lb.	Raisins	$2\frac{1}{2}$ ct. per lb.
Lemons	$1\frac{1}{2}$ ct. per lb.	Rice	$1\frac{1}{2}$ ct. per lb.
Mustard	10 ct. per lb	Salt (in pkgs.)	11 ct. per 100 lb.
Olives	15 ct. per gal.	Vinegar	$7\frac{1}{2}$ ct. per gal.

Using the above rates of duty, find the specific duty on:

1. 50 bu. of apples.
2. 100 bu. of pears.
3. 200 bu. of peaches.
4. 300 gal. of olives.
5. 500 boxes of raisins; the net weight of each box being 22 lb.
6. 100 boxes of lemons; the net weight of each box being 40 lb.
7. 100 bags of salt; 100 lb. in each bag.
8. 600 bu. of potatoes.
9. 120 boxes of rice; net weight of each box, 7 lb.
10. 200 bottles of vinegar, each bottle containing 1 gal., allowing 10 % for leakage.
11. The duty on barley was lowered from 30 ct. per bushel to 15 ct. per bushel. How much is saved on an importation of 250 bu.?
12. The duty on lard was lowered from 2 ct. per pound to $1\frac{1}{2}$ ct. per pound. How much is saved on a shipment of 500 lb.?

To find the Ad Valorem Duty

383. To find the ad valorem duty.

Proceed as in the First Type Problem in Percentage (p. 167).

Suppose the tariff law imposes the following ad valorem duties:

Agricultural Implements	15 %	Cutlery	40 %
Boots and Shoes	15 %	Feathers	20 %
Sewing Machines	30 %	Furs	$27\frac{1}{2}$ %
Shoe Laces	35 %	Leather	15 %
Typewriting Machines	30 %	Sponges	30 %

Using the above rates of duty, find the duty on:

1. A shipment of agricultural implements worth $ 4000.

2. 60 doz. pairs of boots worth $ 2.50 per pair.

3. 10 gross knives worth $ 3.50 per dozen, and 10 gross forks worth $ 3 per dozen.

4. 150 fine feathers valued at $ 6 each.

5. 72 fur coats valued at $ 75 each.

6. A shipment of leather worth $ 6500.

7. 120 sewing machines worth $ 35 each.

8. 70 typewriting machines worth $ 48 each.

9. 60 doz. sponges; 50 % of them worth $ 3 a dozen; the remainder worth $ 5 a dozen.

10. 10 gross shoe laces worth $ 3.60 a gross.

11. 1 doz. sets of fur worth $ 300 each.

12. The duty on shoes was lowered from 25 % to 15 %. How much less duty is paid on 80 boxes, each box containing 12 doz. pairs, worth $ 2.50 per pair?

To find the Duty when both a Specific Duty and an Ad Valorem Duty are Charged

384. Suppose the tariff law imposes the following duties:

Cocoa, 4 ct. per pound and 10 % ad valorem.
Ground Spices, 10 ct. per pound and 30 % ad valorem.
Oilcloth, 12 ct. per square yard and 15 % ad valorem.
Shirts, 35 ct. per dozen and 10 % ad valorem.
Stockings, Cotton, 70 ct. per dozen pairs and 15 % ad valorem.

Using the above rates, find the total duty in each of the following:

1. 250 yd. of oilcloth worth 60 ct. per sq. yd.

2. 72 doz. shirts worth $ 18 per dozen.

3. 120 cans of cocoa, each can containing 1 lb., at 60 ct. per pound.

4. 180 cans of pepper (spices) each can containing $\frac{1}{2}$ lb., at 36 ct. per pound.

5. 288 doz. pairs of cotton stockings valued at 15 ct. per pair.

6. Obtain the latest tariff schedule. Make and solve four problems based on the schedule.

INCOME TAX

FOR READING AND DISCUSSION

385. The government may lay and collect taxes on the incomes of individuals and corporations. This tax is called an **income tax**. The gross or total income may be made of salary and incomes from other sources. Some of these incomes, either whole or in part, are exempt from taxation.*

386. The **net income** is that part of the total income that is left after these exemptions have been made.

387. For the purpose of graduating the tax on incomes, they may be separated into four classes and taxed as follows:

CLASS	INCOME LIMITS	ANNUAL RATE OF TAX ON AMOUNT IN EACH CLASS	EXEMPTION
I	Less than \$20,000	1%	\$3000
II	\$20,000 and less than \$50,000	1% on amt. in Class I 2% on amt. in Class II	\$3000
III	\$50,000 and less than \$100,000	1% on amt. in Class I 2% on amt. in Class II 3% on amt. in Class III	\$3000
IV	\$100,000 or over	1% on amt. in Class I 2% on amt. in Class II 3% on amt. in Class III 4% on amt. in Class IV	\$3000

* In general, incomes that have already been taxed are exempt.

388. A part of the net income, \$3000, of every taxable individual or corporation is exempt from taxation. Additional amounts, \$1000, \$1500, \$2000 or more, depending on circumstances, may also be exempt. In taxation this amount may be deducted from that part of the income in Class I, or that part that is less than \$20,000. For this reason net incomes of \$3000 or less may be exempt from taxation.

EXERCISES.

389. Using the rates given in the table, estimate the tax on the following incomes:

1. Net income a salary \$6000, exemption \$3000.

Process

\$6000 – \$3000 = \$3000
\$3000 × 1 % = \$30.00 Income Tax.

2. Net income \$22,500, exemption \$3000 + \$1000 + \$500.

Process

\$22,500 = \$20,000 + \$2500.
\$20,000 – \$4500 = \$15,500 (Amount in Class I).
\$15,000 × 1 % = \$150.
\$2500 (Amount in Class II) × 2 % = \$50.
Total Income Tax \$200.

	Net Income	Exemption		Net Income	Exemption
3.	\$ 3500	\$3000	6.	\$ 40,000	\$6000
4.	\$ 5000	\$4000	7.	\$ 51,500	\$8000
5.	\$24,500	\$5000	8.	\$110,000	\$9000

INSURANCE

For Reading and Discussion

390. To be insured, one enters into a contract with an insurance company by which he agrees to pay a certain sum of money, and the company agrees to pay damages resulting from fire, accident, or loss during the time the agreement lasts. The written agreement is called the **policy**; the amount of money paid by the person insured is called the **premium**; the amount for which the property is insured is called the **face of the policy**; the **rate of premium,** often called the **rate,** is usually stated as a certain per cent of the face of the policy or as a certain number of cents for each $100 of insurance.

In life insurance, the person insured pays a premium each year for a number of years or until his death. Fire insurance companies usually insure buildings for four fifths of their value.

> Problems in Insurance are solved in the same manner as problems in Percentage.
>
> The **Face of the Policy** corresponds to the **Base.**
>
> The **Rate of Insurance** corresponds to the **Rate.**
>
> The **Premium,** or **Amount Paid,** corresponds to the **Percentage.**

To find the Cost of Insurance

391. 1. A building worth $24,000 is insured for $\frac{2}{3}$ of its value for three years at $\frac{1}{4}$ % per annum. How much is paid for the insurance?

Process

$\frac{2}{3}$ of $24,000 = $16,000. Face of the policy.
$\frac{1}{4}$ % or $\frac{1}{400}$ of $16,000 = $40. Annual premium.
3 × $40 = $120. Cost of policy for 3 yr. *Ans.*

SHORT METHOD

$$\frac{2}{\cancel{3}} \text{ of } \$\overset{60}{\cancel{24000}} \times \frac{1}{\cancel{400}} \times \cancel{3} = \$120. \quad \textit{Ans.}$$

The method of finding the cost of insurance (the premium) is similar to the First Type Problem (p. 167).

ORAL EXERCISES

392. Find the annual premium in each of the following fire insurance policies:

	Face of Policy	Rate		Face of Policy	Rate
1.	$10,000	1 %	6.	$ 2,000	$2\frac{1}{4}$ %
2.	$15,000	2 %	7.	$ 4,000	$1\frac{3}{4}$ %
3.	$ 6,000	$1\frac{1}{2}$ %	8.	$ 4,000	$2\frac{3}{4}$ %
4.	$ 8,000	$1\frac{1}{4}$ %	9.	$16,000	$\frac{1}{2}$ %
5.	$12,000	$2\frac{1}{2}$ %	10.	$18,000	$\frac{1}{4}$ %

WRITTEN PROBLEMS

393. 1. Find the cost of a policy of $15,000 for 2 yr. at $\frac{3}{4}$ % per annum.

2. Find the cost of a policy for $25,000 for 3 yr. at $1\frac{1}{2}\%$ per annum.

3. A house worth $18,000 was insured for $\frac{2}{3}$ of its value for 3 yr. at $\frac{1}{2}\%$ per annum. Find the cost of the policy.

4. A house worth $26,500 was insured for 80 % of its value at $1\frac{3}{4}\%$ per annum. Find the cost of the policy.

5. Mr. Polk owns 2 houses worth $25,000 each. Each house is insured for 3 yr. for 80 % of its value. On one of the houses the insurance rate is 2 % per annum. On the second house the rate is $1\frac{1}{2}\%$ per annum. How much did Mr. Polk pay for the two policies?

6. Mr. Robinson owns 3 houses worth $30,000 each. Two of the houses are insured for 75 % of their value for 3 yr. at $\frac{1}{2}\%$ per annum. The third house is insured for 80 % of its value for three yr. at $\frac{3}{4}\%$ per annum. How much did the 3 insurance policies cost Mr. Robinson?

7. Mr. Walsh owns a house worth $50,000. He had it insured for 80 % of its value for 3 yr. at $1\frac{3}{4}\%$ per annum. When the policy expired, he took out another policy for 3 yr. for 80 % of the value of the house. On this policy the rate was $2\frac{1}{2}\%$ per annum. How much more did the second policy cost than the first?

Rate per $1000 of Insurance

Life Insurance (rates depend upon age):		
Whole Life Policy .	$19.62	annually (age 22)
20-Year Life Policy	$29.84	(age 25)
20-Year Endowment Policy	$48.62	(age 30)
	1 Year	3 Years
Burglary Insurance .	$12.50	$37.50 less 10%

Using the above rates, find the cost of premiums on the following policies:

8. A policy of burglary insurance for $5000 for 1 yr.

9. A policy of burglary insurance for $5000 for 3 yr.

10. A man takes out a Whole Life Insurance Policy for $8000. How much does he pay in premiums in 5 yr.? In 10 yr.?

11. A man takes out a 20-Year Life Insurance Policy for $10,000. How much does he pay in premiums in 20 yr.?

12. A man takes out a 20-Year Endowment Policy for $12,000. How much does he pay in premiums in 5 yr.? In 9 yr.?

To find the Rate of Insurance

394 1. A policy of $28,000 costs $1260 for 3 yr. Find the annual rate.

Process

$1260 ÷ 3 = $420 Annual Premium.

$\frac{420}{28000} = \frac{3}{200} = 1\frac{1}{2}\%$. *Ans.*

Therefore, the method of finding the Annual Premium is similar to the method of finding what per cent one number is of another (Second Type Problem, p. 183).

PROBLEMS

1. The cost of insuring a house worth $12,000 for 3 yr. is $900. Find the annual rate.

2. The cost of insuring a house worth $24,000 at $\frac{2}{3}$ of its value for 3 yr. is $400. Find the annual rate.

3. One policy for $15,000 costs $562.50 for 3 yr. Another policy for $20,000 costs $900 for 3 yr. Which policy is at the lower rate?

4. The cost of a policy for $30,000 on a group of wooden buildings is $1050 for 3 yr. The cost of a policy for $26,000 for the same time on a fire-proof building is $585. The rate on the fire-proof building is what per cent of the rate on the wooden buildings?

To find the Face of a Policy

395. 1. $960 is paid for a 3-yr. policy of insurance at 2% per annum. Find the face of the policy.

Process

$960 ÷ 3 = $320, the Annual Premium.

Therefore, $320 = 2 % of the face of the policy.

The face of the policy = $16,000. *Ans.*

Therefore, the method of finding the face of a policy is similar to the method of finding the base when a part of it is given (Third Type Problem, Case 1, p. 191).

PROBLEMS

396. 1. Find the face of a policy, if the annual premium at $1\frac{1}{2}$ % equals $240.

2. Find the face of a policy, if the cost for 3 yr. at $2\frac{1}{2}$ % equals $1575.

3. A policy issued for 3 yr. on 80 % of the value of a house at 2 % costs $864.

(*a*) Find the face of the policy.

(*b*) Find the value of the house.

4. A policy issued for 2 yr. at 2 % per annum on 50 % of the value of a building costs $550. Find the face of the policy. Find the value of the house.

5. Mr. Brown's house is insured for 3 years for 80 % of its value, at $1\frac{1}{4}$ % per annum. Find the value of the house, if the premium for 3 years is $900.

USE OF THE EQUATION IN THE SOLUTION OF PROBLEMS

Interpretation of Algebraic Expressions

EXERCISES

NOTE TO THE TEACHER. The greatest difficulties in the solution of problems by the Equation Method are found not in the solution of the equation, but in the expression of the conditions and relations in algebraic terms. Continual practice should be given in the interpretation of algebraic expression (as in art. 397) and in the expression of arithmetical relations in algebraic terms (as in art. 398).

397. 1. If a hat costs \$3, what does $4 \times \$3$ represent?

Say, $4 \times \$3$ represents the cost of 4 hats at \$3 each.

2. If a hat costs x dollars, what does $4x$ represent?

Say, $4x$ represents the cost of 4 hats at x dollars each.

The multiplication sign is omitted.

3. If a bicycle weighs 18 lb., what does 5×18 lb. represent?

4. If x represents the weight of a bicycle, what is represented by $3x$? By $9x$?

5. If a workman receives \$35, what does $50 \times \$35$ represent? What does $500 \times \$35$ represent?

6. If x represents the amount of money paid to a workman, what does $20x$ represent? $100x$?

7. If a pound of coffee costs 42 ct., what is represented by 2×42 ct.? By $\frac{1}{2}$ of 42 ct.? By $\frac{1}{4}$ of 42 ct.?

8. If x represents the cost of a pound of coffee, what is represented by $4x$? By $\frac{1}{2}x$? By $\frac{1}{4}x$?

9. A train travels 25 mi. an hour. What does $2\frac{1}{2} \times 25$ represent? What does 10×25 represent?

10. If x represents the number of miles a train travels in an hour, what is represented by $3\frac{1}{2}x$? By $9x$?

11. If Mr. A earns \$30 a week, what is represented by $6 \times$ \$.30? By $52 \times$ \$30?

12. If x represents the amount of money Mr. A earns in a week, what is represented by $5x$? By $30x$?

13. A coat costs \$15; a hat costs \$10. What does \$15 + \$10 represent?

14. A coat costs x dollars; a hat costs \$8. What does $x+8$ represent?

15. A man who had 40 sheep sold 25 of them. What does $40-25$ represent?

16. A man who had x sheep sold 20 of them. What does $x-20$ represent?

17. A vase that weighs 8 lb. is packed in a box that weighs 2 lb. What does $8+2$ represent?

18. A vase that weighs 8 lb. is packed in a box that weighs x lb. What does $8+x$ represent?

19. I bought a chair for \$3.75, and gave the storekeeper a \$10 bill in payment. What does \$10 – \$3.75 represent?

20. I bought a chair for x dollars and gave the storekeeper a \$10 bill in payment. What does \$$10-x$ represent?

21. A hat that cost \$3 is sold at a gain of 25%. What does $\frac{5}{4}$ of \$3 represent?

22. A hat that cost x dollars is sold at a gain of 25%. What does $\frac{5}{4}x$ represent?

Representation of Quantities by Symbols

398. 1. One basket costs 60 ct. Represent the cost of 9 baskets.

Say, the cost of 9 baskets is represented by 9×60 ct.

2. A basket cost x ct. Represent the cost of 5 baskets.

Say, the cost of 5 baskets is represented by $5x$.

3. A family spends \$15 a week. Represent the amount spent in 6 wk.

4. A family spends x dollars a week. Represent the amount spent in 4 wk.

5. If a train travels 18 mi. an hour, how can we represent the distance traveled in 5 hr.?

6. If a train travels x mi. an hour, how can we represent the distance traveled in 8 hr.?

7. A boy who receives \$5 a week (for 6 da.) works only 4 da. Represent the amount of money he receives.

8. A boy receives x dollars a week (for 6 da.). He works only 3 da. Represent the amount he receives.

9. A man who had \$75 spent \$40. Represent the amount he had left.

10. A man who had spent \$75 spent x dollars. Represent the amount he had left.

11. Mr. White had \$200 in the bank. He has increased this by \$150. Represent the amount now in the bank.

12. Mr. White had \$200 in the bank. He has increased this amount by x dollars. How much has he in the bank now?

13. I bought an umbrella for \$5.50 and gave the storekeeper a \$10 bill in payment. Represent the amount I received in change.

14. I bought an umbrella for x dollars and gave the storekeeper a \$20 bill in payment. Represent the amount I received in change.

15. \$50 is spent for shirts at \$2 each. Represent the number of shirts bought.

16. x dollars is spent for shirts at \$2 a dozen. How many shirts are received for the money?

17. 18 lb. of tea cost \$9. Represent the cost of 1 lb.

18. 20 lb. of tea cost x dollars. Represent the cost of 1 lb.

19. \$150 is divided equally between 2 children. Represent the share each received.

20. x dollars is divided equally among 4 children. Represent the share each one receives.

21. One pound of sugar costs 8 ct. Represent the cost of $\frac{1}{2}$ lb. Of $\frac{3}{4}$ lb.

22. One pound of sugar costs x ct. Represent the cost of $\frac{1}{2}$ lb. Of $\frac{3}{4}$ lb.

23. A barrel of apples that cost $3.50 is sold at a gain of 18 %.

(*a*) Represent the gain.

(*b*) Represent the selling price.

24. A barrel of apples that cost x dollars is sold at a gain of 18 %.

(*a*) Represent the gain.

(*b*) Represent the selling price.

25. If x represents the distance a boat sails the first day, and 100 mi. the distance it sails the second day, represent the distance sailed in both days.

26. An ocean steamship is 800 ft. long; the Singer Building is x ft. high. The height of Singer Building is what part of the length of the ocean steamship?

27. A piano that cost $650 is sold at a gain of x dollars. Represent the gain per cent.

28. An automobile that cost $2750 is sold at a loss of x dollars. What part of the cost is lost?

29. The population of a city 10 yr. ago is represented by x. The population has increased 22 % during the last 10 yr.

(*a*) Represent the increase in population.

(*b*) Represent the population to-day.

30. A house that cost x dollars is sold at a loss of 9 %.

(*a*) Represent the loss.

(*b*) Represent the selling price.

31. \$ 200 is loaned for x yr. at 4 %. Represent the interest.

32. \$ 500 is loaned for x yr. at 5 %. Represent the amount.

33. If a pipe fills a tank in 3 hr., what part of it will it fill in 1 hr.?

34. If a pipe fills a tank in x hr., what part of it will it fill in 1 hr.?

35. One pipe can fill a tank in 2 hr.; another pipe will fill the tank in 3 hr. What part of the tank will be filled in 1 hr. by both pipes?

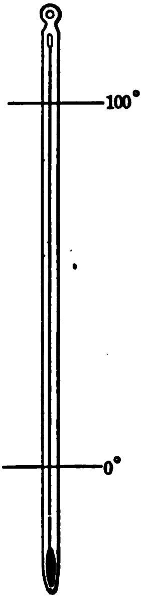
36. If one pipe can fill a tank in 2 hr. and another pipe can fill it in x hr., what part of the tank will be filled in 1 hr. by both pipes?

37. If one pipe can fill a tank in 3 hr. and another can empty it in x hr., what part of the tank will be filled in 1 hr. if both pipes are open?

NEGATIVE NUMBERS

For Reading and Discussion

399. In all the problems you have solved up to the present time, the numbers used were **positive numbers**. The lowest number was zero; all the

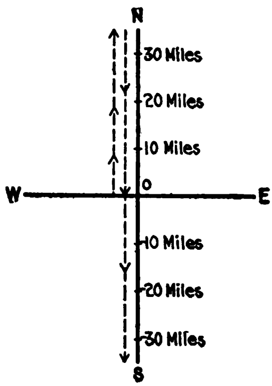


numbers used were greater than zero. But in some problems in arithmetic and algebra **negative** numbers are used.

The following illustrations show the meaning of negative numbers.

First illustration: Suppose that the thermometer shows that the temperature is 30° above 0 at 6 o'clock at night. We would represent the temperature by $+30°$, meaning 30° above 0. If the weather grew colder, the mercury would fall to 20°, to 10°, and then to 0°. Now suppose the weather grew still colder, so that the mercury fell 10° below zero; we would then say that the temperature was $-10°$. If the weather continued to grow colder, the temperature would be represented by $-20°$; $-30°$; etc.

Second illustration: Suppose a boat sails in a northerly direction from 0. If it sails 10 mi. an hour, its distance from 0 at the end of the first hour would be 10 mi.; at the end of the second hour 20 mi. The distance from the starting point, 0, will increase as long as the boat continues to sail in a northerly direction.



If the vessel turns and sails in a southerly direction, its distance from 0, at the end of the third hour, would be

$$10 + 10 - 10 = 10 \text{ mi. from } 0;$$

at the end of the fourth hour,

$$10 + 10 - 10 - 10 = 0 \text{ mi.},$$

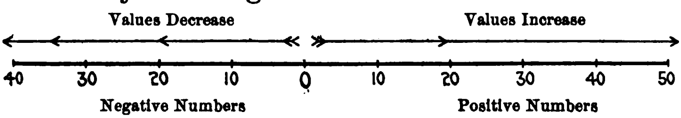
that is, it will be at 0 again.

If the vessel continues to sail toward the south, the distance from 0 at the end of the fifth hour would be -10 mi., that is, 10 mi. from 0, measured in a direction opposite to that in which the positive distances were measured. The next hour the vessel would be -20 mi. from 0, etc.

When no sign is written before a quantity, a plus sign is always understood.

There are as many numbers below zero as there are above zero. Numbers above zero are positive numbers; numbers below zero are negative numbers. The values of positive numbers **increase** from 0; the values of negative numbers **decrease** from 0. Thus $+100$ is greater than $+70$, but -100 is less than -70.

Positive and negative numbers may be represented by this diagram:



EXERCISES

400. 1. A thermometer registered 20° at 2 P.M.; by 3 P.M. it had risen 10°; by 5 P.M. it had fallen 20°. What was the temperature at 5 P.M.?

$$20° + 10° - 20° = ?$$

2. A thermometer registered 15° at 5 P.M.; by 6 P.M. it had fallen 10°; by 7 P.M. it had fallen 15° more. Find the temperature at 7 P.M.

3. A ship sailed from 0 towards the north for 2 hr. at the rate of 10 mi. an hour; then it sailed toward the south for 4 hr. at the rate of 8 mi. an hour. Represent the distance of the ship from 0 at the end of this time.

4. A storekeeper took in \$40 in the morning and \$50 in the afternoon. He paid bills amounting to \$10 and \$15. Represent these transactions by positive and negative numbers.

5. A car started on a trip with 10 passengers. At different stations 15, 25, and 50 people got on the car; also 5, 10, and 20 people left the car. Represent this by positive and negative numbers.

6. A submarine boat was on the surface of the water. It went 5 ft. below the surface; then sank 20 ft. more; then rose 12 ft.; then sank 6 ft.

(*a*) How far was the boat from the surface of the water?

(*b*) Represent the movements of the boat by a diagram.

7. An aeroplane starts from the ground and rises 100 ft.; then it sinks 20 ft. and rises 40 ft.; then it sinks 80 ft., sails parallel to the ground for 100 ft., and sinks 40 ft. more.

(*a*) How far is it from the ground now?

(*b*) Represent the flight by a diagram.

8. A man had $1000 in the bank. He deposited $50, $70, $40, and $200, and withdrew $20, $60, $70, $100. How much is still in the bank?

9. A passenger agent travels 410 mi. north on Monday, 175 mi. north on Tuesday, 350 mi. south on Wednesday, and 275 mi. south on Thursday. How far was he from the starting point? Illustrate graphically.

401. SOLUTION OF EQUATIONS

$3x - 125 = 275$ is an **equation.**

$3x$ is the **unknown quantity** or **unknown number.**

$3x - 125$ is the **first member** of the equation.

275 is the **second member** of the equation.

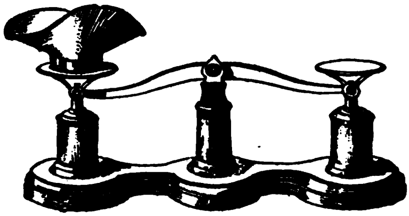
Finding a **value for x** is called **solving** the equation.

402. $3x$ means 3 times x or $3 \times x$; the number **3** is called a **coefficient** of x. When no coefficient is written, the coefficient 1 is understood. x means $1 \times x$ or $1x$.

Important Principles of the Equation

403. 1. Solve: $x - 30 = 75$.

An equation resembles a scale; both sides must be equal.



If 30 lb. are added to the first scale pan, how much must be added to the second scale pan to preserve the balance?

$$\begin{array}{rr} x - 30 = & 75 \\ \text{(Add 30 to each member.)} \quad \underline{30} & \underline{30} \\ x = & 105 \end{array}$$

404. Therefore,

> **First Principle of the Equation**
> **The same number may be ADDED to each member of an equation without affecting the equality.**

2. Solve: $x + 20 = 50$.

Subtract 20 from the first scale pan.

How much is in the first pan now?

How much must be subtracted from the second pan to preserve the equality?

$$\begin{array}{rr} x + 20 = & 50 \\ \text{(Subtract 20 from each member.)} \quad \underline{20} & \underline{20} \\ x = & 30 \end{array}$$

405. Therefore,

> **Second Principle of the Equation**
> **The same number may be SUBTRACTED from each member of an equation without affecting the equality.**

406 In a similar manner the following may be shown:

> **Third Principle of the Equation**
> **Both members of an equation may be MULTIPLIED by the same number without affecting the equality.**

> **Fourth Principle of the Equation**
> **Both members of an equation may be DIVIDED by the same number without affecting the equality.**

EXERCISES

407. Solve each equation. (Prove those marked *.)

*1. $x+5=10$.

2. $x+9=15$.

*3. $x+3=9$.

4. $x-12=4$.

5. $x-3=8$.

6. $x-5=1$.

*7. $x+2=19$.

8. $x-9=4$.

*9. $x+7=3$.

10. $x-2=0$.

*11. $3x=15$.

12. $5x=30$.

13. $10x=75$.

*14. $\frac{x}{2}=6$.

15. $\frac{x}{3}=15$.

16. $\frac{x}{4} = 2\frac{1}{2}$.

17. $6x = 19$.

*18. $\frac{x}{5} = 10$.

19. $7x = 42$.

20. $\frac{x}{6} = 4$.

*21. $3x + 5 = 14$.

22. $2x - 1 = 11$.

23. $\frac{x}{2} + 4 = 5$.

24. $3x + 1 = 22$.

*25. $5x = 17$.

26. $\frac{x}{3} + 4 = 9$.

*27. $2x + 2\frac{1}{2} = 5\frac{1}{2}$.

28. $\frac{x}{4} + 3 = 7.25$.

29. $4x - 12 = 27$.

*30. $6x - 2 = 0$.

PROBLEMS

408. Solve these problems by the Equation Method:

1. 5 blank books cost 35 ct. Find the cost of 1 book.

Let $x =$ cost of 1 book.
Then $5x =$ cost of 5 books,
and $5x = 35$ ct.
$\therefore x = 7$ ct. *Ans.*

2. If a boy walks 9 mi. in 3 hr., how far does he walk in 1 hr.?

3. A steamship burns 1800 T. of coal in 5 da. How much coal is used each day?

4. A man is paid $254 for working 30 da. How much does he receive a day?

5. A machine prints 300 papers in 6 min. How many papers are printed each minute?

6. An expressman must deliver 1200 packages. His wagon can carry only 50 packages. How many trips must be made to deliver all the packages?

7. If 1920 soldiers march 16 abreast, how many rows of soldiers will there be?

8. Find the cost of 1 doz. blank books, if 18 doz. cost $10.80.

9. A train travels 175 mi. in 7 hr. How far does it travel in an hour?

10. How many baskets are required to hold 456 bottles, if each basket holds 24 bottles?

CLEARING EQUATIONS OF FRACTIONS

409. 1. Find the value of x in the equation $\frac{x}{4} = 16$.

Multiply each member by 4. $x = 64$. *Ans.*
(See Principle number 3, p. 289.)

Proof. $\frac{x}{4}$ should equal 16. $\frac{1}{\cancel{4}}$ of $\overset{16}{\cancel{64}} = 16$.

2. Solve: $\frac{x}{3} = 12\frac{1}{2}$.

Multiply each member by 6 (the l. c. m. of the denominators).

$$(\overset{2}{\cancel{6}} \times \frac{x}{\cancel{3}} = 2x;\ 6 \times 12\tfrac{1}{2} = 75.)$$

$$2x = 75.$$

$$x = 37\tfrac{1}{2}. \quad \textit{Ans.}$$

Proof. $\frac{x}{3}$ should equal $12\frac{1}{2}$.

410. RULE. **To clear equations of fractions: MULTIPLY EVERY TERM in the equation by the L. C. M. of the denominators of the fractions.**

EXERCISES

411. Clear these equations of fractions; then find values for x. (Prove those marked *.)

* **1.** $\frac{x}{2}=15$

2. $\frac{x}{3}=8$

3. $\frac{x}{5}=4$

4. $\frac{2x}{3}=6$

* **5.** $\frac{5x}{6}=20$

6. $\frac{3x}{2}=9$

7. $\frac{x}{2}=4\frac{1}{2}$

8. $\frac{x}{4}=3\frac{1}{2}$

9. $\frac{x}{7}=1\frac{1}{4}$

* **10.** $\frac{2x}{5}=6\frac{1}{2}$

11. $\frac{4x}{9}=15$

12. $2x+\frac{x}{2}=5$

* **13.** $2x-\frac{x}{3}=10$

14. $3x-\frac{x}{2}=14$

15. $6x-\frac{2x}{3}=17$

16. $\frac{9x}{10}=72$

* **17.** $1.25x=15$

18. $2.50x=25$

19. $1.10x=33$

* **20.** $3\frac{1}{2}x=21$

21. $\frac{x}{2}+1.25x=3\frac{1}{2}$

PROBLEMS

412. Solve the following problems by the Equation Method:

1. $\frac{1}{2}$ lb. of tea costs 25 ct. Find the cost of 1 lb.

2. $\frac{1}{4}$ lb. of pepper costs 9 ct. Find the cost of 1 lb.

3. $\frac{2}{3}$ of the height of a building is 90 ft. How high is the building?

4. $\frac{3}{4}$ of the number of boys in a school is 336. How many boys are there in the school?

5. $\frac{5}{6}$ of the weight of a machine is 65 lb. Find the weight of the machine.

6. $3\frac{1}{2}$ yd. of ribbon cost 21 ct. Find the cost of 1 yd.

7. If a man earns $ 30 in $\frac{2}{3}$ of a week, how much will he earn in a week?

8. $\frac{1}{2}$ the length of a room is $15\frac{1}{2}$ ft. Find the length of the room.

9. $\frac{3}{4}$ of the amount of the money collected for a present is $ $82\frac{1}{2}$. How much money was collected?

10. The money gained by selling a desk is $ $8\frac{1}{2}$. This is $33\frac{1}{3}\%$ of the cost. Find the cost.

11. The advance in the price of meat was $7\frac{1}{2}$ ct. a pound. This was 25 % of its former price. Find the former price.

12. The width of a plot of ground is 225 % of the length. The width is 81 ft. Find the length.

13. The number of bushels of wheat produced on a field this year is $12\frac{1}{2}\%$ greater than last year. If the number produced this year is 1260 bu., how many bushels were produced last year?

14. By using a new kind of furnace, a company required $16\frac{2}{3}\%$ less coal this year than last year. If the company used 240 T. this year, how many tons were used last year?

15. The number of girls in a school is $\frac{1}{2}$ as great as the number of boys. The total number of pupils is 600. How many girls are there? How many boys?

TRANSPOSING TERMS IN AN EQUATION

413. The **first member** of an equation should contain only **unknown quantities**; the **second member** should contain only **known quantities.**

For example, in $3x = 21$, $3x$ is an unknown quantity and is in the first member; 21 is a known quantity and is in the second member.

Frequently equations contain quantities that are not in the proper member, as

$$3x + 5 = 26, \text{ or } 3x - 7 = x + 21.$$

Here 5 should be in the second member; 7 should be in the second member; x should be in the first member.

Subtract 5 from each member (Principle 2, p. 289).

$$\begin{array}{rcl} 3x + 5 & = & 26 \\ 5 & & 5 \\ \hline 3x & = & 26 - 5 \end{array}$$

We see that 5 may be changed or **transposed** from the first to the second member by **changing its sign.**

	$3x - 7 = x + 21$
Add 7 to each member (Principle 1, p. 289).	$\underline{\quad 7 \qquad 7}$
	$3x \quad = x + 21 + 7$
(Subtract x from each member.)	$\underline{\ x \qquad x}$
	$3x - x = 21 + 7$
	or
	$2x \quad = 28$

7 is transposed from the first to the second member by changing its sign.

x is transposed from the second to the first member by changing its sign.

Therefore,

414. RULE FOR TRANSPOSING TERMS. **Any term may be transposed from one member of an equation to the other, provided its sign is changed.**

415. Solve the following equations. (Prove those marked *.)

*1. $x + 4 = 7$

2. $x - 6 = 15$

3. $2x + 3 = 9$

4. $3x - 5 = 10$

5. $2x = 7 - x$

*6. $3x = 10 + x$

7. $2x + 5 = x = 9$

8. $4x - 2 = x + 7$

9. $5x + 6 = 2x + 19$

*10. $3x - 5 = 8 - 2x$

11. $\dfrac{x}{2} + 3 = 15$

12. $\dfrac{3x}{4} - 2 = 10$

*13. $\frac{2x}{3}+\frac{1}{2}=\frac{x}{2}+9$

14. $2x+\frac{3}{4}=x+6$

15. $5x+2=\frac{x}{2}+4$

16. $10x=28-3x$

*17. $4x+\frac{1}{2}=2x+1$

18. $3x+350=2x+500$

19. $3x+9=\frac{x}{3}+12$

*20. $5x-6=\frac{2x}{3}+20$

PROBLEMS

Solve the following problems by the Equation Method:

1. William and John have $200; John has $50. How much money has William?

2. 5 new pupils were admitted to a class, making 42 pupils on register. How many pupils were in the class at first?

3. 6 pupils left a class, making the register 40. How many pupils were in the class at first?

4. The weight of a statue and a wooden base is 25 lb. The base weighs 6 lb. Find the weight of the statue.

5. A man drew $250 from a bank and had $625 left. How much did he have in the bank originally?

6. The total distance sailed by a ship in two days was 235 mi. If it sailed 110 mi. the first day, how many miles did it sail the second day?

7. William and John have $200; John has $50 more than William. How much money has each?

8. A log of wood was 18 ft. long. A piece was cut off, leaving the log $12\frac{1}{2}$ ft. long. Find the length of the piece that was cut off.

9. A piece of wood 7 ft. long was cut from a log, leaving the log $10\frac{1}{2}$ ft. long. How long was the log before the piece was cut off?

10. A log was cut into two pieces, one $5\frac{1}{2}$ ft. long, the other $9\frac{3}{4}$ ft. long. How long was the log before it was cut?

11. The length of a string was increased to 16 ft. by tying on a piece $4\frac{1}{2}$ ft. long. How long was the string originally?

12. The population of a city increased 75,000 during the last 10 yr., thereby making the population 650,000. Find the population of 10 yr. ago.

13. A train left New York with a certain number of passengers. At the first station 60 more people got on the train. There were then 110 people on the train. How many were on the train when it left New York?

14. 48 pupils graduated from a school and 100 new pupils were admitted. After these changes the register was 900. How many pupils were on register before the changes had been made?

15. A wagon that weighed 125 lb. carried 40 packages. The total weight of the wagon and all the packages is 325 lb. Find the weight of each package.

REMOVAL OF PARENTHESES

Changing of Signs

416. A **parenthesis** is a sign, ( ), used to group two or more quantities. All the quantities in a parenthesis are to be considered as one quantity.

417. A parenthesis preceded by a PLUS sign may be removed without changing the signs of any of the quantities within the parenthesis.

1. Solve: $x+(x+6)=10.$

Therefore, $x+(x+6)=10$ becomes

$$x+x+6=4.$$
$$2x=4.$$
$$x=2. \quad Ans.$$

418. A parenthesis preceded by a MINUS sign may be removed if the SIGN of EACH QUANTITY within the parenthesis IS CHANGED (from + to − or from − to +).

2. Solve: $3x-(x+6)=6.$

Therefore, $3x-(x+6)=6$ becomes

$$3x-x-6=6.$$
$$2x=6+6=12.$$
$$x=6. \quad Ans.$$

419. A parenthesis preceded by a COEFFICIENT may be removed if each quantity within the parenthesis IS MULTIPLIED by the coefficient. The sign of each quantity must be changed if the coefficient is preceded by a minus sign.

3. Solve: $5(x-2)=20$.

Therefore, $5(x-2)=20$ becomes

$$5x-10=20.$$
$$5x=30.$$
$$x=6. \quad \textit{Ans.}$$

4. Solve: $\frac{1}{2}(2x-8)=4$.

Multiplying each quantity within the parenthesis by $\frac{1}{2}$,

$$x-4=4.$$
$$x=8. \quad \textit{Ans.}$$

5. Solve: $\frac{3}{4}(4x-8)=2x+1$.

Multiplying each quantity within the parenthesis by $\frac{3}{4}$,

$$3x-6=2x+1.$$
$$3x-2x=6+1.$$
$$x=7. \quad \textit{Ans.}$$

EXERCISES

420. Solve each of the following. (Prove those marked *).

*1. $2x+(x-2)=3$

2. $2x+(x+6)=9$

3. $5x-(x-3)=6$

4. $3x-(2x+4)=4$

5. $4x-(3x-5)=5$

*6. $2(x+3)=7$

7. $5(x-6)=10$

*8. $(x+4)+(x+2)=14$

9. $(x-9)+(x+5)=8$

*10. $2(x+3)+2(x-1)=8$

11. $\frac{1}{2}(4x-2)=8$

12. $\frac{1}{4}(8x-12)=6$

*13. $3x+\frac{1}{2}(x+4)=11$

14. $2x-\frac{1}{3}(3x+9)=5$

15. $2x+4(x-3)=3x+15$

*16. $24-x=15-3(x-10)$

17. $\frac{3}{4}(4x-12)=\frac{1}{2}(x+6)$

18. $\frac{2}{3}(6x-9)=\frac{1}{4}(12x+4)$

*19. $\frac{1}{2}(x-9)=\frac{1}{3}x$ 20. $.25(x-2)=.5$

PROBLEMS

421. 1. A boat sails 300 mi. in 3 da. The second day it sails 25 mi. more than the first day; the third day it sails 35 mi. more than the first day. How far did it sail the first day?

HINT. Let x = number of miles it sailed the first day.

2. A boat sails 300 mi. in 3 da. The second day it sails 40 mi. more than the first day; the third day it sails 20 mi. more than the second day. How far did it sail the first day?

HINT. Let x = number of miles sailed the first day.

3. A boy took a bicycle trip of 90 mi. The second day he rode 8 mi. more than the first day; the third day he rode 5 mi. less than the second day. How many miles did he ride each day?

4. The length of a rectangle is 4 ft. greater than the width. The perimeter of the rectangle is 24 ft. Find the length and the width.

5. A grocer sold 25 lb. of sugar and 30 lb. of flour for \$4.45. The sugar was sold for 2 ct.

more than the flour. Find the cost of a pound of flour and a pound of sugar.

HINTS. Let x = cost of 1 lb. of flour.
Then $(x + ?)$ = cost of 1 lb. of sugar.
And $(25x)$ = cost of 25 lb. of flour.
And $30(x + ?)$ = cost of 30 lb. of sugar.
Therefore, $25x + 30(x + ?) = \$4.45$.

6. There are two qualities of tea; the better quality costs 10 ct. more than the cheaper quality. Find the cost of 1 lb. of each quality, if 10 lb. of the better kind and 10 lb. of the poorer kind are sold for $ 9.*

7. 15 men and 30 boys receive $ 390 for one weeks' work. The men receive $ 8 more per week than the boys. How much does each boy receive per week?

8. At the beginning of a school term, the number of girls on register was 250 greater than the number of boys. During the term 70 new boys and 90 new girls were admitted, making a new register of 800. How many boys and how many girls were on register at the beginning of the term?

9. Change the conditions in problem 8 so as to make a new problem. Solve the problem.

* NOTE. Remember that both members of an equation must be of the same denomination; if the first member is cents, the second member must also be cents.

SIMPLE INTEREST

For Reading and Discussion

422. In business it often becomes necessary to borrow money. Banks, Trust Companies, and individuals make a business of lending money to persons offering good security. For the use of this money, interest is paid. Moreover, Banks and Trust Companies, themselves, pay interest to depositors on savings and balances left on deposit for the required time.

423. Money for the use of which interest is paid is called the **Principal**.

The **rate of interest** is the number of hundredths paid for the use of the principal for 1 yr.

The number of years, months, and days for which interest is paid is called the **time**.

Money paid for the use of the principal is called **interest.**

The **amount** is the sum of the principal and interest.

Note. The most common rate of interest is 6 %. Money is frequently loaned at 3 %, 4 %, 5 %, or $5\frac{1}{2}$ %. Savings banks usually pay 3 %, $3\frac{1}{2}$ %, or 4 % interest. Large sums of money are sometimes loaned at low rates of interest, *e.g.* 2 % or $2\frac{1}{2}$ %.

In many states it is illegal to charge more than 6 % interest. The charging of more than the legal rate of interest is called **usury**.

TO FIND THE SIMPLE INTEREST AND THE AMOUNT

The Aliquot Parts Method for Years and Months

424. 1. Find the simple interest and amount of $800 for 3 yr. at 6%.

1st Process	2d Process
$800	Int. for 1 yr. = .06 of principal.
.06	Int. for 3 yr. = .18 of principal.
$48.00 Int. for 1 yr. at 6%.	$800
3	.18
$144.00 Int. for 3 yr. at 6%.	$144 Int. for 3 yr. at 6%.
$800 + $144 = $944, Amt. *Ans.*	$800 + $144 = $944, Amt. *Ans.*

RULE. **To find the interest.**

Interest = Principal × Rate × Number of Years.

2. Find the interest and amount of $3000 for 1 yr. 3 mo. at 4%.

1st Process	2d Process
1 yr. 3 mo. = $1\frac{1}{4}$ yr.	1 yr. 3 mo. = $1\frac{1}{4}$ yr.
$3000	Int. at 4% for $1\frac{1}{4}$ yr. = .05 of principal.
.04	$3000
$120. Int. for 1 yr.	.05
30 Int. for 3 mo. or $\frac{1}{4}$ yr.	
$150 Int. for 1 yr. 3 mo. *Ans.*	$150 Interest. *Ans.*
$3000 + $150 = $3150 Amt.	$3000 + $150 = $3150 Amt.

Aliquot Parts Method applied to Days

425. 1. Find the interest on $1200 for 2 yr. 6 mo. 15 da. at 4%.

Process

Int. on $1200 at 4% for 1 yr.	=	$ 48
Int. on $1200 at 4% for 1 yr. more	=	48
Int. on $1200 at 4% for 6 mo. = $\frac{1}{2}$ of $48	=	24
Int. on $1200 at 4% for 15 da. = $\frac{1}{2}$ of $\frac{1}{12}$ of $48	=	2
Int. for 2 yr. 6 mo. 15 da.		$122 *Ans.*

NOTE. For computing interest, a month contains 30 da.

Cancelation Method

426. 1. Find the interest on $900 for 3 mo. 15 da. at 4%.

Process

Change time to days, counting 30 da. to the month, or 360 da. to the year.

$$\begin{aligned} 3 \text{ mo.} &= 90 \text{ da.} \\ 15 \text{ da.} &= \underline{15} \text{ da.} \\ \text{Total time} &= 105 \text{ da.} \end{aligned}$$

$$\$\overset{9}{\cancel{900}} \times \frac{4}{\cancel{100}} \times \frac{\overset{21}{\cancel{105}}}{\underset{\substack{\cancel{40} \\ \cancel{10} \\ 2}}{\cancel{360}}} = \$10.50. \quad \textit{Ans.}$$

NOTE. The rate is written as a common fraction ($\frac{4}{100}$). Do not reduce the rate to lowest terms.

2. Find the interest on $225 for 47 da. at 5%.

Process

$$\$\overset{2.25}{\cancel{225}} \times \frac{\cancel{5}}{\cancel{100}} \times \frac{47}{\underset{72}{\cancel{360}}} = \frac{\$105.75}{72} = \$1.468^{+} \text{ or } \$1.47. \quad \textit{Ans.}$$

NOTE. 100 in the denominator is canceled by pointing off two decimal places to the left in the numerator.

To find the Difference between Dates

427. In solving problems in interest, it is frequently necessary to find the exact time between dates.

When the Time between two Dates is Less than a Year

1. Find the interest on $ 150 at 6% from March 5 to Nov. 26.

Process

Number of days left in first month	26
Number of days in intervening months	
(30 + 31 + 30 + 31 + 31 + 30 + 31)	214
Number of days in last month	26
Total number of days	266

NOTE. This method gives the exact number of days, when the time is less than 1 yr. If the exact number of days is not desired, the difference between the dates may be found by subtraction, as shown in examples number 4 and 5 on page 104.

Using the "Cancelation Method":

$$\$150 \times \tfrac{6}{100} \times \tfrac{266}{360} = \$6.65. \quad \textit{Ans.}$$

When the Time between two Dates is Greater than a Year

2. Find the interest on $ 350 at 4% from Jan. 5, 1910, to Nov. 3, 1913.

Process

From Jan. 5, 1910, to Jan. 5, 1913, is 3 yr.
From Jan. 5, 1913, to Oct. 5, 1913, is 9 mo.
From Oct. 5, 1913, to Nov. 3, 1913, is 29 da.
Total time, 3 yr. 9 mo. 29 da.

x

NOTE. The method of finding the difference between dates when the time is greater than one year is the legal method in New York. The difference in time can also be found by subtraction as in numbers 4 and 5, page 104.

To find the answer, use either the "Aliquot Parts Method," p. 303, or the following method:

Six Per Cent Method

428. 1. Find the interest on $500 for 2 yr. 6 mo. 18 da. at 6%.

This method is based on the following:

At 6%, the interest on $1 for 1 yr. is $.06.

At 6%, the interest on $1 for 1 mo. is $\frac{1}{12}$ of $.06 or $.005.

At 6%, the interest on $1 for 1 da. is $\frac{1}{30}$ of $.005 or $.000$\frac{1}{6}$.

Process

Interest on $1 for 2 yr. = 2 × $ .06 = $.12
Interest on $1 for 6 mo. = 6 × $ .005 = $.03
Interest on $1 for 18 da. = 18 × $.000$\frac{1}{6}$ = $.003
Interest on $1 for 2 yr. 6 mo. 18 da. = $.153
Interest on $500 for 2 yr. 6 mo. 18 da. = 500 × $.153
= $76.50. *Ans.*

2. Find the interest on $400 for 4 yr. 3 mo. 24 da. at 4%.

Process

Interest on $1 for 4 yr. = 4 × $.06 = $.24
Interest on $1 for 3 mo. = 3 × $.005 = $.015
Interest on $1 for 24 da. = 24 × $.000$\frac{1}{6}$ = $.004
$.259

Interest on $400 for 4 yr. 3 mo. 24 da. at 6 % = 400 × $.259 = $103.60.

Therefore, interest at 4 % = $\frac{2}{3}$ of $103.60 = 69.06\frac{2}{3}$ or $69.07. *Ans.*

RULE for finding interest by the Six Per Cent Method.

First: **Find the interest on $1 for the given time at 6 %.**

Second: **Multiply the interest on $1 for the given time at 6 %, by the principal.**

Third: **For rates other than 6 %, proceed as in Art. 429.**

429. For rates other than 6 % :

For 2 %, take $\frac{1}{3}$ of the interest at 6 %.

For 3 %, take $\frac{1}{2}$ of the interest at 6 %.

For 4 %, subtract $\frac{1}{3}$ of the interest at 6 %.

For 5 %, subtract $\frac{1}{6}$ of the interest at 6 %.

ORAL EXERCISES

430. Find the interest on $200 at 6 %

1. For 1 yr.	4. For 2$\frac{1}{2}$ yr.	7. For $\frac{1}{2}$ yr.
2. For 2 yr.	5. For 5 yr.	8. For $\frac{3}{4}$ yr.
3. For 1$\frac{1}{2}$ yr.	6. For 3$\frac{1}{2}$ yr.	9. For $\frac{2}{3}$ yr.

EXERCISES

431. Find the interest on :

	Principal	Time	Rate		Principal	Time	Rate
1.	$600	1 yr.	5 %.	6.	$800	2 yr.	3 %.
2.	$800	1 yr.	6 %.	7.	$800	2 yr.	5 %.
3.	$1200	1 yr.	3 %.	8.	$600	1 yr.	2 %.
4.	$200	1 yr.	2 %.	9.	$600	1 yr.	5$\frac{1}{2}$ %.
5.	$700	1 yr.	4 %.	10.	$350	2 yr.	3$\frac{1}{2}$ %.

	Principal	Time	Rate		Principal	Time	Rate
11.	$900	1 yr.	4%.	16.	$2500	3 yr.	3%.
12.	$900	3 yr.	4%.	17.	$1500	2 yr.	5%.
13.	$900	2 yr.	6%.	18.	$2000	4 yr.	3%.
14.	$900	3 yr.	6%.	19.	$2000	4 yr.	4%.
15.	$900	3 yr.	5%.	20.	$2000	4 yr.	5%.

EXERCISES

432. Find the interest and amount on each of the following:

1. $500 for 2 yr. 6 mo. at 5%.
2. $935 for 3 yr. 6 mo. at 5%.
3. $750 for 2 yr. 3 mo. at 6%.
4. $760 for 4 yr. 2 mo. at 4%.
5. $1050 for 2 yr. 4 mo. at 4%.
6. $275 for 3 yr. 8 mo. at 6%.
7. $295 for 2 yr. 8 mo. at 3%.
8. $415 for 2 yr. 1 mo. at 3%.
9. $350 for 2 yr. 9 mo. at 2%.
10. $490 for 3 yr. 9 mo. at 7%.

EXERCISES

433. Find the interest and amount in each of the following:

1. On $2400 from May 1, 1909, to May 1, 1912, at 4%.

2. On $1800 from May 1, 1909, to Nov. 1, 1912, at 6%.

3. On $1250 from Jan. 15, 1904, to Aug. 15, 1907.

4. On $2100 from April 10, 1909, to Oct. 10, 1912.

5. On $790 from Nov. 20, 1910, to Jan. 10, 1913.

6. On $475 from Oct. 1, 1908, to April 1, 1911.

7. On $1340 from Aug. 25, 1905, to Dec. 25, 1909.

8. On $2650 from Feb. 14, 1910, to Jan. 14, 1914.

9. On $3275 from Dec. 20, 1910, to June 20, 1913.

10. On $5500 from Sept. 25, 1910, to July 25, 1914.

EXERCISES

434. Find the interest and the amount on

1. $1000 for 84 da. at 5%.
2. $360 for 180 da. at 5%.
3. $1980 for 250 da. at 6%.
4. $240 for 6 mo. 15 da. at 6%.
5. $720 for 8 mo. 21 da. at 4%.
6. $950 for 2 yr. 4 mo. 15 da. at 4%.
7. $1425 for 1 yr. 6 mo. 10 da. at 3%.
8. $320.25 for 1 yr. 2 mo. 20 da. at 3%.
9. $510.75 for 1 yr. 4 mo. 12 da. at 2%.

10. \$825.50 for 2 yr. 9 mo. 20 da. at 2%.
11. \$1400 for 1 yr. 3 mo. 18 da. at 6%.
12. \$2450 for 1 yr. 9 mo. 24 da. at 4%.
13. \$1760 for 1 yr. 4 mo. 15 da. at 5%.
14. \$1275 for 3 yr. 2 mo. 12 da. at 3%.
15. \$4500 for 2 yr. 10 mo. 6 da. at 7%.
16. \$14,000 for 4 yr. 1 mo. 18 da. at $2\frac{1}{2}$%.
17. \$27,500 for 2 yr. 6 mo. 12 da. at $5\frac{1}{2}$%.
18. \$30,000 for 3 yr. 3 mo. 21 da. at $3\frac{1}{2}$%.
19. \$19,750 for 3 yr. 7 mo. 15 da. at $4\frac{1}{2}$%.
20. \$26,900 for 2 yr. 9 mo. 18 da. at 7%.

EXERCISES

435. In each of the following find first the number of days, then the interest:

	Principal	Rate	Time
1.	\$ 125	4 %	From Jan. 15 to Dec. 10.
2.	\$ 640	6 %	From Dec. 20 to July 15.
3.	\$ 335	4 %	From Aug. 16 to March 20.
4.	\$ 728	5 %	From Feb. 20 to Nov. 25.
5.	\$ 960	6 %	From July 15 to Feb. 20.
6.	\$ 340	5 %	From Oct. 10 to May 12.
7.	\$ 685	$3\frac{1}{2}$%	From March 25 to Dec. 15.
8.	\$1215	$4\frac{1}{2}$%	From Sept. 1 to March 15.
9.	\$ 395	6 %	From April 12 to Oct. 22.
10.	\$ 275	$5\frac{1}{2}$%	From May 18 to Sept. 30.

EXERCISES

436. Find the interest and the amount on:

	Principal	Rate	Time
1.	$ 800	4 %	From Jan. 5, 1909, to Nov. 26, 1912.
2.	$1100	6 %	From Oct. 20, 1911, to Jan. 15, 1915.
3.	$1950	5 %	From March 10, 1908, to Dec. 23, 1911.
4.	$2225	3 %	From Dec. 15, 1910, to March 12, 1913.
5.	$ 740	3½ %	From Feb. 20, 1909, to Oct. 20, 1912.
6.	$1290	4½ %	From July 10, 1912, to Dec. 15, 1915.
7.	$1500	2½ %	From Nov. 10, 1910, to May 20, 1913.
8.	$ 975.50	5½ %	From April 25, 1910, to Sept. 15, 1912.
9.	$ 650.25	6 %	From May 5, 1907, to April 10, 1910.
10.	$1240.75	5 %	From June 17, 1909, to July 20, 1912.

PROBLEMS

437. 1. How much greater is the interest on $2000 for 2 yr. at 6 % than at 4 %?

2. How much greater is the interest on $1500 for 2 yr. 6 mo. at 6% than at 3%?

3. How much less is the interest on $1250 for 3 yr. 2 mo. at 3% than at 5%?

4. Mr. Jones buys a house worth $25,000. He pays $15,000 cash and gives a mortgage for the balance at 4%. How much interest is paid each 6 mo.?

5. I loan $60,000 for 2 yr. On $\frac{2}{3}$ of it I receive 4% interest; on the balance I receive 5% interest. Find the total interest.

6. How much greater is the interest on $3600 for 3 yr. 6 mo. at 6% than for 2 yr. 3 mo. at 6%?

7. A savings bank lowered the rate of interest it paid from 4% to $3\frac{1}{2}$% per annum. How much less interest is paid for 6 mo. to a man who has $1950 on deposit?

8. A company had been paying 6% interest on $250,000 each year. It borrowed $250,000 from a man who was willing to accept only $4\frac{1}{2}$% interest. How much will the company save in 5 yr. with lower rate of interest?

9. I loaned $1200 to Mr. Furness on July 16, 1909. The money was repaid on Oct. 24, 1911, with interest at 4%. How much was repaid?

10. 360 tables worth $15 each were bought on May 20, 1909. The bill was paid on Nov. 25, 1909, with interest at 6%. Find the amount paid.

11. A bill for $500 should have been paid on April 20, 1911. It was paid on Aug. 30, 1911, with interest at 5%. How much was paid?

12. A debt of $2500 was due on March 15, 1909. It was paid on Sept. 30, 1910, with interest at 4%. How much was paid on Sept. 30, 1910?

13. A man bought $2000 worth of goods on May 1, 1910. If he had paid cash, he would have received a discount of 2%. Instead of paying cash, he paid the debt on Oct. 30, 1910, with interest at 4%. How much did he lose by not paying cash?

14. A man bought 10 lots at an average price of $1500 per lot. He paid $\frac{1}{3}$ of the cost in cash, and allowed the balance to remain on mortgage at $5\frac{1}{2}$% per annum. How much interest will he pay on the mortgage in 3 yr.?

EXACT INTEREST OR ACCURATE INTEREST

438. Most of the methods of computing interest used by business men are based on 30 da. in a month or 360 da. in a year. These methods are, therefore, slightly inaccurate, but business men use them because they are convenient.

The United States Government computes interest on the basis of 365 da. to the year. This interest is called Exact Interest or Accurate Interest.

RULE. **Find the interest for the number of whole years by the "Aliquot Parts Method" (p. 303), and for the remaining fraction of a year by the "Cancelation Method," using 365 days to the year, or in a leap year, 366 days.**

1. Find the exact interest on \$450 for 135 da at 4%.

Process

$$\$\overset{4.50}{\cancel{450}} \times \frac{4}{\cancel{100}} \times \frac{\overset{27}{\cancel{135}}}{\underset{73}{\cancel{365}}} = \$6.657 \text{ or } \$6.66.$$

EXERCISES

Find the exact or accurate interest on:

1. \$320 for 140 da. at 6%.

2. \$540 for 300 da. at 4%.

3. \$275 for 275 da. at 3%.

4. \$160.40 from May 26, 1913, to Oct. 9, 1915, at 5%.

5. \$280.75 from Jan. 18, 1911, to July 15, 1914, at 6%.

6. \$350 from Feb. 4, 1912, to Aug. 1, 1912, at 6%.

7. \$725 from March 19, 1911, to Nov. 16, 1912, at 4%.

8. \$1900 from May 6, 1910, to Dec. 21, 1910, at 5%.

9. $ 1750 from April 5, 1912, to Oct. 19, 1916, at 2%.

10. $ 2570 from Jan. 10, 1909, to July 31, 1914, at $3\frac{1}{2}$%.

11. How much greater is the exact interest on $ 2250 for 1 yr. at 6% than the simple interest for the same time at the same rate?

12. If you borrowed money, would you prefer to pay simple interest on the basis of 360 da. to a year, or exact interest? Why?

INDIRECT CASES OF INTEREST

To find the Rate Per Cent when the Principal, Interest, and Time are Given

439. 1. At what rate must $ 300 be loaned, to yield $ 42 interest in 2 yr. 4 mo.?

This problem corresponds to First Type Problem, page 183.

Process

Interest on $ 300 for 2 yr. 4 mo. at **1** % = $ 7.*

Therefore, the rate per cent required to yield $ 42 interest is as much as $ 42 ÷ 7 or 6. 6 %. *Ans.*

RULE. **To find the rate when the principal, interest, and time are given: Divide the** GIVEN INTEREST **by the** INTEREST **on the given principal for the given time at 1 %.**

EQUATION. **Rate = Interest ÷ Interest on Given Principal at 1 %.**

* First find 1 % of $ 300; then multiply by $2\frac{1}{3}$.

EXERCISES

440. 1. At what rate will $300 yield $67.50 interest in 4 yr. 6 mo.?

2. At what rate will $450 yield $76.50 interest in 2 yr. 10 mo.?

3. At what rate will $480 yield $50.40 interest in 2 yr. 4 mo.?

4. At what rate will $540 yield $85.59 interest in 2 yr. 7 mo. 21 da.?

5. At what rate will $2250 yield $648 interest in 6 yr. 4 mo. 24 da.?

6. At what rate will $880 earn $82.28 interest in 1 yr. 8 mo. 12 da.?

7. At what rate will $570 earn $54.34 interest in 2 yr. 11 mo. 6 da.?

8. At what rate will $880 earn $73.04 interest from May 14, 1910, to Oct. 2, 1911?

9. A man borrowed $810 on Sept. 24, 1909. On April 2, 1913, he paid $142.65 interest. What was the rate?

10. A merchant borrowed $1260 on Aug. 20, 1908. On Dec. 16, 1910, he paid $1391.67 in settlement of the debt. What rate of interest did he pay?

To find the Time when the Principal, Interest, and Rate Per Cent are Given

441. 1. In what time will $450 yield $63.90 interest at 6%?

This problem corresponds to Second Type Problem, page 89.

Process

Interest on $ 450 for **1 yr.** at 6 % = $ 27.

Therefore, the number of years required to yield $ 63.90 interest is as many as $ 63.90 ÷ $ 27 = $2\frac{11}{30}$.

$2\frac{11}{30}$ yr. *Ans.*

Changing to months and days, the time is 2 yr. 4 mo. 12 da.

442. RULE. **To find the time.**

Divide the GIVEN INTEREST by the INTEREST on the given principal for 1 YEAR at the given rate per cent.

EQUATION. **Time = Interest ÷ Interest on Given Principal at Given rate for 1 YEAR.**

EXERCISES

443. 1. In what time will $ 5000 yield $ 350 interest @ 6 %?

2. In what time will $ 480 yield $ 56 interest @ 5 %?

3. In what time will $ 350 yield $ 30.80 interest @ 4 %?

4. In what time will $ 750 yield $ 20.25 interest @ $4\frac{1}{2}$ %?

5. In what time will $ 375 yield $ 74.25 interest @ $5\frac{1}{2}$ %?

6. In what time will $420 yield $36.75 interest @ $3\frac{1}{3}$%?

7. In what time will $6500 yield $614.25 interest @ $3\frac{1}{2}$%?

8. A man borrowed $4175 on May 1, 1906, at $4\frac{1}{2}$% interest. On the day of settlement, he paid $4609.20. What was the date of payment?

9. On May 26, 1907, a man obtained a mortgage for $33,000 at 5% interest. On settlement, $36,960 was paid. When was the mortgage paid?

10. A debt of $1150 was incurred on Aug. 4, 1910. On settlement $1301.80 was paid, including the interest at $5\frac{1}{2}$%. Find the date of settlement.

444. To find the Principal when the Interest, the Rate Per Cent, and the Time are Given

1. I loaned a sum of money for 6 mo. at 6%. I received $12 interest. Find the sum loaned (principal).

This problem corresponds to Third Type Problem (Case 1), page 191.

Process

Interest on $1 for 6 mo. at 6% = $.03.

Therefore, the number of dollars that will yield $12 interest is as many as $12 ÷ $.03 or $400. *Ans.*

2. What principal will yield $82.20 interest in 2 yr. 3 mo. 12 da. at 5%?

Process

Interest on $1 for 2 yr. 3 mo. 12 da. at 6 % = $.137.
Interest on $1 for 2 yr. 3 mo. 12 da. at 5 % = $.114$\frac{1}{6}$.
Therefore, number of dollars that will yield $82.20 interest at 5 % is

$82.20 ÷ $.114$\frac{1}{6}$.

Multiplying dividend and divisor by 6

$493.20 ÷ .685 = $720 Principal. *Ans.*

445. RULE. **To find the principal: Divide the GIVEN INTEREST by the INTEREST ON $1 for the given time at the given rate.**

EQUATION. **Principal = Interest ÷ (INTEREST ON $1 for given time at given rate).**

EXERCISES

446. Find the principal in each of the following:

	Time	Rate	Interest
1.	3 yr.,	6 %,	$180.
2.	2 yr.,	4 %,	$96.
3.	2 yr. 6 mo.,	4 %,	$150.
4.	2 yr. 4 mo.,	6 %,	$105.
5.	1 yr. 10 mo.,	5 %,	$82.50.
6.	1 yr. 6 mo.,	3 %,	$36.
7.	1 yr. 3 mo.,	3$\frac{1}{2}$ %,	$70.
8.	3 yr. 3 mo.,	4 %,	$312.
9.	2 yr. 8 mo.,	5 %,	$280.
10.	1 yr. 3 mo.,	2 %,	$47.50.
11.	1 yr. 6 mo. 12 da.,	6 %,	$184.

12.	1 yr. 8 mo. 24 da.,	3 %,	$650.
13.	2 yr. 6 mo. 12 da.,	4 %,	$1672.
14.	2 yr. 4 mo. 18 da.,	$4\frac{1}{2}$ %,	$2145.
15.	3 yr. 9 mo. 21 da.,	5 %,	$3541.75.

16. From March 2, 1912, to June 6, 1912, 6 %, interest, $80.

17. From May 25, 1911, to Dec. 11, 1911, 4 %, interest, $60.

18. From Jan. 4, 1912, to Nov. 8, 1912, 3 %, interest, $123.60.

19. From Feb. 20, 1910, to July 8, 1912, 2 %, interest, $171.60.

20. From May 16, 1910, to March 22, 1913, 4 %, interest, $342.

447. To find the Principal when the Amount, Rate Per Cent, and Time are Given

1. I loaned a sum of money for 1 yr. 6 mo. at 6 %. When the debt was paid, I received $218. Find the sum loaned (principal).

Process

This problem corresponds to Third type problem, page 391.

$1 at 6 % for 1 yr. 6 mo. amounts to $1.09. Therefore, the number of dollars that will amount to $218 is as many as $218 ÷ $1.09 or 200. $200 *Ans.*

448. Rule. **To find the principal when the amount, rate per cent, and time are given:**

Divide the given amount by the amount of $1 for the given time, at the given rate.

Equation. **Principal = Amount ÷ amount of $1 for given time, at given rate.**

EXERCISES

449. Find the principal in each of the following:

	Time	Rate	Amount
1.	1 yr. 6 mo.,	6%,	$ 490.50.
2.	1 yr. 3 mo.,	4%,	$ 819.
3.	1 yr. 2 mo.,	3%,	$ 952.20.
4.	2 yr. 3 mo. 12 da.,	6%,	$1705.50.
5.	2 yr. 4 mo. 18 da.,	4%,	$1971.60.
6.	3 yr. 6 mo. 6 da.,	6%,	$2906.40.

7. From Jan. 5, 1910, to July 20, 1912, 6%, amount, $1452.15.

8. From May 1, 1908, to Aug. 5, 1911, 4%, amount, $2034.80.

9. From July 9, 1911, to Aug. 18, 1913, 5%, amount, $795.90.

10. From March 4, 1912, to Aug. 20, 1914, $3\frac{1}{2}$%, amount, $8211.21.

REVIEW EXERCISES IN INTEREST

450. Find values for ?

	Principal	Rate	Time yr.	mo.	da.	Interest	Amount
1.	$1240	$4\frac{1}{2}$%	2	5	18	?	?
2.	$ 675	?	3	6	20	$144.00	——
3.	$ 990	4 %		?		$ 90.09	——
4.	?	$2\frac{1}{2}$%	1	7	6	$ 40.40	——
5.	?	4 %	2	2	18	——	$1355.39
6.	$ 450	4 %	2	7	6	?	?
7.	$ 468	?	1	7	22	$ 38.48	——
8.	$ 760	$4\frac{1}{2}$%		?		$ 61.18	——
9.	?	6 %	1	8	15	$ 31.16	——
10.	?	$2\frac{1}{2}$%	2	8	24	——	$2211.45
11.	$1240	$2\frac{1}{2}$%	2	10	15	——	?
12.	$ 672	?	3	3	5	——	$ 737.80
13.	——	5 %		?		$ 85.40	$ 925.40
14.	?	4 %	2	2	18	$110.39	——
15.	?	$3\frac{1}{2}$%	1	7	6	——	$ 554.40
16.	$ 780	4 %	1	10	15	——	?
17.	$ 450	?	2	4	24	——	$ 487.80
18.	——	4 %		?		$ 47.77	$ 812.77
19.	?	$4\frac{1}{2}$%	2	0	20	$ 67.34	——
20.	?	3 %	2	0	24	——	$3748.86
21.	$1320	$3\frac{1}{2}$%	3	3	6	?	?
22.	——	?	1	3	26	$ 33.32	$ 453.32
23.	$ 432	$3\frac{1}{2}$%		?		——	$ 459.30
24.	?	4 %	2	5	1	$113.23	——
25.	?	5 %	3	10	24	——	$ 924.93
26.	$3720	3 %	3	3	21	?	?
27.	——	?	1	7	13	$ 34.98	$ .574.98
28.	$ 560	3 %		?		——	$ 620.34

PRESENT WORTH AND TRUE DISCOUNT

451. Suppose I am under obligation to pay Mr. B $2100, 1 year from to-day, but that I desire to pay the debt at once. How much should I pay Mr. B to-day, to discharge the debt, if money is worth 5%?

It is evident that the amount to be paid to-day is a sum that if put at interest for 1 year at 5%, will amount to $2100.

Therefore, the problem is the same as Art. 447, namely, to find the principal when the amount, rate per cent, and time are given.

Amount of $1 for 1 yr. at 5% = $1.05.

$2100 ÷ $1.05 = $2000.

Therefore, I can discharge the debt by paying $2000 to-day.

$2000 is the **present worth** or **cash value** of the debt. The present worth corresponds to the principal in Art. 423.

$100, the difference between the values of the sum at a future time and the present worth of the sum, is called the **true discount**. The true discount corresponds to the interest on the present worth, for the given time and given rate.

PROBLEMS

452. 1. A merchant buys goods amounting to $5000, on 90 days' credit. If money is worth 4%, how much should he pay at once to discharge the debt?

2. A balance of $1980 should be paid on the purchase price of a house 2 years from to-day. Find the present worth of the sum at 5%. Find the true discount.

Find the present worth and true discount of the following:

	Face of debt	Time	Rate
3.	$3296	1 yr.	3%.
4.	$1590	$1\frac{1}{2}$ yr.	4%.
5.	$3245	3 yr.	6%.
6.	$11,000	2 yr.	5%.
7.	$40	60 da.	6%.
8.	$25	30 da.	4%.

9. How much money must be invested at the present time at 6%, simple interest, to amount to $8260 in 3 years?

10. A man will inherit $18,600 in 4 years. Find the value of that sum at the present time if money is worth 6%.

11. I hold a man's note for $120, due in 60 days, without interest. How much will I receive if I get the present worth of this note?

12. A man bought a city lot for $1250, paying cash. He then sold the lot for $1000 and a non-interest bearing note for $425, due in 3 months. What was his net gain, money being worth 6% at true discount?

453. COMPUTING SIMPLE INTEREST BY TABLES

THIS TABLE SHOWS THE NUMBER OF DAYS FROM ANY DAY OF ANY MONTH TO THE SAME DAY OF ANY MONTH NOT MORE THAN ONE YEAR LATER

FROM	TO JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	30	61	91	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

EXERCISES

454. 1. Find the number of days from May 10 to Dec. 15 of the same year. Use the table.

Process

By the table, from May 10 to Dec. 10 = 214 da.
From Dec. 10 to 15 = 5 da.
Total 219 da.

2. Find the number of days from July 1 to Dec. 24 of the same year.

3. Find the number of days from Feb. 20 to Sept. 19 of the same year.

455. SIMPLE INTEREST TABLE. INTEREST AT 6%

DA.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	DA.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	0.167	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	5
6	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	1.167	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	8
9	0.150	0.300	0.450	0.600	0.750	0.900	0.050	1.200	1.350	1.500	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	1.833	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	2.167	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	2.333	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	2.667	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	2.833	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	3.167	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	3.500	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	3.667	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	3.833	23
24	0.400	0.800	1.200	1.600	2.000	2.400	2.800	3.200	3.600	4.000	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	4.167	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	4.333	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	4.500	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	4.667	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.384	3.867	4.350	4.833	29
Mo.	**$100**	**$200**	**$300**	**$400**	**$500**	**$600**	**$700**	**$800**	**$900**	**$1000**	**Mo.**
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	20.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	25.000	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	30.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	35.000	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	45.000	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49 500	55.000	11
YR.	**$100**	**$200**	**$300**	**$400**	**$500**	**$600**	**$700**	**$800**	**$900**	**$1000**	**YR.**
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00	1
2	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	120.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	180.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	240.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	300.00	5

EXERCISES IN USING TABLES

456. 1. Find the interest on $650 from Jan. 4, 1913, to May 26, 1913.

Process

From the tables, from Jan. 4 to May 4 = 120 da.
From May 4 to May 26 = 22 da.
Total = 144 da. = 4 mo. 22 da.

From the tables, interest on

$600 for 4 mo. at 6 %	=	$12.00
$50 for 4 mo. at 6%	=	1.00
$600 for 22 da. at 6 %	=	2.20
$50 for 22 da. at 6 %	=	.183
Total interest at 6 %		$15.383 *Ans.*

By using the tables, find the interest on following:

2. $450 for 2 yr. 7 mo. at 6%.

3. $630 from March 25 to Oct. 30 at 6%.

4. $740 for 1 yr. 5 mo. at 6%.

5. $840 from May 12, 1911, to Feb. 19, 1911, at 6%.

6. $420 for 2 yr. 3 mo. 5 da. at 6%.

7. $525 from June 8 to Dec. 5 at 4%.

8. $520 for 2 yr. 7 mo. 12 da. at 3%.

9. $1200 from Jan. 9 to Nov. 26 at 3%.

10. $360 for 127 da. at 6%.

11. $2750 from May 25 to Dec. 19 at 5%.

12. $1500 for 220 da. at 4%.

13. $3500 from March 4 to Nov. 16 at 6%.

14. $3200 for 92 da. at 4%.

15. $1450 from Feb. 15, 1910, to Jan 5, 1912, at 6%.

16. $1750 for 128 da. at 5%.

17. $2840 from March 18 to July 22 at 3%.

18. $1375 for 116 da. at 3%.

19. $1760 from April 9 to Aug. 14 at 4%.

20. $965 for 320 da. at 2%.

21. $2920 from Jan. 20 to Nov. 15 at 6%.

COMPOUND INTEREST

For Reading and Discussion

457. Mr. Harris deposited $600 in a savings bank on July 1, 1912. At the end of 6 mo., he was entitled to interest at the rate of 4% per annum. Instead of drawing out the interest ($12), Mr. Harris allowed it to remain in the bank. At the end of the next 6 months, the bank allowed him interest on the original $600, and on the $12 interest which he had left in the bank. Therefore, he was credited with interest at 4% on $612, or $12.24, for the second 6 mo. This made the total to his credit, $624.64. The total interest was $24.24.

This kind of interest is called **compound interest**; it is interest on the original principal and

on the interest which has not been paid; it is computed at regular intervals, sometimes annually, sometimes semi-annually or quarterly. Unless stated otherwise, interest is compounded annually.

458. 1. Find the compound interest on $400 for 2 yr. 6 mo. at 6 %, compounded annually.

Process

Interest for first year	= .06 of $400 = $24.00.
Amount at end of first year	= $400 + $24 = $424.00.
Interest for second year	= .06 of $424 = $25.44.
Amount at end of second year	= $424 + $25.44 = $449.44.
Interest for 6 mo.	= .03 of $449.44* = $13.48.
Amount at end of 2 yr. 6 mo.	= $462.92.
Compound int. for 2 yr. 6 mo.	= $462.92 – $400 = $62.92. *Ans.*

459. RULE. **To find the compound interest:**

First: **Find the amount of the given principal at the given rate for the first period of time** (*i.e.* **for the first year, or half year).**

Second: **Make this amount a new principal; compute the interest and amount of this new principal, at the given rate, for the second period of time.**

Third: **Proceed in this manner for each successive period of time.**

Fourth: **Subtract the original principal from the last amount; the remainder is the compound interest.**

* NOTE. In actual practice, interest is not allowed on cents; therefore, interest for 6 mo. would be computed on $449, not on $449.44.

EXERCISES

460. 1. What is the compound interest of $600 for 3 yr. @ 6%, compounded annually?

2. Find the amount of $720 for 3 yr. 6 mo., if interest @ 4% is added to the principal every 6 mo.

3. Find the compound interest on $640 for 1 yr. 6 mo. @ 4%, compounded semiannually.

4. Find the amount of $375 for 3 yr. 9 mo. @ 4%, compound interest.

5. On Jan. 1, 1911, I deposited $1000 in the bank. If interest is added every 6 mo. at the rate of 4% per annum, how much is there to my credit on Jan. 1, 1913?

6. Find the compound interest on $840 for 1 yr. 6 mo. @ 4%, compounded quarterly.

7. Find the compound interest of $1200 for 3 yr. 6 mo. @ 4%, payable semiannually.

8. Find the amount of $800 for 2 yr. 9 mo. @ 6%, payable quarterly.

9. Find the compound interest and the amount of $2500 for 2 yr. 9 mo. @ 4%, interest payable quarterly.

10. Find the compound interest and amount of $1500 for 3 yr. 6 mo. at 3%, interest payable semiannually.

COMPUTING COMPOUND INTEREST BY TABLES

461. A COMPOUND INTEREST TABLE

Periods	1 Per Cent	1½ Per Cent	2 Per Cent	2½ Per Cent	3 Per Cent	3½ Per Cent	4 Per Cent	4½ Per Cent	5 Per Cent	6 Per Cent	Periods
1	1.010000	1.015000	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	1.147523	1.169859	1.192518	1.215506	1.262477	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686	1.216653	1.246181	1.276281	1.338226	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279	1.315932	1.360861	1.407100	1.503630	7
8	1.082857	1.126493	1.171660	1.218403	1.266770	1.316809	1.368569	1.422100	1.477455	1.593848	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	1.362900	1.423312	1.486095	1.551328	1.689479	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	1.410600	1.480244	1.552969	1.628895	1.790848	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945	1.979931	2.260904	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	1.675348	1.800944	1.935282	2.078928	2.396558	15
16	1.172579	1.268985	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370	2.182875	2.540351	16
17	1.184304	1.288020	1.400241	1.521618	1.652847	1.794676	1.947901	2.113376	2.292018	2.692773	17
18	1.196147	1.307341	1.428246	1.559659	1.702433	1.857489	2.025817	2.208478	2.406619	1.854339	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714	2.653298	3.207136	20

DIRECTIONS FOR USING COMPOUND INTEREST TABLES

462. To find the amount of a given principal for a given number of years.

RULE. **When the interest is compounded annually:**

First: **Find from the table the amount of $1 for the given number of years at the given rate per cent.**

Second: **Multiply the principal by this amount.**

NOTES. If the interest is compounded semiannually, find the amount of $1 for one half the rate for twice the time; if the interest is compounded quarterly, find the amount of $1 for one quarter of the rate for 4 times the time. Then multiply the principal by the amount.

When the principal is $100 or less, use 4 of the decimal places in the table. When the principal is $1000 or less, use 5 decimal places, etc.

EXERCISES IN THE USE OF THE TABLES

463. 1. Find the compound interest and amount of $400 for 3 yr. at 4%, interest being compounded semiannually.

Process

The amount of $1 for **6** periods at **2%**. is $1.12616.
The amount of $400 is 400 times

$1.12616 = $450.46 Amount.

Compound Interest = $450 46 – $400 = $50.46.

2. Mr. White deposits $500 in a bank on Jan. 1 and July 1 of each year for 4 yr.; the bank pays 4% compound interest. How much money has he on deposit at the end of the 4th year?

Explanation

The 1st \$500 is on deposit for 8 periods at 2%.
The 2d \$500 is on deposit for 7 periods at 2%.
The 3d \$500 is on deposit for 6 periods at 2%.
The 4th \$500 is on deposit for 5 periods at 2%.
The 5th \$500 is on deposit for 4 periods at 2%.
The 6th \$500 is on deposit for 3 periods at 2%.
The 7th \$500 is on deposit for 2 periods at 2%.
The 8th \$500 is on deposit for 1 period at 2%.

Process

Amount of \$1 for 8 periods at 2% = \$1.171660
Amount of \$1 for 7 periods at 2% = \$1.148686
Amount of \$1 for 6 periods at 2% = \$1.126162
Amount of \$1 for 5 periods at 2% = \$1.104081
Amount of \$1 for 4 periods at 2% = \$1.082432
Amount of \$1 for 3 periods at 2% = \$1.061208
Amount of \$1 for 2 periods at 2% = \$1.040400
Amount of \$1 for 1 period at 2% = \$1.020000
\$8.754629

Total amount may be found by multiplying each separate amount by 500 and adding the products.

A shorter method is to add the amounts, as shown above, and multiply the sum (\$8.754629) by 500.

Total amount on deposit \$4377.31.

EXERCISES IN COMPOUND INTEREST TABLES

Solve the following by use of the Table:

464. 1. Find the amount of \$250 for 2 yr. at $3\frac{1}{2}$%.

2. Find the amount and compound interest of \$350 for 4 yr. at $4\frac{1}{2}$%.

3. If a man deposits $ 100 every 6 mo. for 3 yr. in a bank that pays 4 % per annum, compound interest, how much will he have on deposit at the end of 3 yr.?

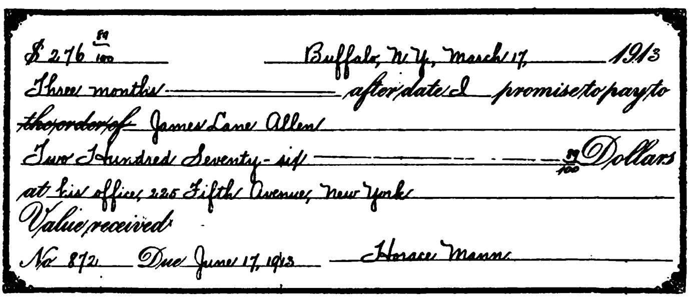
4. A father gives his son $ 125 every birthday, beginning with the 15th birthday. This money is deposited in a savings bank every year. How much has the boy to his credit on his 20th birthday, if none of the interest or principal has been withdrawn (compounded semiannually at 4 %)?

5. How much greater is the compound interest on $ 1200 for 2 yr. 6 mo. at 4 %, compounded semiannually, than the simple interest on the same sum for the same time and rate?

PROMISSORY NOTES

465. A **promissory note** is an unconditional promise to pay a specified sum of money on a specified day.

Form I, Non-negotiable note.


$276 $\frac{84}{100}$ Buffalo, N.Y., March 17 1913

Three months after date I promise to pay to

~~the order of~~ James Lane Allen

Two Hundred Seventy-six $\frac{84}{100}$ Dollars

at his office, 225 Fifth Avenue, New York

Value received

No. 872 Due June 17, 1913 Horace Mann

Form II, Interest-bearing note; negotiable without indorsement.

$50 $\frac{00}{100}$ Akron, Ohio, June 15, 1913
Ninety days after date I promise to pay to
the order of Bearer
Fifty $\frac{00}{100}$ Dollars
at The Fifth National Bank, Akron, Ohio
Value received with interest at 5%
No. 224 Due Sept. 13, 1913 John Henderson

Form III, Demand note, negotiable by indorsement.

$100 $\frac{00}{100}$ New York, N.Y., May 20, 1913
On demand ~~after date~~ we promise to pay to
the order of Charles E. Fields
One Hundred $\frac{00}{100}$ Dollars
at our office
Value received
No. 901 Due Allen and Hayes
J.C.A.

466. The **face** of a note is the specified amount of money.

467. There are two principal "parties" or "persons" to a note, the **maker** and the **payee**.

The **maker** is the party who signs the note.

The **payee** is the party to whom the money is to be paid.

468. The **date of maturity** is the day on which the note becomes due and payable.

In times past, it was a custom sanctioned by law to allow the maker of a note three days beyond the specified time for payment before resorting to process to compel payment. These three days are known as "**days of grace.**" With the improved means of communication afforded by the railroad and the telegraph, the custom is dying out. It has already been abolished in most of the states.

QUERY. Does the law allow days of grace in your state?

A **demand note**, Form III, is due and payable on any day that the holder chooses to present it for payment. Some of our paper money consists of "demand notes" made "legal tender" by statute.

Interest-bearing notes contain the words "with interest." If the rate is not specified, the legal rate is charged.

A non-interest-bearing note, if "dishonored" upon maturity, draws interest at the legal rate from maturity until settlement.

The words "value received" usually appear upon a note.

If the place of payment is not specified, the note must be presented for payment at the place of business or at the residence of the maker.

469. A **negotiable note** is one that may be transferred by the holder to another party.

The holder is the party that holds the note.

470. Indorsement is the signing of an order on the back of a note before transferring the note to another party.

The party that indorses a note is called an indorser.

A note drawn payable "to bearer" is negotiable without indorsement.

A note drawn payable "to the order of" the payee is negotiable by "indorsement."

A note that does not contain the words "to bearer" or "to the order of" is not negotiable; *i.e.* it is payable only to the party named in the note as payee.

471. There are several kinds of indorsement.

Indorsement in full assigns the note to a specified person and guarantees the payment of the note.

> Pay to the order of
> Robert Blair.
> CHAS. E. FIELD.

Indorsement in blank assigns the note "to bearer" and guarantees its payment. The note is thereafter negotiable without indorsement.

> CHAS. E. FIELD

Restrictive indorsement converts a negotiable note into a non-negotiable note.

> Pay to Robert Blair.
> CHAS. E. FIELD.

Indorsement "without recourse" discharges the indorser from liability for non-payment of the note. It is sometimes called a "qualified indorsement."

> Without recourse.
> CHAS. E. FIELD.

If the maker fails to pay the note when it is presented for payment upon maturity, the note is said to be **dishonored.**

In order to take advantage of the guarantee of payment made by previous indorsers of the note, the holder of a dishonored note must have the note "protested" and must serve "notice" on the previous indorsers.

472. Protest is the declaration made before a notary public that the note has been presented to the maker for payment and that payment has been refused.

473. Notice is the notification of protest sent to the indorsers of a note within twenty-four hours after the day of maturity.

474. To find the date of maturity of a note, the number of months or of days, as expressed in the note is to be counted forward from the date of the note.

To illustrate, a three months' note dated Dec. 31, 1912, is due on March 31, 1913; and a ninety days' note is also due March 31; but a two months' note of Dec. 31, 1912, would be due Feb. 28, 1913, and a sixty days' note on March 1, 1913.

EXERCISES

475. 1. Make the following non-negotiable, non-interest-bearing notes, writing before the date the place in which the note is signed:

Date	Maker	Payee	Face	Term	Place of Pay't
Sept. 5, 19—	Yourself	R. F. Adams	$ 308.61	2 mo.	Payee's office
Jan. 4, 19—	T. Y. Loftus	Yourself	$ 5000.00	90 da.	Your office

2. Write a negotiable note for $645.40 payable by J. F. Andrews to you sixty days after date, bearing the legal rate of interest.

3. Write a negotiable non-interest-bearing note for $1000 payable by yourself to Henry Stoddard six months after date, at the maker's office.

4. Write a non-negotiable note: maker, yourself; payee, Jacob Schmidt; make it payable sixty days after date, with interest at 5 per cent.

5. Write a negotiable demand note for $225, and indorse it in full to J. H. Brown.

6. Write a non-negotiable demand note for $850 payable by F. P. Adams to yourself, without interest. Write an indorsement "without recourse."

7. Make a negotiable note for $1250 in payment of merchandise sold to Wright and Evans by yourself, payable in sixty days.

BANK DISCOUNT

For Reading and Discussion

476. The business of buying notes and other negotiable paper is largely committed to commercial banks. When a merchant borrows money from a bank, he usually gives a negotiable, non-interest-bearing note for the time agreed upon. The bank discounts the note at the rate of interest agreed upon, usually the legal rate, and pays to him the

face of the note less the interest or discount. In computing the **bank discount** (or interest) the value of the note at maturity is taken as the base. In the case of an interest-bearing note, the **amount** of the note is the base. In other words, the bank receives **interest in advance** upon the money value of the note when due.

477. **Bank discount** is the simple interest paid in advance upon the amount due on a note or draft at maturity.

478. The **proceeds** of a note is the amount of the note or draft at maturity less the bank discount.

If a note does not bear interest, the amount is the face of the note.

479. The **term of the note** is the time between the date of the note and its maturity.

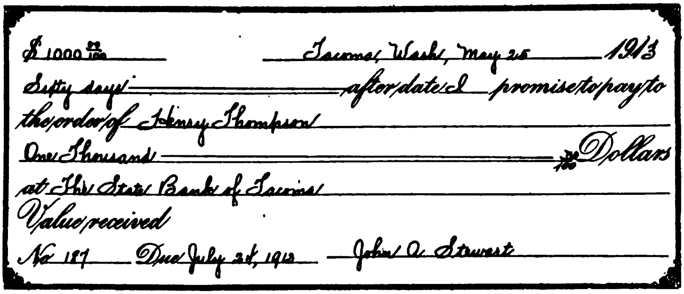
480. The **term of discount** is the number of days that the note has to run from the day of discount to the day of maturity.

In states that allow days of grace, three days additional should be included in the "term of discount" and in the "term of the note."

481. Suppose that Henry Thompson has received the following note from John A. Stewart in payment of a bill of goods. He may keep the note until it matures and then present it to Mr. Stewart

for payment; or he may keep it for a part of the time and then discount it at the bank; or he may discount it the very day that he receives it.

Let us suppose that the note is discounted on May 25, at 6 %.


$1000 00/100 Tacoma, Wash., May 25 1913
Sixty days after date I promise to pay to
the order of Henry Thompson
One Thousand 00/100 Dollars
at The State Bank of Tacoma
Value received
No 187 Due July 24, 1913 John A. Stewart

The **date of maturity** is sixty days after May 25, which is July 24.

The **term of discount** is 60 da.

The **bank discount** is the simple interest on \$1000 for 60 da. at 6%, or \$10.

The **proceeds** is \$1000 minus \$10, or \$990.

NOTE. If the above note were to read "Two months after date" instead of "Sixty days after date," the date of maturity would be July 25, and the term of discount 61 da. The term of discount is always computed as the exact number of days from discount till maturity.

If the above note were discounted on June 1, the term of discount would be 29 da. + 24 da., or 53 da., the bank discount \$8.83, and the proceeds \$991.17.

If the above note were made in Alabama or in some other state which allows days of grace, the date of maturity would be July 27 and the term of discount 64 da.

EXERCISES

482. Find the bank discount and the proceeds of the following notes:

1. For \$3500, payable in 90 da., discounted at 6%.

2. For \$5000, payable in 30 da., discounted at 9%.

3. For \$150, payable in 60 da., discounted at $4\frac{1}{2}$%.

4. For \$500, payable in 42 da., discounted at 7%.

5. For \$640, payable in 33 da., discounted at 5%.

6. For \$400, payable Dec. 31, discounted Nov. 13 at 6%.

7. For \$1500, payable on March 15, 1912, discounted Jan. 28, 1912, at 8%.

8. For \$2150, payable on Jan. 3, discounted on Aug. 21 at $7\frac{1}{2}$%.

9. For \$875, dated April 19, due Aug. 19, discounted on date drawn at 5%.

10. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a note for \$600, drawn June 18, payable in 120 da., discounted on June 18 at $4\frac{1}{2}$%.

Find the proceeds of the following notes:

11. \$300, drawn Jan. 15, 1913, payable in 60 da., discounted on Feb. 1, 1913, at 4 %.

12. \$750, drawn Oct. 24, payable in 3 mo., discounted on Oct. 24 at 6 %.

13. For \$75, drawn July 5, payable in 2 mo., discounted on July 15 at 6 %.

14. For \$2800, drawn Dec. 31, 1912, payable in 2 mo., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

15. For \$2800, drawn Dec. 31, 1912, payable in 60 da., discounted on Jan. 9, 1913, at $4\frac{1}{2}$ %.

16. For \$2000, with interest at 5 %, for 60 da., discounted on date drawn at 5 %.

HINT. The amount of the note at maturity is the base on which bank discount is computed.

17. For \$375, with interest at 7 %, for 90 da., discounted on date drawn at 8 %.

18. For \$1250, with interest at $4\frac{1}{2}$ %, dated Aug. 10, payable in 30 da., discounted Aug. 15 at 5 %.

19. For \$1800, with interest at 6 %, dated Sept. 12, payable in 3 mo., discounted Sept. 18 at 6 %.

HINT. Compute the "interest" for *3 mo.*, and the "discount" for the *exact number of days in the term of discount.*

20. For $10,000, with interest at 7%, dated Jan. 4, 1912, payable in 6 mo., discounted Feb. 1, 1912, at 5%.

21. For $340.75, with interest at 5%, dated Nov. 1, 1913, payable in 4 mo., discounted Feb. 4, at 7%.

22. For $58,647.29 with interest at 3%, dated Nov. 30, 1913, payable in 3 mo., discounted at 4% on day made.

23. For $200,000 with interest at $2\frac{1}{2}$%, dated July 31, payable in 2 mo., discounted Aug. 30 at $5\frac{1}{2}$%.

This note was made in Indiana, which allows 3 da. of grace.

To find the Face of Note

483. For what sum must I draw my note at 60 da. to obtain $200 in cash, if the rate of discount is 6%?

Process

$\$1 - .01 = .99 =$ proceeds of $1.

$\$200 \div \$.99 = 202.02$

$\$1 \times 202.02 = \$202.02$. *Ans.*

Explanation

Since 99 ct. is the proceeds of a note for $1, $200 is the proceeds of a note for as many dollars as 99 ct. is contained times in $200, which is 202.02 times.

$202.02 \times \$1 = \$202.02.$

EXERCISES

484. 1. For how much must a note at 90 da. be drawn so as to produce $ 1000 when discounted at 5 %?

2. I wish to borrow $ 500 from the bank on a 30-da. note. For what sum must I draw the note, if the rate of discount is 9 %?

3. Find the face of a 60-da. note which will yield $ 250 when discounted at 7 %.

4. What was the face of the note at 90 da., on which I obtained $ 875 in cash, after it was discounted at 6 %?

5. What must be the face of a note dated June 28, and drawn at 2 mo., which will produce $ 450 when discounted at $4\frac{1}{2}$ %?

6. Find the sum for which a note at 2 mo. must be drawn so as to produce $ 10,000 when discounted at 8 %.

7. I received $ 1486.67 as proceeds of a note discounted for 64 da. at 5 %. What was the face of the note?

8. A merchant bought goods for $ 500 on terms 5/10, n/90. In order to take advantage of the discount offered for payment within 10 da., he borrowed $ 500 from the bank on his 90-da. note. For how much was the note drawn, the rate of discount being 6 %? How much did he gain by paying his bill in this way?

EXCHANGE

485. Exchange in arithmetic deals with methods of making payments and of collecting credits at distant places without the sending of money.

Domestic exchange or **inland exchange** is exchange between places in the same country.

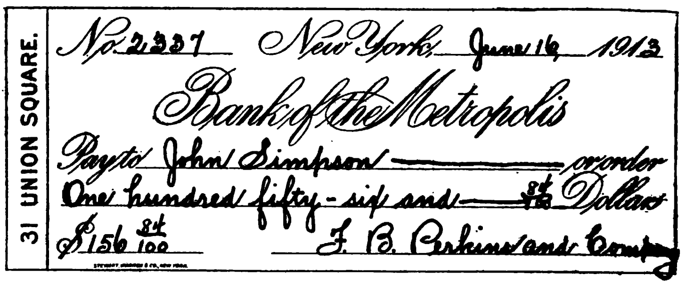
Foreign exchange is exchange between places in different countries.

486. Business men employ various simple **methods of remitting funds** to creditors in distant places without the actual transfer of money, *e.g.* by **postal money order, by express money order** or by **telegraphic money order.**

NOTE TO TEACHERS. Specimen copies of these money orders should be exhibited, and, if practicable, pupils should fill out money order blanks, and calculate the fees to be charged on various amounts.

487. The **personal check** is a convenient means of sending money.

F. B. Perkins & Co.'s check in favor of John Simpson should be drawn as follows:

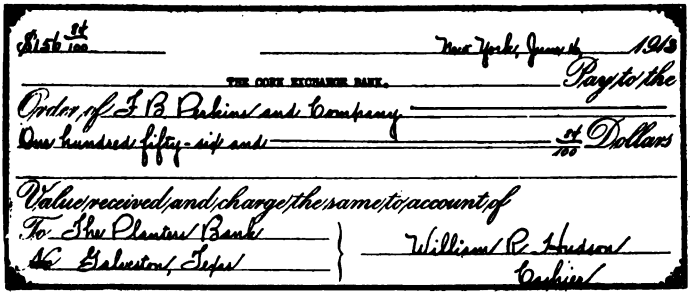

31 UNION SQUARE.
No. 2337 New York, June 16 1913
Bank of the Metropolis
Pay to John Simpson —— or order
One hundred fifty-six and 84/100 Dollars
$156 84/100 F. B. Perkins and Company

488. A check is an order drawn by a depositor on his bank to pay on demand the sum specified to the person specified or to his order.

In this check, Perkins & Co. are the *drawers*, the Bank of the Metropolis is the *drawee*, and John Simpson is the *payee*.

John Simpson will indorse this check and deposit it in his bank, the Planters Bank of Galveston, Texas.

489. Instead of sending a personal check, Perkins & Co. may buy **a bank draft or bill of exchange**, indorse it, and send it to Simpson.


$156 84/100 New York, June 16 1913

THE CORN EXCHANGE BANK. Pay to the

Order of J. B. Perkins and Company

One hundred fifty-six and 84/100 Dollars

Value received and charge the same to account of

To The Planters Bank

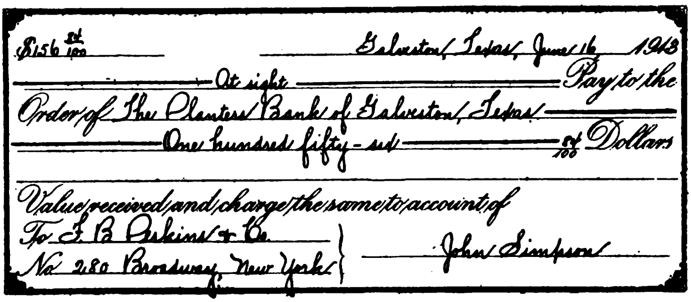
No. Galveston, Texas

William R. Hudson

Cashier

490. A **bank draft** is a check drawn by a bank on one of its "correspondent banks."

491. In business, the **commercial draft** is a convenient means of securing funds or collecting money.


$156 84/100 Galveston, Texas, June 16 1913

At sight Pay to the

Order of The Planters Bank of Galveston, Texas

One hundred fifty-six 84/100 Dollars

Value received and charge the same to account of

To J. B. Perkins & Co.

No. 280 Broadway, New York

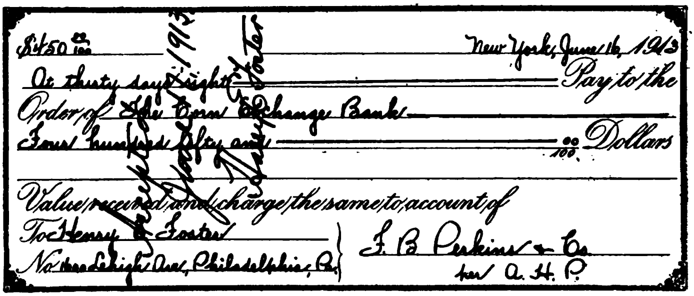
John Simpson

492. A **draft** is a written order, by which one person directs another person to pay a specified sum of money to a third person, or to his order.

A **sight draft** is payable "at sight" or on demand.

Checks, bank drafts, and commercial drafts are three distinct kinds of drafts. The rules of negotiability and indorsement are the same for all drafts as for promissory notes.

493. Time Drafts. Suppose that F. B. Perkins & Co. sell to Henry E. Foster, Philadelphia, a bill of goods for $985.50 on the following terms: 30-day draft for $450, balance at 60 days. The following draft would accompany the bill:


$450 00/100 New York, June 16, 1913

At thirty days sight Pay to the

Order of The Corn Exchange Bank

Four hundred fifty and 00/100 Dollars

Value received and charge the same to account of

To Henry E. Foster

Lehigh Ave, Philadelphia, Pa.

F. B. Perkins & Co.

per A. H. P.

Time Draft

Mr. Foster writes in red ink across the face of the draft, "Accepted, June 17, 1913, Henry E. Foster," thereby promising to pay the draft 30 days later, or on July 17, 1913. He then returns the draft to F. B. Perkins & Co. The draft is then called **an acceptance**, and it has the legal character of a promissory note. Perkins & Co. may hold it to collect at maturity, or may have it discounted at their bank.

A **time draft** is a draft payable on a specified day, or at a specified time after date, or after acceptance.

Acceptances, being similar to non-interest-bearing promissory notes, may be discounted like such notes.

The date of maturity is postponed three days in the case of drafts payable in states which allow three days of grace.

Sometimes the drafts, before acceptance, are deposited by the drawer for collection, or even for discount. Of course the drawer guarantees acceptance and payment.

494. Rates of Exchange. Bank drafts on other cities are frequently sold by banks at prices slightly above or below their face value or par value. Thus, New York exchange on San Francisco may be at $\frac{1}{10}$ % premium, or $\frac{1}{10}$ % above par; while at the same time San Francisco exchange on New York will be at $\frac{1}{10}$ % discount, or $\frac{1}{10}$ % below par.

495. Commercial drafts are sold to banks by merchants who draw against their credits in other cities. As buyers, the banks pay something less than the market rates of exchange which they charge as sellers.

WRITTEN EXERCISES

1–5. Draw five checks. Write indorsements on each check, showing two or more transfers.

496. Draw bank drafts as follows:

	Amount	Bank Drawer	Bank Drawee	Payee
6.	$6890	1st National, Savannah	Northern, St. Paul	J. R. Sands
7.	$351.45	Commercial, Boston	5th National, Denver	M. I. Jenkins
8.	$800	Miners, Spokane	Central, Chicago	Ourselves
9.	$98.25	2d National, Atlanta	Marine, Savannah	Southern Railway Co.
10.	$510.18	Corn Exchange, N. Y.	Alliance, San Francisco	Cal. Fruit Co.

11–15. Find the cost of each of the drafts described above, the rates of exchange per $1000 being:

On No. 6, 50 cents premium; on No. 7, 60 cents premium; on No. 8, 40 cents discount; on No. 9, $\frac{1}{10}$% discount; on No. 10, 30 cents premium.

Write the following commercial sight drafts, inserting the names of different cities in the United States as the places of drawer and drawee:

	Amount	Drawer	Drawee	Payee
16.	$1000.00	H. J. Rogers	E. L. Poe	R. M. Anderson
17.	$205.50	Hope Mfg. Co.	Sands & Co.	J. I. Taylor
18.	$5000.85	Elmer & Sons	The Speer Co.	Amer. Steel Co.
19.	$640.28	Knight Bros.	Va. Coke Co.	Knight Bros.
20.	$2000.00	W. T. Kidd	Merchants Bk., New Orleans	Southern Fruit Co.

21–25. Assume that the five sight drafts just written were sold by the drawers to their local bankers. How much was received for each, if the banks made the following rates per $1000?

On No. 16, $\frac{1}{10}$% discount; on 17, 50 cents premium; on 18, 75 cents discount; on 19, 25 cents premium; on 20, $\frac{3}{8}$% discount.

Find the Value of a Time Draft when Exchange is not at Par

497. 1. What is the cost of a sixty-day draft for $2500 on New Orleans, exchange being $\frac{1}{4}$% discount, and money worth 6%?

Solution

Discount for 60 da. at 6 %	$.01
Discount for exchange	.0025
Total discount	.0125
Cost of draft for $1	.9875
Cost of draft for $2500	

2500 × $.9875 = $2468.75. *Answer.*

2. What is the cost of a ninety-day draft for $6250 on San Francisco when the exchange is $\frac{1}{2}$% premium, and money is worth 6%?

Solution

Discount for 90 da. at 6 %	$.015
Premium for exchange	.005
Net discount	.01
Cost of draft for $1	.99
Cost of draft for $6250	

6250 × $.99 = $6187.50. *Answer.*

3. I drew a thirty-day draft against Albert Johnson, of St. Louis, $16,459, and discounted it at 6% at the City National Bank. Exchange being at a premium of 75 cents per $1000, and "collection" being $\frac{1}{10}$%, what were the proceeds of the draft?

Discount for 30 da.	$.005
"Collection"	.001
	$.006
Proceeds of draft of $1	$.994
Proceeds of draft of $1000 . . .	$994.
Premium per $1000	.75
Total proceeds per $1000	$994.75
Proceeds of draft of $16,459 . . .	

16,459 × $.994 = $16,372.59. *Answer.*

Write the following time drafts and their acceptances, allowing two days for the payee to receive the draft and to present it for acceptance:

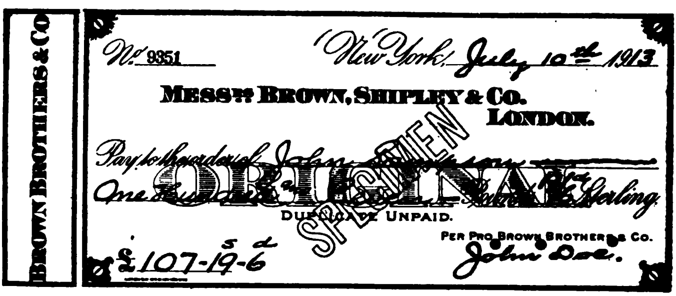
	Amount	Drawer	Payée	Drawee	Time
4.	$36000	I. F. Folk	R. T. Page	I. N. Mills	60 days
5.	$8575	J. A. Sinns	L. S. Short	M. T. Lee	90 days
6.	$40829	G. L. Ames	2d Nat'l Bk.	H. A. Marr	2 mo.
7.	$210056	N. S. Pitt	T. E. Ball	J. O. Noble	4 mo.
8.	$367610	T. F. Clark	Garfield Bk.	R. L. Moss	30 days

9–13. Assume that these notes were discounted at 6% on the day of acceptance, and find the proceeds of each, exchange rates being:

On 4, 50 cents discount; on 5, 75 cents premium; on 6, $\frac{1}{4}$ discount; on 7, par; on 8, 25 cents discount.

Foreign Exchange

498. Foreign exchange is made for small amounts by means of (1) international money orders, issued by the post-office and by some express companies, and (2) by travelers' checks and letters of credit, issued by bankers. All large transactions, however, are settled by means of foreign bills of exchange and foreign commercial drafts.



Foreign bills of exchange are usually issued by bankers in sets of two similar drafts, called the first and the second of exchange. The purchaser sends these to the payee by different mails so as to avoid delay if one draft be lost.

499. The **par of exchange** is the value of the standard unit of money of a foreign country expressed in terms of United States money; *e.g.* £1 = $4.8665, 1 franc = $.193, 1 mark = $.238.

500. The **market quotations for exchange** fluctuate about the par value so that foreign exchange is sometimes at a premium and sometimes at a discount.

In quoting exchange in England, the value of £1 is given, *e.g.* $4.86.

In quoting exchange in France, the value of $1 is given, *e.g.* 5.17 fr.

In quoting exchange in Germany, the value of 4 reichsmarks is given, *e.g.* $96\frac{1}{2}$ cents.

Different prices are quoted for "Cables" (or telegraphic orders), for checks (or sight drafts), and for sixty-day and ninety-day bills. The last are the cheapest, because the bankers have the use of the purchasers' money for a long period.

PROBLEMS

1. What will a 60-day draft for £1260 on London cost, exchange being $4.8305?

2. How large a sight draft in sterling currency can be bought for $5000, exchange being $4.8685?

3. Find the cost of a draft on Berlin for 27,690 M, at $93\frac{3}{16}$.

4. I wish to pay a bill in Paris amounting to 18570 fr., which is due 60 days hence. How much must I pay for a draft, exchange being $5.22\frac{1}{2}$?

5. Find the cost of a "Paris Cable" for $850 at $5.18\frac{1}{8}$.

Find the cost of the following bills:

6. £1928 15*s*. @ $4.868. 7. 2500 M @ $96\frac{1}{8}$.

8. £972 8*s*. @ $4.88. 9. 7645.50 fr. @ $95\frac{7}{8}$.

How large a draft can be purchased for $15,000, exchange as quoted?

10. On London @ $\$4.82\frac{3}{4}$. 11. On Paris, @ $5.22\frac{3}{4}$.

12. On Geneva, @ $5.17\frac{1}{2}$. 13. On Hamburg @ $95\frac{3}{8}$.

PARTIAL PAYMENTS

501. A **partial payment** is a payment of part of the amount due on a note, a mortgage, or some other interest-bearing obligation.

Partial payments on a note are usually acknowledged on the back of the note.

United States Rule

502. The following is the method of computing the amount due on a note on which a number of payments have been made, according to the rule laid down by the Supreme Court of the United States.

1. A note $2000, bearing interest at 6%, is dated March 4, 1909. The following payments were made: April 10, 1910, $400; Jan. 22, 1911, $50; Dec. 19, 1911, $250. How much was due on the note on March 4, 1912?

Process

Face of note		$2000.00
Interest at 6 % from March 4, 1909, to April 10, 1910 (1 yr. 1 mo. 6 da.)		132.00
Amount due April 10, 1910		$2132.00
Payment made April 10, 1910		400.00
New principal April 10, 1910		$1732.00
Interest from April 10, 1910, to Jan. 22, 1911 (9 mo. 12 da.)	$81.40	
Interest exceeds the payment; therefore, a new principal is not formed.		
Interest from Jan. 22, 1911, to Dec. 19, 1911 (10 mo. 27 da.)	$94.39	
Total interest from April 10, 1910, to Dec. 19, 1911		175.79
Amount due Dec. 19, 1911		$1907.79
Sum of second and third payments		300.00
New principal Dec. 19, 1911		$1607.79
Interest from Dec. 19, 1911, to March 4, 1912 (2 mo. 15 da.)		20.10
Amount due March 4, 1912		$1627.89

The method requires that payments shall be applied first to the payment of interest; then the balance is applied to the reduction of the principal.

503. United States Rule. **Find the amount of the principal to a time when the payment, or the sum of two or more payments, equals or exceeds the interest due.**

Deduct the payments or the sum of the payments from this amount.

Consider the remainder a new principal and proceed as before, until the date of final payment.

Merchants' Rule

504. A different method of finding the amount due at the time of settlement is usually followed when interest-bearing notes run for a year or less than a year. This method, called the **Merchants' Rule,** is shown in the following:

1. A note for $600, bearing interest at 6%, was dated Jan. 8, 1912. The following payments were made: Feb. 14, 1912, $100; April 26, 1912, $50; May 26, 1912, $150. How much is due on the note on Sept. 20, 1912?

Process

Face of note		$600.00
Interest at 6% from Jan. 8, 1912, to Sept. 20, 1912		25.20
Amount of note, at date of settlement		$625.20
Payment Feb. 14, 1912	$100.00	
Interest on first payment (Feb. 14, 1912, to Sept. 20, 1912)	3.60	
Payment April, 26, 1912	50.00	
Interest on second payment (April 26, 1912 to Sept. 20, 1912)	1.20	
Payment May 26, 1912	150.00	
Interest on third payment (May 26, 1912, to Sept. 20, 1912)	2.85	
Total of payments and interest on payments		307.65
Balance due Sept. 20, 1912, date of settlement		$317.55

The Merchants' Rule requires:

First: that interest shall be charged against the debtor from the date the note is drawn to the date of settlement.

Second: that interest shall be credited to the debtor on each payment from the date of payment to the date of first settlement.

505. Merchants' Rule. **Find the amount of the face of the note from the date it is drawn to the date of settlement, without regard to payments made.**

Find the amount of each payment from the date it is made to the date of settlement. Subtract the sum of the amounts of the payments from the amount of the face of the note: the remainder is the balance due at the date of settlement.

EXERCISES

United States Rule

506. 1. A note of $ 1200, dated March 12, 1910, bearing interest at 6 %, had the following payments indorsed on it: July 15, $ 300; Nov. 20, $ 400; Jan. 8, $ 150; March 18, $ 200. How much was due on April 18, 1911?

2. A note of $ 1500 was dated July 15, 1911, and, drawing interest at 4 %, had payments indorsed as follows: Oct. 12, $ 200; Jan. 18, 1912, $ 150; March 22, $ 300; June 10, $ 400; Aug. 14, $ 300. How much was due on Sept. 12, 1912?

3. A 6 % interest-bearing note of $ 2400 was dated May 10, 1911. How much was due on Sept. 12, 1912, if the following payments were indorsed: July 22, 1911, $ 200; Nov. 16, $ 150; March 10, $ 30; June 7, 1912, $ 500; Aug. 7, $ 200?

4. A 4% note for $450, dated April 7, 1909, had the following payments indorsed: July 13, $50; Nov. 19, $100; Jan. 22, $30; May 10, $50; Aug. 16, $100. How much is due on Oct. 25?

5. $1500. NEW YORK, N.Y., Feb. 1, 1911.

Fifteen months after date, I promise to pay to Bacey & Co., or order, Fifteen Hundred Dollars, at their offices, 1226 John St., with interest at 6% for value received. Due April 1, 1912.

A. B. STUART.

On the above note the following payments were indorsed: May 19, $350; July 6, $100; Sept. 3, $50; Jan. 15, $20; Feb. 15, $600. How much is due on April 1, $1912?

Merchants' Rule

507. 1. On Sept. 1, 1911, a man borrowed $400 for 9 mo. with interest at 6% and gave his promissory note. He made the following payments: Nov. 15, $25; Jan. 18, $100; March 12, $50; and May 9, $100. How much was due on the note on the day of maturity?

2. On May 7, Oliver James borrowed $650 with interest at 4%. He made the following payments: June 4, $85; Aug. 1, $100; Dec. 13, $50; Jan. 10, $100. How much was due on Feb. 1?

3. On a 4 % note of $ 700, dated Oct. 1, the following payments were made: Dec. 7, $ 100; Feb. 13, $ 50; May 19, $150; June 1, $ 100. How much was due on July 15?

4. A 7 mo. note of $ 600, dated May 15, with interest at 4 %, had the following payments indorsed: June 6, $ 100; July 9, $ 40; Oct. 12, $ 150. How much was due on Dec. 15?

STOCKS AND BONDS

508. **Capital** is money and other property invested in manufactures, trade, or other business.

A person may engage in business by himself; he is then entitled to all the profits and is personally responsible for all debts. Or, a person may form a partnership with one or more other persons under a firm name, *e.g.* Hewitt & Powell; the partners then share profits and losses according to agreement, but each partner is singly responsible for all the debts of the firm. Or, three or more persons may form a *stock company* or *corporation* in which each person participates according to the amount of capital subscribed by him.

509. A **corporation** is an association of persons authorized by law to do business as a single individual.

510. The capital of a corporation is called its **capital stock** or its **stock**.

511. Capital stock is divided into equal parts called **shares.**

512. A share of stock has a fixed **face value** or **par value,** usually $ 100.

513. Unless otherwise stated, the par value of a share is assumed to be $ 100.

Thus a corporation having a capital stock of $ 100,000 may divide it into 1000 shares of $ 100 each.

Certificates of stock are issued to each shareholder stating the number of shares owned by him and the *par value* of the shares. These certificates or "stocks" may be bought and sold like other property.

Corporations are managed by officers elected by the stockholders, each stockholder having as many votes as he owns shares.

514. The profits of a corporation when distributed among the stockholders are called a **dividend.**

Suppose that the capital stock is $ 100,000, and that the net profit for six months is $ 3500, and suppose that $ 1500 of this profit is set aside as a "reserve fund" or "surplus," then there remains $ 2000 to be distributed as a dividend, which is 2 % of the capital stock, or $ 2 per share of $ 100 par value. A stockholder owning a certificate for 150 shares would receive a dividend of $ 300.

515. An **assessment** is a tax levied by a corporation upon its stockholders to make up losses in the business.

An assessment, like a dividend, is expressed as a certain per cent of the par value of the capital stock.

516. If the profits are large, the stock of the corporation becomes a desirable investment and investors will pay more than the par value to pur-

chase it; the stock is then **above par** or **at a premium**. On the other hand, if the business is not profitable, stockholders are willing to sell it for less than par value; the stock is then **below par** or **at a discount**. Stock is usually bought and sold through an association of stockbrokers, called a **stock exchange**.

517. The **market value** of stock is the price at which it is sold on a stock exchange.

This price, or market value, may change daily according to business conditions.

518. A **bond** is an interest-bearing note issued by a corporation and secured by the property of the corporation.

Bonds are issued by railroad companies, steamship companies, manufacturing companies, and other industrial corporations, and also by governments of cities, of states, and of nations. The denominations are usually $1000 or $100, with a fixed rate of interest payable annually, or semi-annually, or quarterly.

They are usually made payable in 20, 25, or 50 years after the date of issue. Like stocks, they have a market value which may be *at par*, or *at a premium*, or *at a discount*, according to their rate of interest, the security they offer, and the conditions of the money market.

519. In every large city having a stock exchange, the prices at which stocks and bonds are sold are printed daily in the local newspapers. These prices are called the **market quotations**.

The following market quotations taken from a New York newspaper show the prices at which the stocks and bonds mentioned were sold the previous day on the New York Stock Exchange:

STOCKS

Canadian Pacific R.R.	$229\frac{1}{2}$
General Electric Co.	$138\frac{7}{8}$
U. S. Steel Co.	$61\frac{3}{8}$

BONDS

N. Y. City 4's 1957	$96\frac{1}{2}$
U. S. 2's reg. 1930	$100\frac{7}{8}$
Adams Express 4's	$81\frac{3}{4}$

NOTES. The only fractions used in the quotations are eighths, quarters, and halves. The quoted price $229\frac{1}{2}$ means $229\frac{1}{2}$ % of the par value.

"N. Y. City 4's 1957" means that the bonds were issued by New York City and pay 4 % interest annually, and that they are redeemable (or payable) in the year 1957.

"U. S. 2's reg. 1930" means that these U. S. bonds pay 2 % interest annually and that they are registered bonds redeemable in 1930.

The market values for bonds fluctuate much less than the market values of stocks, because the interest on a bond is always the same whether the corporation is prosperous or not, while the dividend on stock depends on the profits of the business, which naturally change from year to year.

Stocks and bonds are collectively referred to as **securities**.

Brokerage is the commission charged by stockbrokers for buying or selling stocks and bonds.

The customary rate of brokerage is $\frac{1}{8}\%$ of the par value of the securities dealt in.

Thus, a purchaser of Canadian Pacific R.R. stock at the market price of $229\frac{1}{2}$ would pay $229\frac{1}{2}\% + \frac{1}{8}\%$ brokerage, or $229\frac{5}{8}\%$ of the par value, \$100; that is, the stock would cost \$$229\frac{5}{8}$ per share. On the other hand, if he sold the stock at $229\frac{1}{2}$, he would receive \$$229\frac{3}{8}$ per share, the broker retaining $12\frac{1}{2}$ ct. per share.

PROBLEMS

NOTE. Unless stated otherwise, consider par value \$100. Do not include brokerage, unless it is mentioned in the problem.

520. 1. Find the total cost of 100 shares of stock at \$$82\frac{1}{2}$ per share, brokerage at $\frac{1}{8}\%$.

2. I sold 100 shares of stock at \$$92\frac{1}{4}$ per share. How much did I receive, allowing for brokerage at $\frac{1}{8}\%$?

3. How many shares of stock at \$$62\frac{1}{2}$ per share may be purchased for \$8020? (Include brokerage at $\frac{1}{8}\%$.)

4. How many shares of stock at \$$220\frac{1}{4}$ per share may be purchased for \$26,445? (Include brokerage at $\frac{1}{8}\%$.)

521. Find the number of shares and the annual income on the following:

5. \$5000 invested in a 5% stock at par.

Solution: \$5000 will buy 50 shares at par; the annual income will be 50 times \$5, or \$250.

6. $8000 invested in a 4% stock at $80 per share.

7. $3000 invested in 2% bonds at $75 each.

8. What is the annual income from 350 "Southern Pacific 5's"? From 280 "Wabash 4's"?

9. (*a*) How many U. S. 2's will give an annual income of $5000, the par value being $1000?

(*b*) How much will they cost at $100\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

10. At what price must 4% stock be purchased to afford 5% interest on the investment?

Find value for ?:

	Stock	Price per Share	Dividend Rate	Net Interest in Per Cent
11.	Am. Sugar Co.	80	4	?
12.	Baltimore R.R.	110	6	?
13.	General Electric	?	8	5
14.	U. S. Rubber	67	4	?

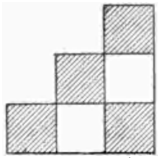
15. I bought 50 shares of a 4% stock at $60 per share, paying $\frac{1}{8}\%$ brokerage. At the end of 2 yr. I sold it for $80 per share, paying $\frac{1}{8}\%$ brokerage. Find my total gain.

16. I bought 100 shares of a 6% stock at $110 per share, paying $\frac{1}{8}\%$ brokerage. At the end of 3 years, I sold it for $85 per share, paying $\frac{1}{8}\%$ brokerage. Find my net loss.

RATIO AND PROPORTION

Ratio

$$\begin{array}{rrr} & & 4 \\ & 4 & 4 \\ 4 & 4 & 4 \\ \hline 4 & 8 & 12 \end{array} \qquad \begin{array}{rrr} & & 10 \\ & 10 & 10 \\ 10 & 10 & 10 \\ \hline 10 & 20 & 30 \end{array}$$



A *B* *C*

522. Comparing *A*, *B*, and *C*, we may say:

A is $\frac{1}{2}$ of *B*, or "the ratio of *A* to *B* is $\frac{1}{2}$."
B is 2 times *A*, or "the ratio of *B* to *A* is 2."
A is $\frac{1}{3}$ of *C*, or "the ratio of *A* to *C* is $\frac{1}{3}$."
C is 3 times *A*, or "the ratio of *C* to *A* is 3."
B is $\frac{2}{3}$ of *C*, or "the ratio of *B* to *C* is $\frac{2}{3}$."
C is $\frac{3}{2}$ of *B*, or "the ratio of *C* to *B* is $\frac{3}{2}$."
Also, the ratio of 4 to 12 is $\frac{4}{12}$, or $\frac{1}{3}$.

What is the ratio of 4 to 8? 8 to 4? 12 to 4?

What is the ratio of 10 to 20? 20 to 10? 20 to 30? 30 to 20?

What is the ratio of 3 to 6? 8 to 4? 10 to 15?

What is the ratio of 25 to 75? 50 to 100? 60 to 40?

523. Ratio is the relation between two like numbers expressed by the quotient of the first divided by the second.

524. All ratios are abstract numbers.

525. Writing Ratios. A ratio may be written as a fraction, as a division, or with the sign of

ratio; thus the ratio of 3 to 4 may be written $\frac{3}{4}$, or $3 \div 4$, or $3:4$. The colon : is the ratio sign.

The ratio sign : is the division sign $\div$ with the horizontal line omitted. If the ratio is a whole number, it is regarded as a fraction having 1 for a denominator; thus, the ratio 2 means $2:1$, or $\frac{2}{1}$.

526. Reading Ratios. A ratio may be read in several ways. $3:4$ may be read:

1. The ratio of 3 to 4.
2. The ratio of $\frac{3}{4}$.
3. The ratio of $3 \div 4$.
4. *As* 3 *is to* 4.

The last expression "As 3 is to 4" is the common mode of expressing a ratio.

527. A ratio has two terms, an **antecedent** and a **consequent**. The antecedent is the first term of the ratio; the consequent is the second term of the ratio. In the ratio $3:4$, 3 is the antecedent, and 4 is the consequent.

When the ratio is expressed as a fraction, the numerator is the antecedent, and the denominator is the consequent; *e.g.* $\frac{3}{4} = \frac{\text{antecedent}}{\text{consequent}}$.

When the ratio is expressed as a division, the dividend is the antecedent, and the divisor is the consequent; *e.g.* $3 \div 4 = \text{antecedent} \div \text{consequent}$.

A fraction may be regarded in three aspects:

1. $\frac{3}{4}$ may mean a whole (unit) divided into four parts of which 3 are taken.

2. $\frac{3}{4}$ may mean "3 divided by four."

3. $\frac{3}{4}$ may mean "the ratio of 3 to 4."

In whichever way a fraction is regarded, its value in arithmetical computations is always the same.

528. Since a ratio may be regarded as a fraction, the following principle is true:

Multiplying or dividing both terms of a ratio by the same number does not change the value of the ratio. (See arts. 64 and 87.)

Thus, $3:6 = 6:12$, just as $\frac{3}{6} = \frac{6}{12}$.
Also, $3:6 = 1:2$, just as $\frac{3}{6} = \frac{1}{2}$.

ORAL EXERCISES

529. Give in lowest terms the ratios of:

1. 8 to 16	2. 5 to 15	3. 6 to 24
4. 12 to 4	5. 15 to 5	6. 24 to 6
7. 9 to 12	8. 9 to 15	9. 9 to 30
10. $12\frac{1}{2}$ to 100	11. $37\frac{1}{2}$ to 100	12. $87\frac{1}{2}$ to 100
13. 100 to 25	14. 100 to $62\frac{1}{2}$	15. 100 to $33\frac{1}{3}$
16. $\frac{1}{2}$ to 1	17. $\frac{3}{8}$ to 1	18. $\frac{5}{6}$ to 1
19. $\frac{1}{4}$ to $\frac{1}{2}$	20. $\frac{1}{4}$ to $\frac{3}{4}$	21. 2 to $\frac{1}{2}$
22. .2 to .4	23. .03 to .30	24. .05 to .25
25. 25% to 1	26. 50% to 1	27. 40% to 1
28. 1 to $12\frac{1}{2}$%	29. 1 to 60%	30. 1 to 20%

Reduce to lowest terms the following ratios:

31. $20:25$	32. $18:45$	33. $27:36$
34. $48:60$	35. $24:72$	36. $36:24$
37. $\frac{25}{40}$	38. $\frac{60}{36}$	39. $\frac{36}{30}$
40. $30 \div 50$	41. $80 \div 60$	42. $100 \div 80$

ORAL EXERCISES

530. 1. What is the ratio of \$5 to \$10? \$10 to \$5?

2. What is the ratio of 1 ft. to 1 yd.? 3 in. to 1 ft.?

3. What is the ratio of 1 lb. to 4 oz., avoirdupois?

4. What is the ratio of 1 ft. 6 in. to 1 yd.?

5. What is the ratio of a yard to a meter? Of a meter to a yard?

WRITTEN EXERCISES

531. 1. What is the value of the ratio of $\frac{2}{3}$ to $\frac{4}{5}$?

Process: $\frac{2}{3} : \frac{4}{5} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$. *Ans.*

Find the value of:

2. $\frac{1}{2} : \frac{1}{5}$	3. $\frac{3}{4} : \frac{5}{6}$	4. $\frac{5}{8} : \frac{3}{10}$
5. $2\frac{1}{2} : 3\frac{1}{3}$	6. $6\frac{1}{4} : 1\frac{2}{3}$	7. $62\frac{1}{2} : 50$
8. .28 : .42	9. .125 : .05	10. .08 : .002

11. \$2.25 : \$15.75 12. 2 gal. 2 qt. : 10 gal.

13. 5 da. : 60 hr. 14. 2 pk. 4 qt. : 5 bu.

Proportion

For Reading and Discussion

532. Name two numbers that have the ratio of 1 to 2.

Name two numbers that have the ratio of 2 to 3. Of 3 to 2. Of 3 to 1. Of 15 to 3.

What number has the same ratio to 10 that 3 has to 5? That 2 has to 4?

To what number has 12 a ratio equal to the ratio of 6 to 2? 4 to 2? 3 to 4?

Name a ratio equal to 3 : 1; 4 : 8; 8 : 2.

Name a ratio equal to the ratio $\frac{3}{5}$; to $\frac{5}{6}$; to 3; to $\frac{4}{3}$.

Insert the true consequent:

1. $3:2=6:?$
2. $2:6=8:?$
3. $20:10=12:?$
4. $3:5=12:?$

To express the fact that one ratio is equal to another ratio, the double colon (: :) may be used instead of the equality sign. Thus, $3:4=6:8$ may be written $3:4::6:8$ and may be read "3 is to 4 as 6 is to 8." It may also be written $\frac{3}{4}=\frac{6}{8}$.

The expression $3:4::6:8$ is a proportion; also $\frac{3}{4}=\frac{6}{8}$.

533. A proportion is an equation of ratios.

If the ratio on one side of the colon is not equal to the ratio on the other side, we say that the proportion is false.

534. The first and last terms of a proportion are called the **extremes** and the second and third terms are called the **means.** Thus, in $3:5::6:10$ the extremes are 3 and 10; and the means are 5 and 6.

Multiply the two extremes of the following proportion; and multiply the two means and compare the two products.

$$3:6::4:8.$$

In each of the following proportions, compare the product of the extremes with the product of the means:

1. 2 : 4 : : 5 : 10 2. 4 : 5 : : 8 : 10
3. 2 : 3 : : 4 : 6 4. 6 : 2 : : 9 : 3

535. Hence, we have the following

PRINCIPLE. **In any proportion, the product of the extremes is equal to the product of the means.**

An easy way to test a proportion is to reduce both ratios to lowest terms; if the ratios when so reduced are equal, the proportion is true.

ORAL EXERCISES

536. Read the following proportions and show in the case of each that the ratios are equal: first, by reducing both ratios to lowest terms; second, by comparing the product of the **extremes** with the product of the **means.**

1. 3 : 2 : : 6 : 4 2. 20 : 10 : : 12 : 6
3. 2 : 6 : : 8 : 24 4. 3 : 5 : : 12 : 20
5. 10 : 5 : : 2 : 1 6. 25 : 5 = 5 : 1
7. 4 : 5 = 12 : 15 8. $\frac{3}{4}=\frac{9}{12}$
9. $\frac{10}{6}=\frac{5}{3}$.

EXERCISES

537. Find missing terms by inspection:

1. 12 : 4 : : ? : 7 2. 18 : ? : : 36 : 4
3. 5 : 15 : : ? : 60 4. 11 : 22 : : 9 : ?

5. $9 : ? :: 24 : 8$

6. $4 : 16 :: 32 : ?$

7. $? : 4 :: 15 : 3$

8. $24 : 4 :: ? : 3$

9. $28 : 7 :: ? : 2$

EXERCISES

538. Write ten true proportions and prove that each is true.

Thus: $16 : 36 :: 4 : 9$ $\qquad$ $16 \times 9 = 36 \times 4$

$$144 = 144$$

539. By means of the principle of art. 535, we can find the missing term of any proportion if the other three terms are given.

The letter x is used to indicate the missing term.

1. If $4 : 3 :: 20 : x$, what is the value of x?

Process

4 times $x = 3$ times 20.	The product of the extremes equals the product of the means.
$4x = 60.$	
$x = 15.$	Dividing by 4.

2. If $15 : 35 :: x : 7$, what is the value of x?

Process

$35x = 15 \times 7.$	The product of the extremes equals the product of the means.
$x = \dfrac{\overset{3}{\cancel{15}} \times \cancel{7}}{\underset{\cancel{5}}{\cancel{35}}} = 3.$ *Ans.*	Dividing by 35.

WRITTEN EXERCISES

540. Find the value of x in the following:

1. $36:24::3:x$
2. $27:45::x:10$
3. $x:25::32:40$
4. $50:x::125:20$
5. $112:4::21:x$
6. $30:12::x:16$
7. $x:4::88:121$
8. $90:36::15:x$
9. 24 yd. : 9 yd. : : \$16 : \$$x$*
10. \$25 : \$15 : : 12 yd. : x yd.
11. 200 rd. : 300 rd. : : 50 A. : x A.
12. 320 T. : 1000 T. : : 18 hr. : x hr.
13. $\frac{225}{x}=\frac{15}{16}$
14. $\frac{\$1.80}{\$3.20}=\frac{60 \text{ mi.}}{x \text{ mi.}}$

WRITTEN EXERCISES

541. 1. Solve $\frac{2}{3}:\frac{5}{8}::6\frac{2}{3}:x$.

$$\frac{2}{3}x=\frac{5}{8}\times\frac{20}{3}.$$

Multiplying by 3, $2x=3\times\frac{5}{8}\times\frac{20}{3}.$

Dividing by 2, $x=\frac{1}{2}\times\cancel{3}\times\frac{5}{\underset{2}{\cancel{8}}}\times\frac{\overset{5}{\cancel{20}}}{\cancel{3}}=\frac{25}{4}.$

$$x=\frac{25}{4}. \quad \textit{Ans.}$$

2. $\frac{3}{4}:\frac{9}{10}::15:x$
3. $2\frac{1}{2}:6\frac{3}{4}::37\frac{1}{2}:x$
4. $8\frac{1}{3}:\frac{5}{6}::\frac{9}{25}:x$
5. $\frac{7}{8}:\frac{15}{16}::\$1200:\$x$
6. $57\frac{1}{7}:77::x:21$
7. $\frac{3}{11}:\frac{4}{7}::\frac{5}{8}:x$
8. $x:12\frac{1}{2}::\frac{5}{6}:\frac{7}{8}$
9. $1\frac{3}{4}\%:x::\$62\frac{1}{2}:\$87\frac{1}{2}$

* Since all ratios are abstract, disregard the denominations.

PROBLEMS IN PROPORTION

When two quantities are so related that an increase (or decrease) in one quantity causes a corresponding increase (or decrease) in the other, the two quantities are in **direct proportion**.

The following are instances of quantities directly proportional:

1. The greater the quantity, the greater the cost.
2. The smaller the quantity, the smaller the cost.
3. The faster the speed, the greater the distance covered.
4. The higher the object, the longer the shadow.
5. The more men working, the more work done.
6. The greater the base, the more the commission, profit, interest, etc.

Direct Proportion

542. A bookseller asks $ 30 for 25 copies of a certain book. How much must I pay for 45 copies of the book at the same rate?

Statement

25 bk. . . . $ 30
45 bk. . . . $ x

$25 : 45 :: 30 : x.$

$25x = 45 \times 30.$

$$x = \frac{\overset{9}{\cancel{45}} \times \overset{6}{\cancel{30}}}{\underset{\cancel{5}}{\cancel{25}}} = 54.$$

$ x = $ 54. *Ans.*

Steps in Solution

1° State the problem in the usual form, putting x last (see art. 45, note) and "like numbers" together (art. 535).

2° The greater the quantity, the greater the cost. Hence, the quantities are in direct proportion.

3° Solve for value of x.

1. If 5 T. of coal cost \$ 31.25, what will 8 T. cost?

2. If a steamer makes 15 mi. an hour, how long will it take the steamer to go 560 mi.?

3. If a bullet travels 1500 ft. in one second, in what time will it go 100 yd.?

4. How many yards of carpet can be bought for \$ 414 if 96 yd. cost \$ 138?

5. If each cylinder of a 4-cylinder motor boat engine consumes one gallon of gasoline every 24 mi., how many gallons will be necessary for a cruise of 200 mi.?

6. If a stick 4 ft. long casts a shadow of 30 in., how high is the flagpole whose shadow is 33 ft.?

7. A crate of eggs containing 30 doz. costs \$ 6.50. How much will 4 doz. eggs cost?

8. The proportion of gold in an ore averages one part in four hundred. How much gold may be expected in 2 T. of the ore?

9. If a clock ticks 120 times a minute, how many times will it tick in $7\frac{1}{3}$ hr.?

10. If sound travels 1089 ft. per second, how long will it take the report of a shot fired 1000 yd. away to reach us?

11. A man sells $\frac{2}{5}$ of his land for \$ 6250. At the same rate what would $\frac{1}{6}$ of it be worth?

NOTE. All denominate numbers must be reduced to the same denomination before ratios can be formed.

12. If the interest on $ 12,500 amounts to $ 187.50 in 3 mo., what will be the amount accruing on $ 8000 for the same time and at the same rate?

13. A car going 40 mi. an hour traveled from A to B in $2\frac{1}{3}$ hr. How long will it take a car going 32 mi. an hour to cover the same distance?

14. In making a dozen aprons, a seamstress used $40\frac{1}{2}$ yd. of gingham. How many similar aprons can she make out of 100 yd. of the same material?

15. Eight men dug a trench 150 ft. long yesterday. This morning 3 of the men did not report for work. How many feet of trench may the other 5 men be expected to dig to-day?

16. A salesman working on commission earned $ 256.50 in commissions in one month on sales amounting to $ 20,520. The following month his sales amounted to $ 16,000. What was his commission for the latter month?

17. The flywheel on an engine makes 2200 revolutions in 2 min. 10 sec. How many revolutions does it make in 8 sec.?

18. The municipal tax on a building appraised at $ 168,000 was $ 2973.60. At the same rate what should be the tax on a building appraised at $ 21,000?

19. Solve by proportion the problems given in the table on page 31.

Inverse Proportion

When two quantities are so related that an increase (or decrease) in one causes a corresponding decrease (or increase) in the other, the two quantities are in **inverse proportion**.

Instances of quantities inversely proportional are:

1. The faster the speed, the shorter the time required.
2. The slower the speed, the longer the time required.
3. The more men working, the shorter the time required.
4. The fewer men working, the longer the time required.
5. The higher the rate of interest, the shorter the time required to earn a certain amount of interest.

Notice that in all of these instances of inverse proportion there enters the question of time. There are, however, instances of inverse proportion in which the question of time does not enter; *e.g.* the greater the principal, the lower the rate required to earn a certain amount of interest; generally, the larger the unit (or rate), the fewer units needed.

If 8 men can build a wall in 15 da., how many days will 12 men require to build it?

Statement	Steps in Solution
8 men . . . 15 da. 12 men . . . x da.	1°. State the problem in the usual form, putting x last, and like numbers together.
$12 : 8 :: 15 : x$.	2°. The larger the number of men, the fewer the days required. Hence the quantities are in inverse ratio. Inverting the first ratio, write the proportion.
$12x = 120$. $x = 10$. x da. $= 10$ da. *Ans.*	3°. Solve for value of x.

1. It took 2000 bricks with surface 8 in. by 4 in. to lay a sidewalk. How many concrete blocks will be needed to lay a sidewalk of the same size if the concrete blocks have a surface 14 in. by 10 in?

2. An express train moving 58 mi. an hour went from A to B in 18 hr. How long would it take a fast "freight" moving 24 mi. an hour to make the same trip?

3. An army camp is provisioned for a garrison of 240 men for 28 da.; but only 112 men are sent to the camp. How long should the provisions last?

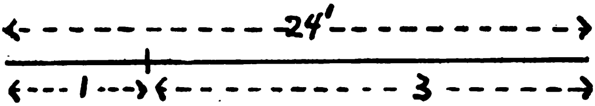
4. A supply pipe having a capacity of 8 gal. per minute requires 18 hr. to fill a reservoir. In order to fill the reservoir in 10 hr., what should be the capacity of the supply pipe?

5. A reservoir has a supply pipe of 300 gal. per minute and a discharge pipe of 250 gal. per minute. If it takes 24 hr. to fill the reservoir, how long will it take to empty it?

6. By saving 9 ct. a day, a young man can accumulate the price of a bicycle in 360 da. How much a day must he save to accumulate the price in 200 da.?

7. A train takes 26 hr. to go from New York to Chicago, moving at the rate of 35 mi. an hour. How fast must the train go per hour to make the journey in 20 hr.?

8. Divide a line 24 ft. long into two parts having the ratio 1 : 3.



Explanation: The smaller segment is $\frac{1}{4}$ of 24'. The larger segment is $\frac{3}{4}$ of 24'.

9. Divide a line 24' long into two segments having the ratio of 1 : 2; 5 : 1; 7 : 1; 3 : 5; 7 : 5.

10. Separate into parts having the ratio 2 : 3, the following numbers: 25, 60, 100, 250, 75, 15000.

Examples 8 to 14 are in **partitive proportion.** Formulate the rule for solving such problems.

11. A certain spraying mixture consists of 3 parts kerosene, and 47 parts water. How much kerosene will be needed to make 200 gal. of the mixture?

12. Two farmers agree to construct a ditch, Mr. A to pay twice as much as Mr. B. If the work cost \$ 217.60, how much was Mr. A obliged to pay?

13. Three partners share their profits of \$ 21,500 for the year. What is each partner's share, their interests being in the ratio 2 : 3 : 4?

14. Two dealers unite in ordering a carload of 40 tons of coal. The freight bill is \$ 24. The first dealer takes 15 tons of coal. What is his share of the freight bill?

MULTIPLES AND POWERS

543. Compare these expressions:

1. $3 \times 5 = 15$.
2. $5 \times 5 = 25$.
3. $2 \times 2 \times 3 \times 5 = 60$.
4. $5 \times 5 \times 5 = 125$.

In the first expression, 15 is a multiple of 5, and of 3. In the second expression, 25 is a multiple of 5. In the third expression, 60 is a multiple of 2, of 3, and of 5. In the fourth expression, 125 is a multiple of 5.

A **multiple** of a number is the **product** obtained by multiplying that number by another number.

15 is a multiple of 5 and 3; 30 is a multiple of 6 and 5.

544. In the second expression, 25 is obtained by multiplying 5 by **itself**, or by taking 5 as a factor twice. In the fourth expression, 125 is obtained by multiplying 5 by itself twice, or by taking 5 as a factor three times.

25 and 125 are called **powers** of 5.

A **power** of a number is the **product** obtained by multiplying that number by itself a certain number of times.

545.

Since $2 \times 2 = 4$, **4** is the **second power** of 2.

$2 \times 2 \times 2 = 8$, **8** is the **third power** of 2.

$4 \times 4 = 16$, **16** is the **second power** of 4.

$4 \times 4 \times 4 = 64$, **64** is the **third power** of 4.

The second power of a number is called the **square** of the number.

The third power of a number is called the **cube** of the number.

We also speak of the 4th power, 10th power, etc. Powers are named from the number of times a number is taken as a factor.

546. Difference between Multiples and Powers

A Multiple is the product obtained by multiplying a number by **any** other number.	Power is the name given to those products only that arise from the multiplication of a number by **itself**, a certain number of times.

547. The power to which a number is to be raised is indicated by a small figure called the **exponent**.

$(3)^2 = 3 \times 3 = 9$. The small 2 is the exponent.

$(2)^5 = 2 \times 2 \times 2 \times 2 \times 2$. The small 5 is the exponent.

548. To raise a fraction to a given power, raise both the numerator and the denominator to the given power.

$$(\tfrac{1}{2})^2 = \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4}.$$
$$(\tfrac{3}{4})^2 = \tfrac{(3)^2}{(4)^2} = \tfrac{9}{16}.$$

549. Any power of 1 is 1, because 1×1 any number of times always equals 1.

Every number is considered to be the first power of itself; the exponent 1 is not written.

550. To **square** a number is to raise it to the second power; 4 is the square of 2.

To **cube** a number is to raise it to the third power; 8 is the cube of 2.

EXERCISES

551. 1. Square 2; 3; 5; 8; 4; 10; 12

2. Cube 2; 3; 5; 6; 4; 1; 7; 10; 9

EXERCISES

552. Raise each number to the power indicated by the exponent.

1.	$2^2 = ?$	$2^3 = ?$	6.	$5^2 = ?$	$5^3 = ?$
2.	$2^4 = ?$	$2^5 = ?$	7.	$6^2 = ?$	$6^3 = ?$
3.	$3^2 = ?$	$3^3 = ?$	8.	$7^2 = ?$	$7^3 = ?$
4.	$3^4 = ?$	$3^5 = ?$	9.	$8^2 = ?$	$8^3 = ?$
5.	$4^2 = ?$	$4^3 = ?$	10.	$9^2 = ?$	$9^3 = ?$

EXERCISES

553. Find values for:

	Number	Second Power	Third Power		Number	Second Power	Third Power
1.	9	?	?	6.	30	?	?
2.	12	?	?	7.	50	?	?
3.	15	?	?	8.	10	?	?
4.	20	?	?	9.	100	?	?
5.	25	?	?	10.	1000	?	?

EXERCISES

554. Raise the following fractions to the powers indicated by the exponents:

1.	$(\frac{1}{2})^2 = ?$	$(\frac{1}{2})^3 = ?$	11.	$(\frac{5}{6})^2 = ?$	$(\frac{5}{6})^3 = ?$
2.	$(\frac{1}{2})^4 = ?$	$(\frac{1}{2})^5 = ?$	12.	$(\frac{3}{5})^2 = ?$	$(\frac{3}{5})^3 = ?$
3.	$(\frac{1}{3})^2 = ?$	$(\frac{1}{3})^3 = ?$	13.	$(\frac{4}{5})^2 = ?$	$(\frac{4}{5})^3 = ?$
4.	$(\frac{1}{3})^4 = ?$	$(\frac{1}{3})^5 = ?$	14.	$(\frac{2}{5})^2 = ?$	$(\frac{2}{5})^3 = ?$
5.	$(\frac{1}{4})^2 = ?$	$(\frac{1}{4})^3 = ?$	15.	$(\frac{9}{10})^2 = ?$	$(\frac{9}{10})^3 = ?$
6.	$(\frac{2}{3})^2 = ?$	$(\frac{2}{3})^3 = ?$	16.	$(\frac{3}{20})^2 = ?$	$(\frac{3}{20})^3 = ?$
7.	$(\frac{2}{3})^4 = ?$	$(\frac{2}{3})^5 = ?$	17.	$(\frac{7}{12})^2 = ?$	$(\frac{7}{12})^3 = ?$
8.	$(\frac{3}{4})^2 = ?$	$(\frac{3}{4})^3 = ?$	18.	$(\frac{3}{100})^2 = ?$	$(\frac{3}{100})^3 = ?$
9.	$(\frac{3}{4})^4 = ?$	$(\frac{3}{4})^5 = ?$	19.	$(\frac{2}{25})^2 = ?$	$(\frac{2}{25})^3 = ?$
10.	$(\frac{3}{8})^2 = ?$	$(\frac{3}{8})^3 = ?$	20.	$(\frac{7}{30})^2 = ?$	$(\frac{7}{30})^3 = ?$

EXERCISES

555. Raise the following to the powers indicated:

1.	$(.5)^2 = ?$	$(.5)^3 = ?$	6.	$(1.5)^2 = ?$	$(1.5)^3 = ?$
2.	$(.05)^2 = ?$	$(.05)^3 = ?$	7.	$(2.5)^2 = ?$	$(2.5)^3 = ?$
3.	$(.10)^2 = ?$	$(.10)^3 = ?$	8.	$(.03)^2 = ?$	$(.03)^3 = ?$
4.	$(.25)^2 = ?$	$(.25)^3 = ?$	9.	$(1.1)^2 = ?$	$(1.1)^3 = ?$
5.	$(.01)^2 = ?$	$(.01)^3 = ?$	10.	$(7.5)^2 = ?$	$(7.5)^3 = ?$

EXERCISES

556. Raise the following to the powers indicated:

1.	$(1\frac{1}{2})^2 = ?$	$(\frac{3}{2})^2 = ?$	6.	$(1\frac{1}{10})^2 = ?$	$(1\frac{1}{10})^3 = ?$
2.	$(2\frac{1}{2})^2 = ?$	$(2\frac{1}{2})^3 = ?$	7.	$(2\frac{1}{5})^2 = ?$	$(2\frac{1}{5})^3 = ?$
3.	$(1\frac{1}{4})^2 = ?$	$(1\frac{1}{4})^3 = ?$	8.	$(1\frac{1}{8})^2 = ?$	$(1\frac{1}{8})^3 = ?$
4.	$(1\frac{1}{3})^2 = ?$	$(1\frac{1}{3})^3 = ?$	9.	$(3\frac{1}{2})^2 = ?$	$(3\frac{1}{2})^3 = ?$
5.	$(2\frac{1}{3})^2 = ?$	$(2\frac{1}{3})^3 = ?$	10.	$(2\frac{1}{4})^2 = ?$	$(2\frac{1}{4})^3 = ?$

FACTORS AND ROOTS

557. Compare these expressions:

1. $15 = 3 \times 5$. 2. $25 = 5 \times 5$.

3. $60 = 2 \times 2 \times 3 \times 5$. 4. $125 = 5 \times 5 \times 5$.

In the first expression, 3 and 5 are factors of 15.

In the second expression, 5 is a factor of 25.

In the third expression, 2, 3, and 5 are factors of 60. In the fourth expression, 5 is a factor of 125.

A **factor** of a number is **one** of the **whole numbers** which, when multiplied by another number, produces that number. 3 is a factor of 15, because 3 multiplied by $5 = 15$.

Every exact divisor of a number is a factor of that number. The quotient obtained by dividing a number by one factor is the other factor; 2 is a factor of 18; $18 \div 2 = 9$; 9 is the second factor of 18. (See also p. 49.)

558. In the second expression, the two factors of 25 are equal. In the fourth expression, the three factors of 125 are equal.

Therefore, 5 is said to be **a root** of 25, and 5 is a **root** of 125.

A **root** of a **number** is **one** of the **equal factors** which, when multiplied together, produce the number.

559. Since $2 \times 2 = 4$, 2 is called the **square root** of 4.

$2 \times 2 \times 2 = 8$, 2 is called the **cube root** of 8.

The **square root** of a number is **one** of the **two equal factors** of the number.

The **cube root** of a number is **one** of the **three equal factors** of the number.

We also speak of the fourth root of a number; the fifth root, etc. Roots are named from the number of equal factors necessary to produce the number.

$2 \times 2 \times 2 \times 2 \times 2 = 32$; therefore, 2 is the fifth root of 32.

Difference between Factors and Roots

560. A factor is **any one** of the whole numbers which, when multiplied, produces a number.

A **root** of a number is **one** of the equal factors of a number.

The root which is to be taken is indicated by a **root sign** or **radical** sign, $\sqrt{\quad}$, and a small figure called the **index** of the root.

$\sqrt[3]{8}$ means that the cube root of 8 is to be taken; 3 is the index of the root.

$\sqrt[5]{32}$ means that the fifth root of 32 is to be taken; 5 is the index of the root.

When the square root is to be taken the index 2 is usually omitted; $\sqrt{4}$ means the square root of 4.

561. Any root of 1 is 1, because 1 multiplied by itself any number of times always equals 1.

Every number is considered to be the first root of itself.

562. Proof. To prove the work, raise the root to the required power; if the original number is obtained, the answer is correct.

EXERCISES

563. Find the roots indicated. (Prove those marked *.)

1. $\sqrt{9}$	6. $\sqrt{16}$	11. $\sqrt[3]{8}$
2. $\sqrt{100}$	*7. $\sqrt{36}$	12. $\sqrt[3]{125}$
*3. $\sqrt{25}$	8. $\sqrt{64}$	13. $\sqrt[4]{16}$
4. $\sqrt{49}$	9. $\sqrt[3]{27}$	14. $\sqrt{1}$
5. $\sqrt{81}$	*10. $\sqrt[3]{64}$	15. $\sqrt[3]{1000}$

EXERCISES

564. Find the roots indicated. (Prove those marked *.)

1. $\sqrt[4]{256}$

Process

Factoring, $\sqrt[4]{256} = \sqrt[4]{4 \times 4 \times 4 \times 4} = 4.$ *Ans.*

2. $\sqrt{625}$	6. $\sqrt[4]{81}$	10. $\sqrt[4]{10,000}$
3. $\sqrt[4]{625}$	*7. $\sqrt{400}$	11. $\sqrt[6]{729}$
4. $\sqrt[4]{1296}$	8. $\sqrt[3]{1000}$	*12. $\sqrt[3]{512}$
*5. $\sqrt[5]{243}$	9. $\sqrt{10,000}$	13. $\sqrt{4225}$

EXERCISES

1. $\sqrt{\frac{25}{36}} = \frac{5}{6}$?	4. $\sqrt[3]{\frac{27}{64}} = ?$	7. $\sqrt[3]{\frac{64}{729}} = ?$
2. $\sqrt[3]{\frac{1}{8}} = ?$	5. $\sqrt[3]{\frac{8}{125}} = ?$	*8. $\sqrt{\frac{25}{169}} = ?$
*3. $\sqrt{\frac{49}{100}} = ?$	*6. $\sqrt{\frac{100}{144}} = ?$	9. $\sqrt{\frac{625}{16}} = ?$

EXERCISES

565. Find the roots indicated (Prove those marked *.)

1. $\sqrt{.04} = \sqrt{.2 \times .2} = .2.$
2. $\sqrt[3]{.008} = ?$
*3. $\sqrt{.09} = ?$
4. $\sqrt{.0025} = ?$
5. $\sqrt{2.25} = ?$
*6. $\sqrt{1.44} = ?$
7. $\sqrt{12.25} = ?$
8. $\sqrt{.0001} = ?$
*9. $\sqrt{.0004} = ?$
10. $\sqrt[3]{.027} = ?$
11. $\sqrt{16.25} = ?$
12. $\sqrt{.64}$
13. $\sqrt[3]{.064}$
14. $\sqrt[4]{.0016}$
15. $\sqrt{.0016}$

SQUARE ROOT OF NUMBERS

To find the Number of Figures in the Square Root

The square root of 1 is 1. The square root of 81 is 9.	There is 1 figure in the root for 1 or 2 figures in the square.
The square root of 1′00 is 1 0. The square root of 98′01 is 9 9.	There are 2 figures in the root for 3 or 4 figures in the square.
The square root of 1′00′00 is 1 0 0 The square root of 99′80′01 is 9 9 9.	There are 3 figures in the root for 5 or 6 figures in the square.

Therefore, to find the number of figures in the square root of a number, separate the number into periods of two figures each; there will be one figure in the root for each period in the number. (The last left-hand period may have only one figure.)

To find the Square Root

566. 1. Find the square root of 784.

$$\begin{array}{rr|l}
 & & \text{tens units} \\
 & 7'84 & \underline{2 \quad 8} \\
(2\cancel{0})^2 = & 4\,\cancel{0}\cancel{0} & \\
4\cancel{0} & \overline{3\,84} & \\
\underline{8} & 3\,84 & \\
48 & \overline{} &
\end{array}$$

Divide the number into periods of two figures each, beginning at units' place. The square root will be composed of two figures, *tens* and *units*. Extract the square root of the first period; this gives 2 for the tens' figure of the root; square 2 tens (20) = 400; subtract 400 from 784; double 2 tens (20) = 40; divide 384 by 40 as a trial divisor; write 8 as the units' figure of the answer; add 8 to 40, making a complete divisor of 48; multiply 48 by 8 = 384. Subtract; no remainder. Therefore, the square root is 28.

2. Find the square root of 6889.

$$\begin{array}{rr|l}
 & 68'89 & \underline{83} \\
 & 64\,\times & \\
2 \times 8\cancel{0} = 16\cancel{0} & 4\,89 & \\
\underline{3} & 4\,89 & \\
163 & \overline{} &
\end{array}$$

Note. After practice, the $\cancel{0}$ may be omitted, as shown in Number 3.

3. Find the square root of 1.5876.

$$\begin{array}{rr|l}
 & 1.58'76 & \underline{1.26} \\
 & 1\,\times\,\times & \\
22 & 58 & \\
 & \underline{44} & \\
246 & 14\,76 & \\
 & \underline{14\,76} &
\end{array}$$

4. Find the square root of 25.7049.

$$\begin{array}{r|l}
\multicolumn{2}{r}{25.70'49 \;\; \underline{|5.07}} \\
\multicolumn{2}{l}{\;\;\;\;\;\;\;\; 25} \\ \cline{2-2}
1007 & \;\;7049 \\
 & \;\;7049
\end{array}$$

567. RULE. *First:* **Divide the number into periods of two figures each, beginning at units' place or at the decimal point.**

Find the greatest square in the left-hand period; extract the square root; write the figure as the first figure of the root.

Square this root; subtract the result from the first period and bring down the next period. Double the part of the root already found; use the product as a trial divisor and divide the dividend by it, disregarding the right-hand figure of the dividend; write the quotient in the answer, and annex the new figure in the quotient to the trial divisor. Multiply the new divisor by the last figure in the root; subtract, and bring down the next period. Proceed in this manner until all the periods have been used.

NOTE 1. When any divisor is greater than the corresponding dividend, write zero in the answer and bring down the next period.

NOTE 2. In pointing off periods for square roots in a mixed decimal, begin at the decimal point; point off to the **left** for the periods for integers, and to the **right** for periods for decimals; if necessary, annex a decimal cipher to make the last period contain two places.

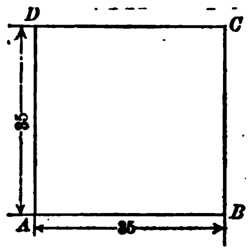
NOTE 3. When the given number is not a perfect square, annex periods of ciphers to the decimal point; ordinarily, it is sufficient if the answer is calculated to one or two places of decimals.

EXERCISES

568. Find the square root of each of the following:

1. 484	4. 9025	7. 110.25
2. 729	5. .4624	8. 5.6644
3. 4761	6. 56.25	9. 163.84

569. Find the length of one side of a square whose area is 1225 square feet.



Since $ABCD$ is a square, its sides are of equal length. Suppose that the length of each side is x feet; then the area of the square is represented by x times x or x^2 square feet. This fact is expressed in the equation:

$$x^2 = 1225$$

To find x, extract the square root of both members of the equation

$$x = \sqrt{1225}$$
$$x = 35.$$

Therefore, the length of each side of the square is 35 feet.

EXERCISES

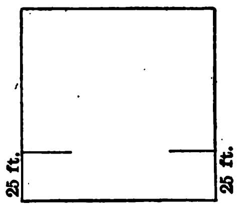
570. Find the length of a side of a square whose area is:

1. 900 sq. ft.	4. 1225 sq. ft.	7. 5184 sq. yd.
2. 576 sq. ft.	5. 1681 sq. ft.	8. 4489 sq. yd.
3. 1024 sq. ft.	6. 5625 sq. ft.	9. 2025 sq. yd.

PROBLEMS

571. 1. The area of one square wall of a room is 324 sq. ft. Find the length of the wall.

2. A square plot of ground having an area of 40,000 sq. ft. is divided into lots, each 25 × 100, as shown on the accompanying diagram. How many lots may be formed?



3. The volume of a square prism 8 ft. high is 288 cu. ft. Find the length of the base.

4. The square floor of a bathroom is covered with tiles, each tile being 3 in. long and 3 in. wide. If the area of the floor is 441 sq. ft., how many tiles are used?

5. A strip of picture molding is placed along the top of a square wall of a room. The area of the wall is 484 sq. ft. Find the cost of the picture molding at 8 ct. per linear foot.

6. A rectangular field is 169 ft. long and 100 ft. wide. Find the length of a side of a square field having the same area.

7. A square field contains 160,000 sq. ft. A man walks around the field.

(*a*) How many feet does he walk?

(*b*) What per cent of a mile does he walk?

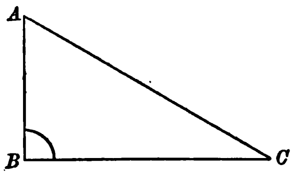
8. A square field has an area of 184,900 sq. ft. Find the cost of the fence at 75 ct. per linear foot.

9. A square wall containing 3600 sq. ft. is covered with tiles, each tile being 6 in. long by 3 in. wide. How many tiles were used for the wall?

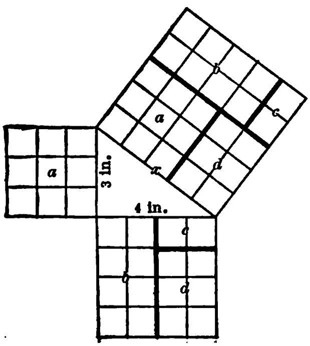
10. A square floor containing 576 sq. ft. is paved with rectangular pieces of marble, each piece being 3 ft. long and 2 ft. wide. How many pieces of marble are used?

Relation between the Hypotenuse of a Right Triangle and the Other Two Sides

The triangle *ABC* is a **right-angled triangle,** or a **right triangle,** because the angle at *B* is a right angle. The side *AC*, opposite the right angle, is the **hypotenuse** of the triangle; the sides *AB* and *BC*, adjacent to the right angle, are the **legs** of the triangle.



If the line *AB* is 3 in. long and *BC* is 4 in. long, how long is *AC*?



Construct a square on each side. The square on *AB* contains 9 sq. in., on *BC* 16 sq. in.

Place the small square on the largest square, as shown in the diagram; then cut the second square and place the pieces on the large square. It will be found that the large square equals the sum of the other two squares. This is true of all right triangles.

Therefore, the large square contains 25 sq. in. and the hypotenuse is 5 in. long.

572. Therefore, the square of a hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

573. FORMULAS. **Let x = length of the hypotenuse, a the length of one side, b the length of the second side.**

Then

$$x^2 = a^2 + b^2.$$
$$x = \sqrt{a^2 + b^2}.$$
$$a^2 = x^2 - b^2.$$
$$a = \sqrt{x^2 - b^2}.$$

574. The square on one of the sides of a right triangle is equal to the square on the hypotenuse minus the square on the other side.

EXERCISES

575. Find the length of the hypotenuse or the side in each of the following right triangles:

Lengths of Sides	Lengths of Hypotenuse and one Side
1. 8 ft. and 6 ft.	9. 10 ft. and 6 ft.
2. 40 ft. and 30 ft.	10. 10 in. and 8 in.
3. 48 ft. and 36 ft.	11. 20 rd. and 16 rd.
4. 9 ft. and 12 ft.	12. 25 ft. and 12 ft.
5. 5 in. and 12 in.	13. 37 in. and 35 in.
6. 85 ft. and 51 ft.	14. 27 in. and 26 in.
7. 15 ft. and 8 ft.	15. 4.5 ft. and 4 ft.
8. 35 ft. and 12 ft.	16. 1.75 rd. and 1.50 rd.

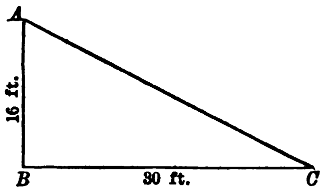
EXERCISES

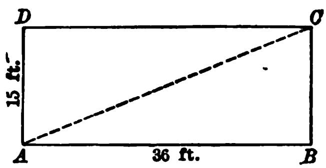
576. Using the formulas given in art. 573, find values for ? in right triangles:

	Base	Altitude	Hypotenuse
1.	5 ft.	5 ft.	?
2.	?	9 ft.	36 ft.
3.	64 yd.	?	100 yd.
4.	15 ft.	?	39 ft.
5.	?	27 yd.	45 ft.
6.	36 ft.	?	20 yd.

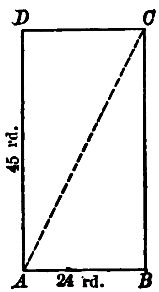
PROBLEMS

577. 1. In this right triangle the base is 30 ft. long, the altitude is 16 ft. How much less is the distance from C to A, than the distance from C to B to A?



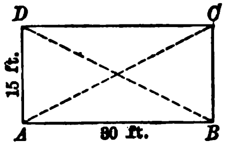


2. A rectangle is 15 ft. by 36 ft. How long is a diagonal (A to C)?



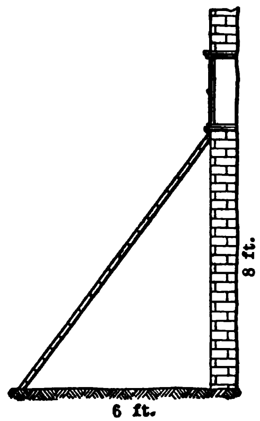
3. A rectangular field is 24 rd. by 45 rd. What distance is saved by walking from A to C instead of walking from A to B to C?

4. This rectangle is 15 ft. long



and 30 ft. wide. Is the distance AC equal to DB or greater than DB or less than DB? Why?

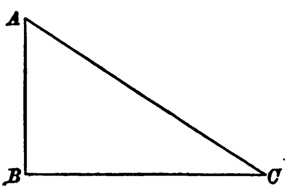
5. A window is 8 ft. above the ground. A ladder is placed as shown in the diagram. The base of the ladder is 6 ft. from the wall. How long is the ladder?



6. A ladder 26 ft. long is placed against a wall. How far is the bottom of the ladder from the base of the wall, if the ladder touches a window 24 ft. above the ground?

7. A ladder 17 ft. long reaches a fire escape 15 ft. above the ground. How far is the base of the ladder from the base of the wall?

8. A derrick 45 ft. high is tied to a post by a rope 51 ft. long. How far is the post from the foot of the derrick?



Represent the distance by x.

9. In the triangle ABC, AC is 10 ft., BC is twice as long as AB. How long is BC? How long is AB? (Right angle at B.)

10. The area of a right triangle is 600 sq. ft. The base is 30 ft. Find the hypotenuse.

11. A baseball diamond is almost a square 90 ft. long. How far is it from 1st base to 3d base?

12. A rectangular football field is 300 ft. long by 100 ft. wide. Find the length of the diagonal.

MENSURATION REVIEWED AND APPLIED

LINES AND ANGLES

578. An **angle** is the **difference** of **direction** of two lines that proceed from the same point.

The point A is called the vertex of the angle.

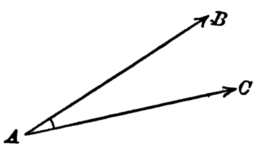


FIG. 1.

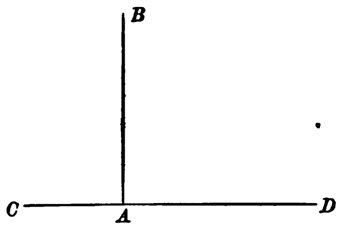


FIG. 2.

579. The angles in Fig. 2 are called **right angles**.

Right angles are sometimes called square corners.

580. The lines AB and CD are said to be **perpendicular** to each other.

PLANE SURFACES: RECTANGLES

581. A **rectangle** is a plane figure bounded by four straight lines and having four right angles.

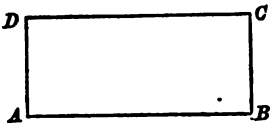


FIG. 3.

AB (or CD) is the **base** of the rectangle.

AD (or BC) is the **altitude** of the rectangle.

582. The **area of a rectangle** is the surface enclosed by the four sides of the rectangle.

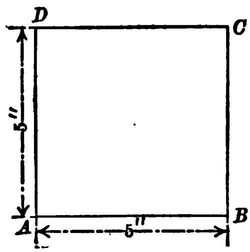
The unit of measurement of small surfaces or areas is the **inch square.** An inch square is a square each side of which is 1 in. long. The surface or area of an inch square is 1 sq. in. Larger areas or surfaces are measured in square feet or square yards, etc.

583. RULE. **The area of a rectangle equals the product of the base by the altitude, expressed in like units.**

EQUATION AND FORMULA. **Area Rectangle** $= b \times a$.

To find the Area of a Square

584. A **square** is a rectangle in which the four sides are equal.



585. EQUATION AND FORMULA.

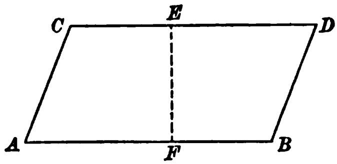
Area Square $= b \times b = b^2$.

NOTE. $b^2 = b \times b$. The small 2 is called an exponent. It means that b is multiplied by itself, or that b is taken twice as a factor; *e.g.* $5^2 = 5 \times 5 = 25$. See also Art. 547.

PARALLELOGRAMS

586. The figure $ABCD$ is a **parallelogram** because the lines AB and CD are parallel (they

never meet) and the lines AC and BD are parallel. AB is the **base**, and EF is the **altitude** of the parallelogram.



Parallelogram. AB and CD are parallel. AC and BD are parallel. EF is the altitude.

587. RULE. **The area of a parallelogram equals the product of the base by the altitude, expressed in like units.**

EQUATION AND FORMULA.

$$\textbf{Area Parallelogram} = b \times a.$$

TRIANGLES

588. These 3 figures are triangles.

A **triangle** is a plane figure bounded by 3 straight lines.

AB is the **base** of the triangle; CD is the **altitude** or height of the triangles in Figs. 1 and 3.

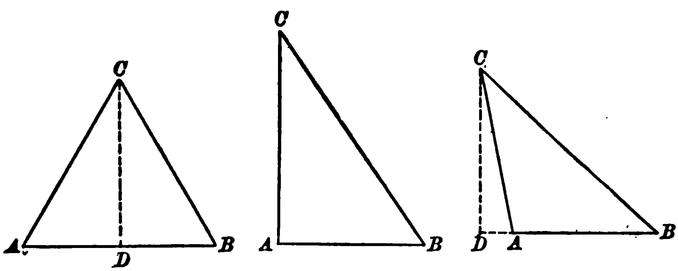


FIG. 1. FIG. 2. FIG. 3.

Fig. 2 represents a right triangle; it contains a right angle at A; in Fig. 2, CA is the altitude or height.

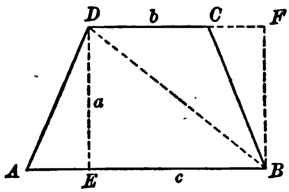
The altitude is always perpendicular to the base (or to the base prolonged).

589. RULE. **The area of a triangle equals one half of the product of the base by the altitude, expressed in like units.**

EQUATION AND FORMULA.

$$\textbf{Area Triangle} = \tfrac{1}{2} b \times a, \text{ or } \frac{b \times a}{2}.$$

TRAPEZOIDS



590. The figure $ABCD$ is a **trapezoid.** The sides AB and CD are parallel. The lines DE and BF represent the altitude.

Draw the diagonal DB; this divides the trapezoid into 2 triangles, BDC and ADB.

Area triangle $BDC = \frac{1}{2}(DC \times \text{Alt. } DE)$.
Area triangle $ABD = \frac{1}{2}(AB \times \text{Alt. } DE)$.
Area trapezoid $= \frac{1}{2}(DC + AB) \times \text{Alt. } DE$.

591. RULE. **The area of a trapezoid equals one half of the product of the altitude by the sum of the parallel sides.**

EQUATION AND FORMULA.

$$\textbf{Area Trapezoid} = \tfrac{1}{2}(b + c) \times a.$$

EXERCISES

592. Find the area of the following rectangles:

	Length	Width		Length	Width
1.	5 ft.	3 ft.	4.	$10\frac{1}{2}$ yd.	$3\frac{1}{2}$ ft.
2.	$3\frac{1}{2}$ ft.	$2\frac{1}{2}$ ft.	5.	6 ft.	9 in.
3.	9 ft. 6 in.	2 ft. 4 in.	6.	12.7 ft.	3.5 yd.

Of each of the following parallelograms:

	Base	Altitude		Base	Altitude
7.	22.7 ft.	16.5 ft.	9.	300 ft.	6 ft. 2 in.
8.	12 ft. 9 in.	5 ft. 4 in.	10.	500 ft.	9.3 yd.

Of each of the following trapezoids:

	Base	Side Opposite the Base	Altitude
11.	8 ft.	12 ft.	4 ft.
12.	25 ft.	18 ft.	9 in.
13.	35 ft.	20 ft.	5 yd.

PROBLEMS

593. 1. The width of a rectangle is 40 ft.; the length is $2\frac{1}{2}$ times the width. Find the area.

2. The base of a rectangle is 50 ft.; the altitude is $9\frac{1}{2}$ ft. Find the area.

3. The base of one rectangle is 50 ft.; the altitude is $9\frac{1}{2}$ ft.; the base and altitude of another rectangle are twice as great as the base and altitude of the first rectangle. Find the area of the second rectangle. The area of the second rectangle is how many times the area of the first rectangle?

4. The area of a rectangle is 750 sq. yd.; the base is 50 yd. Find the altitude.

5. The area of a rectangle is 400 sq. ft. The base is 4 times the altitude. Find the base and the altitude.

EXERCISES

594. Find the area of each of the following triangles:

	Base	Altitude		Base	Altitude
1.	10 in.	6 in.	4.	30 ft. 6 in.	5 ft. 2 in.
2.	$4\frac{1}{2}$ yd.	$2\frac{1}{2}$ ft.	5.	35.6 ft.	4.5 in.
3.	20 ft.	4 in.	6.	52.7 ft.	3.4 in.

PROBLEMS

595. 1. The base of a triangle is 20 ft.; the altitude is $\frac{1}{2}$ the base. Find the area.

2. The base of a triangle is 18 in.; the altitude is $3\frac{1}{2}$ times the base. Find the area.

3. Find the area of a triangle if the base equals 11.5 ft. and the altitude is twice as great as the base.

4. The area of a triangle equals 400 sq. ft. The base is 10 ft. Find the altitude.

5. The area of a triangle is 225 sq. ft. The base is twice as long as the altitude. Find the dimensions of the triangle.

REGULAR POLYGONS

596. Surfaces like those in Figs. 1, 2 and 3 are **polygons.**

A **polygon** is a plane figure bounded by three or more straight lines.

Polygons are named from the number of sides; *e.g.*

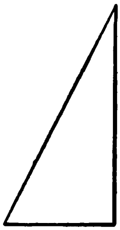


FIG. 1.

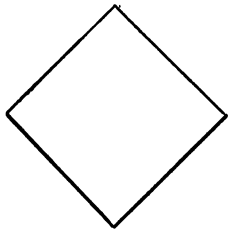


FIG. 2.

A polygon having 3 sides is called a **triangle.**
A polygon having 4 sides is called a **quadrilateral.**
A polygon having 6 sides is called a **hexagon.**
A polygon having 8 sides is called an **octagon.**

597. The entire distance around a figure is the **perimeter** of the figure; the **perimeter** of a polygon is the sum of all its sides.

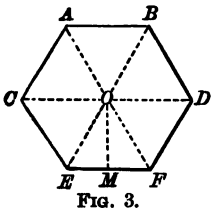


FIG. 3.

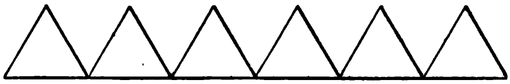
A polygon is called a **regular polygon** when all its sides are equal and all its angles are equal.

The distance OM in Fig. 3 is called the **apothem** of the regular polygon. It is a perpendicular drawn from the center of the polygon to a side EF.

To find the Area of a Regular Polygon

598. Divide the polygon into triangles. How many triangles are there if the polygon has 4 sides? If the polygon has 6 sides?

The area of each triangle equals $\frac{1}{2}$ of the product of the base by the altitude. The base of the triangle is a side of the polygon; the altitude of the triangle is the apothem of the polygon.



Therefore, the area of all the triangles (or the area of the polygon) equals $\frac{1}{2}$ of the product of the perimeter by the apothem.

RULE. **The area of a regular polygon equals $\frac{1}{2}$ of the product of the perimeter by the apothem.**

EQUATION AND FORMULA.

$$\textbf{Area Polygon} = \tfrac{1}{2}\ (\textbf{perimeter} \times \textbf{apothem}).$$

$$\textbf{Area Polygon} = \tfrac{1}{2}\ (\mathbf{p} \times \mathbf{a}).$$

EXERCISES

599. 1. Find the area of a regular hexagon, side 3 ft., apothem, approximately 2.58 ft.

2. Octagon, side 2.28 ft. apothem, approximately 2.77 ft.

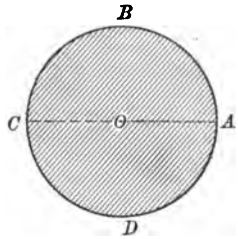
THE CIRCLE

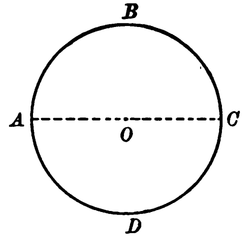
600. The shaded surface $ABCD$ is a **circle.**

The curved line $ABCD$ is the **circumference** of the circle.

The point O is the **center** of the circle. The line OA is the **radius** of the circle. The line AC is the **diameter** of the circle. The diameter is twice as long as the radius.

601. A **circle** is a plane surface, bounded by a curved line, every point of which is equally distant from a point within, called the center.





To find the Relation between the Circumference and the Diameter of a Circle

602. Draw on cardboard a circle having a radius of $3\frac{1}{2}$ inches; then cut out the circle; place a string carefully along the circumference. Measure the length of the string; it is 11 inches. Draw other circles with different diameters. Make a table showing the lengths of the diameters and circumferences as follows:

	Diameter	Circumference	Ratio of Circumference to Diameter (Circumference ÷ Diameter)
1.	$3\frac{1}{2}$ in.	11 in.	?
2.	——	——	——
etc.	etc.	etc.	etc.

It will be found that in each circle the circumference is about $3\frac{1}{7}$ times as long as the diameter. If we measured the length of the circumference more accurately, it would be found that the circumference is 3.1416 times the diameter.

RULE. **The circumference of a circle equals $3\frac{1}{7}$ times the diameter**

FORMULA. $C = 3\frac{1}{7} \times d,$

or, more accurately, $C = 3.1416 \times d.$

The Greek letter π (pī) is used to denote $3\frac{1}{7}$, or 3.1416. r is used to denote the radius of the circle.

603. Therefore, $C = \pi \times d,$

or $C = 2 \times \pi \times r$

(because the diameter equals twice the radius).

To find the Area of a Circle

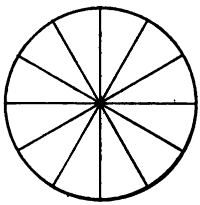


FIG. 1.

604. Draw a circle having a radius of $3\frac{1}{2}$ in. Divide it into 16 portions (called sectors) as shown in Fig. 1.

Cut out the sectors and arrange them as in Figs. 2 or 3.

The sectors form a figure that is almost a parallelogram. The base AB is nearly $\frac{1}{2}$ the circumference; the height of the figure is nearly equal to the radius of the circle. If the circle be divided into an infinitely large number of sectors,

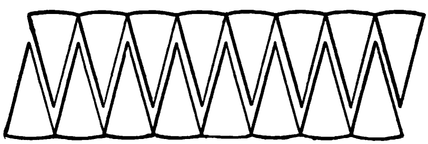


FIG. 2.

the figure $ABCD$ tends to become a parallelogram, having a base equal to $\frac{1}{2}$ the circumference and an altitude equal to the radius.

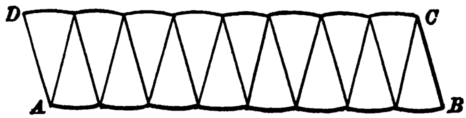


FIG. 3.

Area of parallelogram = base × altitude.

Area of circle = $\frac{1}{2}$ (circumference × radius),

or $\frac{1}{2}(2 \times \pi \times r \times r)$.

The area of a circle equals $\frac{1}{2}$ of the product of the circumference by the radius.

A second rule is obtained from the first by substituting $2 \times \pi \times r$ for circumference (see Art. 603).

EQUATION AND FORMULA.

$$\textbf{Area of Circle} = \frac{1}{2} \times 2 \times \pi \times r \times r, \textbf{ or}$$

$$\textbf{Area of Circle} = \pi \times r^2.$$

605. RULE. **The area of a circle equals 3.1416* times the square of the radius.**

EXERCISES

606. Find the area of a circle whose diameter is:

1. 6 ft.	3. 3 m.	5. 1 ft. 2 in.
2. $2\frac{1}{2}$ in.	4. 15 cm.	6. 2.8 ft.

Find the circumference of a circle whose radius is:

7. 3 in.	9. $4\frac{3}{4}$ in.	11. 70 cm.
8. $3\frac{1}{2}$ ft.	10. 5 m.	12. 4 ft. 2 in.

Find the area of a circle whose radius is:

13. 22 in.	15. 14.7 ft.	17. 19.3 yd.
14. 2 m.	16. $28\frac{1}{2}$ ft.	18. $3\frac{1}{2}$ m.

* NOTE. It is usually sufficiently accurate to consider $\pi = 3\frac{1}{7}$.

PROBLEMS

607. 1. A circular plot of ground is 28 ft. in diameter. Find its value at $3 per sq. ft.

2. The diameter of the inner rim of a wheel is 30 in. How long is the rim?

3. What is the length of the circumference of a circle described by a compass whose points are 12 in. apart?

4. The distance around a circular plot of ground is 440 ft. What is the greatest distance across the plot?

5. (*a*) The diameter of a wheel of a bicycle is 26 in. How far does the wheel go in making one revolution?

(*b*) How many revolutions are necessary to go $\frac{1}{2}$ mi.?

SOLIDS

For Reading and Discussion

608. Anything that occupies space is a solid; *e.g.* a baseball, a book, a pencil, etc. Solids have three **dimensions**, namely, **length**, **breadth** and **thickness**;

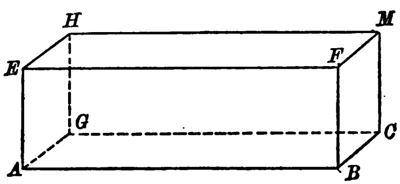


Fig. 1.

surfaces have only two dimensions, length and breadth; lines have only one dimension, length.

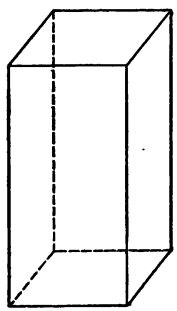
Fig. 1 is not a solid, but it represents a solid.

The lines AB, BC, AE, etc., represent the **edges** of a solid.

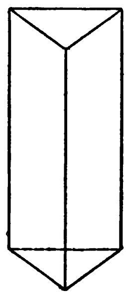
The surfaces $AEFB$, $BCMF$, $CMHG$ *and* $GHEA$ are the **lateral faces** of the solid.

609. A **rectangular solid** is a solid having 6 rectangular faces or surfaces. Fig. 1 represents a rectangular solid. The lower and upper faces ($ABCG$ and $EFMH$) are called the lower and upper **bases** of the solid.

610. A **prism** is a solid whose lateral faces are parallelograms, and whose upper and lower bases are equal and parallel to each other.

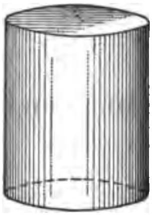


Rectangular Prism



Triangular Prism

Prisms are named from the shape of their bases — as triangular prisms, rectangular prisms, square prisms, hexagonal prisms, etc.



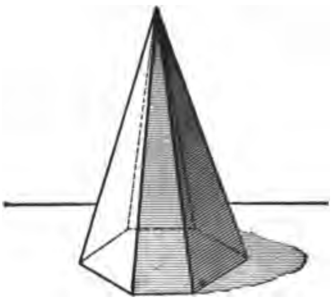
Cylinder

611. By the **convex surface** of a prism is meant the total surface of all the faces except the upper and lower bases.

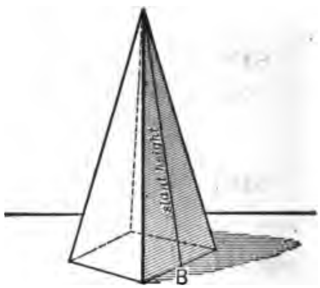
612. A **circular cylinder** is a solid bounded by a uniformly curved surface and having circles parallel to each other for its upper and lower bases.

613. The **convex surface** of a cylinder is the curved surface.

614. The total surface of a prism or cylinder consists of the convex surface plus the upper and lower bases.



Hexagonal Pyramid



Rectangular Pyramid

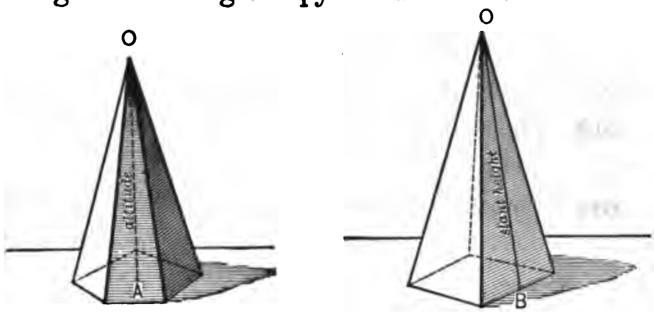
615. A regular **pyramid** is a solid whose base is a polygon and whose lateral faces are equal triangles meeting in a point.

The point in which the lateral faces meet is called the **vertex** of the pyramid.

Pyramids are named from the shape of their bases.

The distance *OA* is called the **altitude** of the pyramid. It is the perpendicular distance from the vertex to the base.

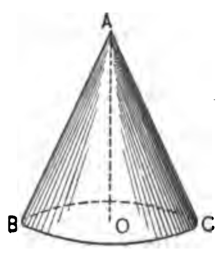
616. The distance *OB* is called the **slant height** of the pyramid. It is the altitude of one of the triangles bounding the pyramid.



Pyramids showing Altitude and Slant Height

617. The **convex surface** of a pyramid consists of the triangles that bound the pyramid. The base is not included in the convex surface.

618. A circular **cone** is a solid whose base is a circle and whose lateral surface is curved and tapers uniformly to a point.

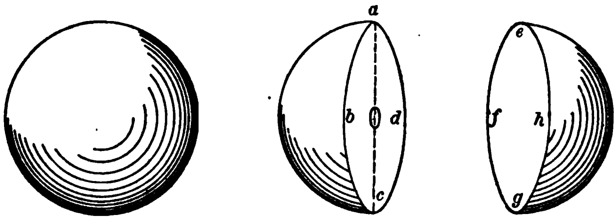


The point *A* to which the surface tapers is the **vertex** of the cone.

The distance *AO* is the **altitude** of the cone.

The distance *AB* is the **slant height** of the cone.

619. A **sphere** is a solid bounded by a curved surface every point of which is equally distant from a point within called the **center.**



620. The line AOC is the **diameter** of the sphere. It passes through the center, O, and terminates at both ends in the surface.

621. The line OA is the **radius** of the sphere. It is the distance from the center to any point on the surface. The radius is one half the diameter.

***AOC* is the Diameter of the Sphere**
***OA* is the Radius of the Sphere**

622. A circle made by dividing a sphere into two parts by a plane that passes through the center, is a **great circle of the sphere.**

SURFACES OF SOLIDS

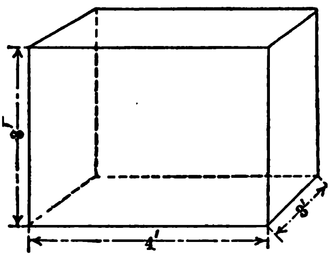
To find the Area of the Convex Surface of a Prism

623. A rectangular solid is 4 ft. long, 3 ft. wide, and 3 ft. high. Find the convex surface.

Process

The convex surface consists of the areas of 4 rectangles.

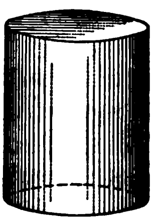
Area of one rectangle, 4 ft. by 3 ft.	= 12 sq. ft.
Area of one rectangle, 4 ft. by 3 ft.	= 12 sq. ft.
Area of one rectangle, 2 ft. by 3 ft.	= 6 sq. ft.
Area of one rectangle, 2 ft. by 3 ft.	= 6 sq. ft.
Convex surface	= 36 sq. ft. *Ans.*



A shorter method is to find the perimeter of the base (12 ft.) and multiply it by the height. The result, expressed in square units of the same denominations as the dimensions, is the convex surface.

624. RULE. **The area of the convex surface of a prism equals the perimeter of the base multiplied by the altitude, expressed in like units.**

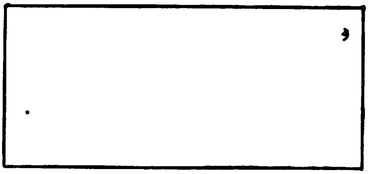
To find the Area of the Convex Surface of a Circular Cylinder



Cylinder

1. A cylinder is 6 in. high; the diameter of the base is 2 in. Find the convex surface.

Wrap a piece of paper around the cylinder so that it will equal the convex surface; then lay the paper as shown here. The paper will have the shape of a rectangle.



The area of the rectangle, which is equal to the convex surface of the cylinder, is found by multiplying the length (the circumference of the base) by the altitude (the altitude of the cylinder).

Convex Surface of a Circular Cylinder $= 3.1416 \times 2 \times 6 = 37.6992$ (sq. in.).

625. RULE. **The area of the convex surface of a circular cylinder equals the circumference of the base multiplied by the altitude, expressed in like units.**

FORMULA. $CS = \frac{1}{2}(2\pi \times r \times h)$.

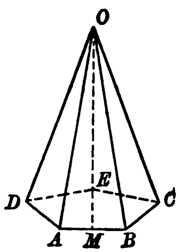
To find the Area of the Convex Surface of a Regular Pyramid

626. The convex surface of a regular pentagonal pyramid consists of the areas of 5 triangles.

Therefore, the convex surface of the pyramid equals

$$\tfrac{1}{2}(AB + AD + DE + EC + CB) \times \text{altitude.}$$

But $\frac{1}{2}(AB + AD + DE + EC + CB) = \frac{1}{2}$ perimeter of base. And the altitude of each triangle equals the slant height of the pyramid. This is true in any regular pyramid.



627. RULE. **The area of the convex surface of a regular pyramid equals $\frac{1}{2}$ of the product of the perimeter of the base by the slant height, expressed in like units.**

To find the Area of the Convex Surface of a Cone

628. RULE. **The area of the convex surface of a circular cone equals $\frac{1}{2}$ of the product of the circumference of the base multiplied by the slant height, expressed in like units.**

$$CS = \pi r \times Sl\,H.$$

To find the Area of a Convex Surface of a Sphere

629. Divide the sphere into two equal parts, called **hemispheres** (Fig. 1).

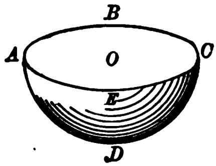
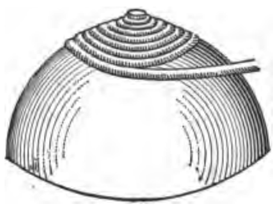


Fig 1. A Hemisphere
The circle *ABCE* is a great circle

Drive a tack into one hemisphere and wind a cord completely over the surface.

Drive a tack into the center of the great circle, and wind a cord completely over the surface of the circle.

Great Circle with Cord wound over Surface



Hemisphere with Cord wound over Surface

Compare the lengths of the two cords. It will be found that the surface of a hemisphere is twice as great as the area of a great circle.

Therefore, the surface of a sphere is 4 times as great as the area of a great circle.

The area of a circle whose radius is r is $\pi \times r^2$.

Therefore, the area of the convex surface of a sphere is

$$4 \times \pi \times r^2.$$

RULE. **The area of the convex surface of a sphere equals the square of the radius multiplied by 4 times π.**

EXERCISES

630. Find the area of the convex surface of the following rectangular prisms:

	Base	Altitude
1.	3 ft. by 6 ft.	5 ft.
2.	2 ft. by 3 ft.	4 ft.
3.	$2\frac{1}{2}$ m. by $3\frac{1}{2}$ m.	$2\frac{1}{2}$ m.
4.	9 dm. by 10 dm.	2 dm.

Find the area of the convex surface of the following prisms:

5. Triangular prism; perimeter of base, 18 ft; altitude, 3 ft.

6. Hexagonal prism; perimeter of base, 12 ft; altitude, 3 ft.

Find the area of the convex surface of the following circular cylinders:

7. Circumference of base, 21 in.; altitude, 6 in.

8. Diameter of base, 2 ft.; altitude, 1 ft.

Find the area of the convex surface of the following regular square pyramids:

	Base	Slant Height
9.	3 ft. by 2 ft.	3 ft.
10.	2 ft. by 4 ft.	5 ft.
11.	8 dm. by 5 dm.	7 dm.
12.	6 m. by 2 m.	3 m.

Find the area of the convex surface of the following regular pyramids:

13. Triangular pyramid: perimeter of base, 16 ft.; slant height, 8 ft.

14. Triangular pyramid: perimeter of base, 12 meters; slant height, 5 meters.

Find the area of the convex surface of the following circular cones:

15. Circumference of base, 25 in.; slant height, 15 in.

16. Diameter of base, 1 m.; slant height, 4 m.

Find the area of the convex surface of the following spheres:

Radius of sphere	Diameter of sphere
17. 4 in.	21. 3 ft.
18. $6\frac{1}{2}$ in.	22. 4 ft.
19. 3 m.	23. 5 m.
20. 9 dm.	24. 8 dm.

PROBLEMS

632. 1. How much sheet iron is necessary to line the sides of a refrigerator 18 ft. long, 12 ft. wide, and 8 ft. high, adding 10 % for waste?

2. Find the total surface of a cube 15 in. long.

3. The distance around a cylindrical gas tank is 154 ft. What is the convex surface of the tank, if the altitude is 40 ft.?

4. How much sheet iron will be needed to make a smokestack 20 ft. high and 7 ft. in diameter?

5. Find the convex surface of a rectangular pyramid whose base is a square 3 ft. long and whose slant height is 6 ft.

6. Find the convex surface of a cone, if the diameter of the base is 12 in. and the slant height is 18 in.

7. The top of an obelisk is a square pyramid, the base of which is 3 ft. long. Find the convex surface of the pyramid, if its slant height is $10\frac{1}{2}$ ft.

8. The slant height of a steeple is 80 ft. and the base is a square 15 ft. on a side. How many square feet of surface on the outside of the steeple?

9. How many square inches in the cover of a baseball 3 in. in diameter?

10. How many square inches of leather are necessary to cover a basket ball 14 in. in diameter?

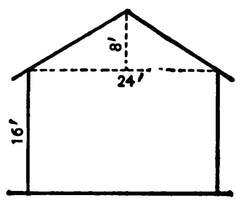
11. The steel drum of a hoisting apparatus is 3 ft. long and 24 in. in diameter. What length of cable $\frac{1}{2}$ in. in diameter may be wound upon the drum?

12. A silo in the form of a cylinder is 12 ft. in diameter and 30 ft. high. Allowing $\frac{1}{5}$ for waste, how many board feet of matched pine will be required to enclose the silo?

13. A church spire is in the form of a hexagonal pyramid, each side of whose base is 6 ft. and whose slant height is 36 ft. How many shingles laid 5 in. to the weather, the first row double, will be required to cover its lateral surface? Allow $\frac{1}{5}$ for waste.

14. How many square yards of canvas are required to make a tent in the form of a regular triangular prism whose base is 18 ft. by 12 ft., and whose altitude is 8 ft.?

15. Find the cost at 45 cents per square yard for a double coat of painting for the walls of a building 48 ft. long and having the end section shown in the cut.



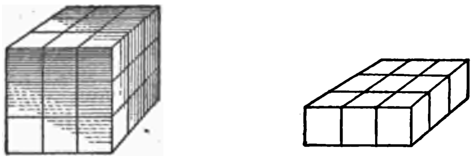
16. How many shingles laid 4 in. to the weather will be required to shingle the roof of the building shown in problem 15, if the eaves project $1\frac{1}{2}$ ft. beyond the sides of the building, and the first row of shingles is laid double?

VOLUMES OF SOLIDS

To find the Volume of a Prism

633. 1. By using inch cubes, build a rectangular prism 3 in. long, 3 in. wide, and 3 in. high.

2. This prism can be divided horizontally into 3 layers, or prisms, each being 3 in. long, 3 in. wide, and 1 in. high. The volume of 1 layer is 9 cu. in.



Therefore, the volume of the whole prism is 3×9 cu. in. $= 27$ cu. in.

634. RULE. **The volume of a prism equals the product of the area of the base by the altitude; the result, expressed as cubic units of the same denomination as the dimensions, is the volume.**

NOTE. The volume of a rectangular prism may be found by multiplying together the length, width, and altitude, the three dimensions being expressed in like units.

EQUATION AND FORMULA.

$$\textbf{Volume Prism} = l \times w \times a.$$

To find the Volume of a Cube

635. In a cube the length, width, and altitude are equal.

RULE. **The volume of a cube equals the product obtained by taking the length 3 times as a factor.**

EQUATION AND FORMULA.

$$\textbf{Volume Cube} = l^3 \textbf{ or } (l \times l \times l).$$

To find the Volume of a Cylinder

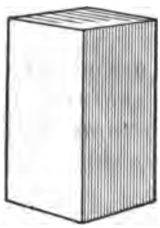
636. RULE. **The volume of a cylinder equals the product of the area of the base by the altitude.**

The two rules for finding the volumes of a prism and a cylinder may be combined into one rule.

637. RULE. **The volume of a prism or a cylinder equals the product of the area of the base by the altitude.**

To find the Volume of a Pyramid

638. Construct with cardboard a prism 6 in. high, whose base is a square 4 in. long.



Square prism 6 in. high, base 4 in. long.

Square pyramid 6 in. high, base 4 in. long.

Also, construct a pyramid 6 in. high, whose base is a square 4 in. long.

Use the pyramid as a measure; fill it with sand; pour the sand into the prism.

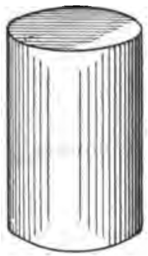
How many times must the pyramid be filled, to fill the prism?

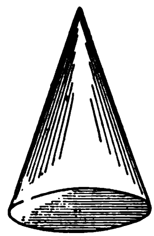
Therefore, the volume of a pyramid is $\frac{1}{3}$ as great as the volume of a prism having the same base and altitude.

639. RULE. **The volume of a pyramid equals $\frac{1}{3}$ of the product of the area of the base by the altitude.**

To find the Volume of a Cone

640. Construct with cardboard a cylinder 6 in. high, the radius of the base being 2 in. Also, construct a cone 6 in. high, the radius of the base being 2 in.





Using the cone as a measure, find the relation between the volume of the cylinder and the volume of the cone.

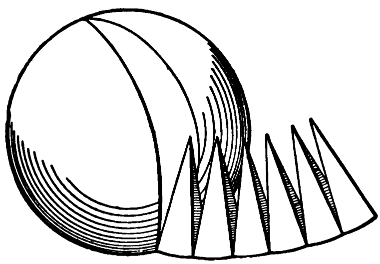
The volume of the cone is $\frac{1}{3}$ as great as the volume of a cylinder having an equal base and an equal altitude.

641. RULE. **The volume of a cone equals $\frac{1}{3}$ of the product of the area of the base by the altitude.**

To find the Volume of a Sphere

Cut an orange as shown in the figure.

The small figures may be considered to be pyramids. The volume of the sphere is equal to the sum of the volumes of the pyramids. The volume of each pyramid is equal to $\frac{1}{3}$ of the product of the area of the base by the altitude.



The areas of the bases of the pyramids compose the convex surface of the sphere; the altitude of the pyramids equals the radius of the sphere. Therefore, the

volume of the sphere equals $\frac{1}{3}$ of the product of the convex surface by the altitude.

But in art. 631 we found that the convex surface of a sphere whose radius is r equals $4 \times \pi \times r^2$.

Therefore,

The volume of a sphere equals $\frac{1}{3} \times 4 \times \pi \times r \times r \times r$ or

$$\textbf{Volume of sphere} = \tfrac{4}{3} \times \pi \times r^3.$$

RULE. **The volume of a sphere equals $\frac{4}{3}$ of π multiplied by the cube of the radius.**

NOTE. r^3 is read "r cube"; it means that r is taken 3 times as a factor; *e.g.* $2^3 = 2 \times 2 \times 2 = 8$.

EXERCISES

642. Find the volume of the following rectangular prisms:

	Length	Width	Altitude		Length	Width	Altitude
1.	3 ft.	2 ft.	1 ft.	2.	3 ft.	3 ft.	4 ft.
3.	9 in.	2 ft.	6 in.	4.	3 m.	2 m.	1 m.

Find the volume of the following prisms:

5. Triangular prism: area of base, 30 sq. ft.; altitude, 4 ft.

6. Hexagonal prism: area of base, $12\frac{1}{2}$ sq. ft.; altitude, $2\frac{1}{2}$ ft.

Find the volume of the following cylinders:

7. Area of base, 16 sq. ft.; altitude 5 ft.

8. Area of base, 25 sq. m.; altitude 4 m.

Find the volume of the following square pyramids:

	Area of Base	Altitude		Area of Base	Altitude
9.	16 sq. ft.	3 ft.	10.	20 sq. yd.	2 yd.
11.	15 sq. m.	2 m.	12.	21 sq. m.	5 m.

Find the volume of the following pyramids:

13. Triangular pyramid: area of base, 15 sq. ft.; altitude 4 ft.

14. Hexagonal pyramid: area of base, 6 sq. m.; altitude, $1\frac{1}{2}$ m.

Find the volume of the following cones:

15. Area of base, 10 sq. ft.; altitude, 5 ft.

16. Area of base, 5 sq. m.; altitude, 3 m.

17. Diameter of base, 6 ft.; altitude, 3 ft.

Find the volume of the following spheres:

18. Radius = 2 in.   19. Diameter = 8 in.

20. Radius = 3 dm.   21. Diameter = $2\frac{1}{2}$ m.

PROBLEMS

643. 1. The dimensions of the base of a rectangular prism are 5 ft. by 3 ft. Find the volume of the prism, if the altitude is 4 ft.

2. The area of the base of a cylinder is 30 sq. ft. The cylinder is 6 ft. high. Find the volume.

3. A column of stone in the form of a rectangular prism is 8 ft. high. The area of the base is 40 sq. ft. Find the weight of the column, if 1 cu. ft. weighs 160 lb.

4. A cylindrical column has a base the area of which is 16 sq. ft.; the altitude of the column is 5 ft. Find the weight of the column if a cubic foot weighs 140 lb.

5. A cylindrical stone column, 5 ft. high, has a base, the area of which is 16 sq. ft. Find the weight of a column, if a cubic foot weighs 140 lb.

6. Find the volume of a square pyramid, if the area of the base is 36 sq. ft. and the altitude is 9 ft.

7. Find the volume of a cone, if the radius of the base is 7 ft. and the altitude is $3\frac{1}{2}$ ft.

8. An ice-cream cone is 6 in. high and 2 in. in diameter. How many cubic inches of ice cream does it hold?

9. The large pyramid at Gizeh, Egypt, is about 450 ft. high and its base is a square 750 ft. long. How many cubic feet of stone does it contain? How many cubic yards does it contain?

10. Find the volume of a hexagonal prism if the area of its base is 748 sq. in. and the altitude of the prism is 3 ft.

11. Iron is 7.6 times as heavy as water. How many pounds of iron will be needed to make 18 iron spheres each 12 in. in diameter?

REVIEW

644. Find the value for ? in each of the following:

	Figure	Base	Altitude	Radius	Circumference	Area
1.	Rectangle	6 ft.	6 in.	——	——	?
2.	Rectangle	?	2 ft.	——	——	18 sq. ft.
3.	Square	?	?	——	——	256 sq. ft.
4.	Parallelogram	1½ ft.	9 in.	——	——	?
5.	Parallelogram	?	12 ft.	——	——	216 sq. ft.
6.	Parallelogram	24 ft.	?	——	——	384 sq. ft.
7.	Rectangle	?	5½ ft.	——	——	41¼ sq. ft.
8.	Rectangle	10½ ft.	?	——	——	26¼ sq. ft.
9.	Circle	——	——	7 ft.	?	?
10.	Circle	——	——	?	?	44 sq. in.
11.	Circle	——	——	?	22 in.	?
12.	Circle	——	——	9½ in.	?	?
13.	Triangle	9½ ft.	2 yd.	——	——	?
14.	Triangle	?	3 yd.	——	——	180 sq. ft.
15.	Triangle	2.8 ft.	?	——	——	9.8 sq. ft.
16.	Triangle	?	7 in.	——	——	9 sq. ft.
17.	Rectangle	12.6 ft.	2.7 yd.	——	——	?
18.	Triangle	5.6 ft.	1.3 yd.	——	——	?
19.	Square	?	?	——	——	42.25 sq. yd.
20.	Parallelogram	?	8.5 ft.	——	——	21.25 sq. ft.
21.	Circle	——	——	2.5 ft.	?	?
22.	Circle	——	——	?	?	36 sq. ft.
23.	Triangle	40.5 ft.	?	——	——	101.25 sq. ft.
24.	Square	?	?	——	——	21.16 sq. ft.
25.	Rectangle	?	8.9 ft.	——	——	30.26 sq. ft.

645. Find the value for ? in each of the following:

	Figure	Perimeter of Base	Altitude	Slant Height	Diameter	Convex Surface
1.	Prism	16 sq. ft.	2 yd.	——	——	?
2.	Prism	24 sq. ft.	?	——	——	120 sq. ft.
3.	Cylinder	18.5 sq. ft.	1½ yd.	——	——	?
4.	Cone	30 sq. ft.	——	9 ft.	——	?
5.	Pyramid	42 sq. ft.	——	2 yd.	——	?
6.	Cylinder	?	3 yd.	——	——	225 sq. ft.
7.	Pyramid	?	——	4 yd.	——	72 sq. ft.
8.	Cone	44 sq. ft.	——	?	——	132 sq. ft.
9.	Sphere	——	——	——	3 ft.	?
10.	Sphere	——	——	——	2½ yd.	?

646. Find the value for ? in each of the following:

	Figure	Area of Base	Altitude	Radius	Volume
1.	Prism	48 sq. ft.	$2\frac{1}{2}$ yd.	——	?
2.	Cylinder	72 sq. in.	$1\frac{1}{2}$ ft.	——	?
3.	Cone	80 sq. ft.	107 ft.	——	?
4.	Sphere	——	——	3 ft.	?
5.	Cylinder	?	4 yd.	——	72 cu. ft.
6.	Pyramid	24 sq. ft.	2.4 yd.	——	?
7.	Prism	——	$\frac{2}{3}$ yd.	——	32 cu. ft.
8.	Cone	9 sq. yd.	?	——	$62\frac{1}{2}$ cu. ft.
9.	Sphere	——	——	3.5 ft.	?
10.	Sphere	——	——	$10\frac{1}{2}$ in.	?

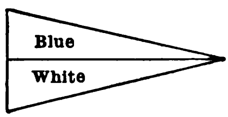
MISCELLANEOUS PROBLEMS IN MENSURATION

647. 1. How many lots, each 25 ft. by 100 ft., can be made from a plot 300 ft. by 200 ft.? (Draw a diagram.)

2. The area of a rectangular plot is 1200 sq. ft. The base is three times as long as the altitude. Find the dimensions of the plot.

3. A rectangular field is 180 yd. long; the width is 40 % of the length. Find the area.

4. A flag is made of two strips of felt of equal size. The base of the flag is 9 in.; the altitude of the flag is 18 in. How many square inches of felt in each strip?



5. Find the area of a trapezoid with bases 6 ft. and 8 ft. and altitude 4 ft.

6. A circular track is 140 ft. in diameter. How many times must a boy run around the track to make a mile? ($R = 3\frac{1}{7}$.) If the diameter of the track were 280 ft., how many laps would it take to make a mile?

7. A bin 10 ft. long, 5 ft. wide, and 5 ft. deep is filled with wheat.

(*a*) How many bushels of wheat are in the bin?

(*b*) Find the value of the wheat at $1.05 per bushel.

8. Find the area of a regular hexagon if the length of one side is 3 in. and the apothem 4 in.

9. Find the area of a regular octagon if the length of one side is 4 in. and the apothem 6 in.

10. A rectangular stone prism is to have all its faces polished except the base. The base of the prism is 3 ft. 4 in. long and 2 ft. 10 in. wide. What will the work cost at 75 ct. a square foot?

11. A cylindrical stone column is 12 ft. high; the area of the base is 12 sq. ft.

(*a*) Find the volume of the column in cubic yards.

(*b*) Find the weight of the column if one cubic yard weighs 42 lb.

(*c*) Find the area of the convex surface of the column.

(*d*) Find the area of the total surface of the column.

12. Draw to a convenient scale the place of

(*a*) A baseball field.

(*b*) A lawn tennis court.

13. Make up and solve three problems about the baseball field and three problems about the lawn tennis court.

14. The top of an obelisk is in the form of a square pyramid. The base of the pyramid is 4 ft. square and the height is 6 ft. Find the volume of the pyramid.

15. A tin funnel (cone) is 6 in. high; the area of the large end is 12 sq. in. Find the volume of the funnel.

16. Find the surface, area, and volume of a sphere 6 in. in diameter.

RELATED PROCESSES

Group I

648. 1. $24\frac{1}{2}$ yd. of cloth are bought for \$$36\frac{3}{4}$. Find the cost of one yard.

2. $30\frac{3}{4}$ yd. of cloth are bought for \$$1\frac{1}{5}$ per yard. Find the cost.

3. $32\frac{1}{2}$ yd. of cloth are bought for \$40.36. Find the cost of $16\frac{1}{4}$ yd.

4. 47 yd. of cloth are bought for \$$70\frac{1}{2}$. How many yards may be bought for \$$211\frac{1}{2}$?

5. Cloth bought at \$$1\frac{3}{4}$ per yard is sold for \$$2\frac{1}{4}$ per yard.
(*a*) Find the gain per cent on 1 yd.
(*b*) Find the gain per cent on 25 yd.

6. Cloth that cost \$$1\frac{3}{4}$ per yard is sold at a gain of 20%. Find the selling price.

7. Cloth is sold at a gain of $33\frac{1}{3}$%. If the selling price is \$2 per yard, what is the cost per yard?

Group II

649. Mr. Joseph Adams keeps a store at 214 East 23d St., New York. He orders the following goods from the American Hat Company, 130 Mercer St., New York:

4 doz. Derby Hats @ \$30 per dozen.
6 doz. Straw Hats @ \$24 per dozen.
10 doz. Boys' Caps @ \$ 9 per dozen.

1. Write, in full, the letter that Joseph Adams sends to the American Hat Company ordering the goods.

2. The American Hat Company answers Mr. Adams, thanking him for the order and stating that the goods will be sent within 5 days. Write the letter.

3. Write, in full, the bill that is sent with the goods. Fill out all the amounts, etc.

4. Mr. Adams paid as soon as the goods were received, and obtained discounts of 10 % and 2 %. How much did he pay?

5. Write the check which Mr. Adams sent to the American Hat Company.

6. The American Hat Company deposits the check in the Corn Exchange Bank.

(*a*) Indorse the check so that if it is lost no one can collect the money.

(*b*) Make out the deposit slip.

7. Write the receipt sent by the American Hat Company to Mr. Adams.

Group III

650. 1. A merchant bought:

120 white shirts @ $12 per dozen.
180 gray shirts @ $18 per dozen.

Find the total cost of the shirts.

2. The white shirts were marked at a gain of 25 %. 75 % of them were sold at that price. How much was received for them?

3. The rest of the white shirts were sold at a reduction of 10 % from the marked price. How much was received for them?

4. The gray shirts were marked at a gain of 20 %. 60 % of them were sold at that price. How much was received for them?

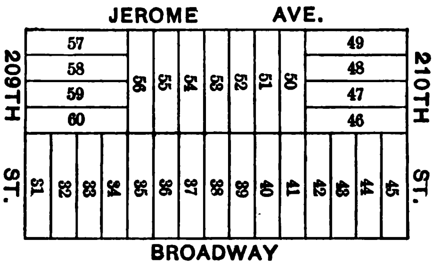
5. The rest of the gray shirts were sold at a reduction of 20 % from the marked price. How much was received for them?

6. How much did the merchant gain or lose on the whole transaction?

7. What per cent did he gain or lose on the whole transaction?

Group IV

651. 1. Find the total area of the entire plot shown in the diagram. The lots were sold at an auction sale. The prices are given below.



31 — 25 × 100 (cor.)	. . . .	$11,800
32, 33, 34 — 25 × 100 (each)	. . . .	4,800
35, 36 — 25 × 100 (each)	. . . .	3,950
37, 38 — 25 × 100 (each)	. . . .	3,850
39, 40 — 25 × 100 (each)	. . . .	4,000
41 — 25 × 100	. . . .	4,150
42, 43, 44 — 25 × 100 (each)	. . . .	4,000
45 — 25 × 100 (cor.)	. . . .	8,000
46, 47, 48 — 25 × 95 (each)	. . . .	1,800
49 — 25 × 95 (cor.)	. . . .	3,200
50, 51, 52, 53, 54 — 25 × 100 (each)	. . . .	1,200
55, 56 — 25 × 100 (each)	. . . .	1,100
57 — 25 × 95 (cor.)	. . . .	2,000
58, 59, 60 — 25 × 95 (each)	. . . .	1,800

2. Find the average price per lot on Broadway.

3. Find the average price of all the lots.

4. The price of lot 31 is what per cent of the price of lot 32?

5. Find the area of lot 45.

6. Find the price per square foot of lot 45.

7. The buyer of lot 37 paid 30 % in cash and allowed the rest to remain on mortgage. Find the face of the mortgage.

8. Find the semiannual interest on the mortgage at 5 %.

9. Find the annual taxes on lot 50 at 2.15 per centum.

10. The buyer of lot 58 erected a house that cost $ 5000. He sold the house and lot at a gain of 18 %. How much did he receive for them? How much did he gain?

Group V

652. 1. A train travels $20\frac{1}{4}$ mi. in an hour. How far does it go in $6\frac{3}{4}$ hr. at that rate?

2. If a train travels $112\frac{1}{2}$ mi. in $4\frac{1}{2}$ hr., how far does it go in one hour at that rate?

3. If a train travels $256\frac{1}{2}$ mi. in $8\frac{1}{2}$ hr., how far does it go in $4\frac{1}{4}$ hr. at that rate?

4. A train travels $99\frac{1}{8}$ mi. less in $5\frac{1}{2}$ hr. than in $8\frac{3}{4}$ hr. Find the number of miles it travels per hour.

5. A train traveled at the rate of 30 mi. per hour for $3\frac{1}{2}$ hr., then it increased its speed by $\frac{1}{8}$ of its former rate and traveled at the faster rate for $2\frac{1}{2}$ hr. How far did it travel in the entire time?

6. A train leaves the first station at 8:20 A.M. and arrives at the fifth station at 12:50 A.M. The distance between the first and fifth stations is 300 miles. At what average rate of speed does the train travel, allowing 15 minutes for all stops?

ECONOMY IN BUYING

653. 1. A family can buy eggs at 2 for 5 ct., or at 28 ct. a dozen. It uses 60 doz. a year. How much will be gained by buying the eggs by the dozen?

2. A dealer buys a case of eggs (30 doz.) for $ 8.50. He sells them at the rate of 38 ct. a dozen.

(*a*) How much does he gain?

(*b*) What per cent does he gain?

3. A grocer buys a tub of butter (56 lb.) at 32 ct. a pound. He sells half the tub at 37 ct. a pound and the rest at 19 ct. a half pound.

(*a*) What per cent does he gain?

(*b*) How much does he gain on 8 tubs of butter?

4. A family buys butter at the rate of 10 ct. a quarter of a pound. If the family uses 12 lb. of butter a month, how much would be saved each month by buying it at 37 ct. a pound?

5. A grocer bought a barrel of flour (196 lb.) for $ 5.80. He put the flour into $3\frac{1}{2}$-lb. bags and sold it at 15 ct. a bag.

(*a*) How much did he gain on a barrel?

(*b*) What per cent did he gain?

6. Instead of buying a ton of coal for $ 6.75 at one time, a family buys it in pails, each pail containing 25 lb., at 15 ct. a pail. How much does the family lose by buying the coal by the pail?

7. A grocer bought 24 bbl. of flour at $ 5.60 a barrel. He sells $\frac{1}{2}$ of the quantity in $\frac{1}{8}$-bbl. sacks at 85 ct. a sack, and the rest in $\frac{1}{4}$-bbl. sacks at $ 1.65 a sack. How much does he gain on the transaction?

8. A family can buy sugar at the rate of $3\frac{1}{2}$ lb. for 19 ct., or 10 lb for 54 ct.

(*a*) Which is the better rate?

(*b*) How much is saved by buying 70 lb. at the better rate?

9. A barrel of vinegar ($31\frac{1}{2}$ gal.) is bought at 40 ct. a gallon. It is sold at 5 ct. a quart.

(*a*) Find the per cent gain.

(*b*) Find the gain on 5 bbl.

10. A family can buy salt in 2-ct. bags containing 1 lb., or in 5-ct. bags containing 3 lb. If the family uses $4\frac{1}{2}$ lb. of salt a week, how much will it save in 40 wk. by buying 5-ct. bags?

11. Mr. Hunt buys 75 pr. of shoes at $ 1.50 per pair and 100 pr. at $ 2 per pair. He sells the cheaper shoes for $ 1.75 per pair and the more expensive shoes at a gain of 50 ct. per pair.

(*a*) How much does he gain on the whole transaction?

(*b*) What per cent does he gain on the cheaper shoes?

12. I gained 15 %, or $ 9.75, by selling a sewing machine. How much did the machine cost?

13. I gained 22 %, or $ 32.20, by selling a rug. How much did I receive for the rug?

14. A clock was sold for $ 81. This was a gain of 8 % on the cost. Find the cost.

15. By selling a load of fruit for $ 60, I lost 40%. Find the cost.

16. A druggist spends 85 ct. for a bottle of medicine. He sells it for $ 1.25. What per cent does he gain?

17. A butcher buys 80 turkeys. The average weight is 15 lb. The turkeys cost 24 ct. a pound and are sold at a gain of $16\frac{2}{3}$ %.

(*a*) How much is received for all the turkeys?

(*b*) Find the per cent profit.

APPENDIX I

TABLES FOR REFERENCE

Long Measure

Long measure, or linear measure, is used in measuring lengths or distances. The standard unit of length is the yard. The standard yard is a metal bar carefully preserved by the government in Washington. An exact copy is kept in each state capital.

Table of Long Measure

12	inches (in.)	= 1 foot (ft.)
3	feet	= 1 yard (yd.)
$5\frac{1}{2}$	yards	= 1 rod (rd.)
$16\frac{1}{2}$	feet	= 1 rod
320	rods	= 1 mile (mi.)
1760	yards	= 1 mile
5280	feet	= 1 mile

One eighth of a mile is called a furlong.

Surveyors, or measurers of land, use the following:

4 rods, or 66 feet, or 100 links (li.) = 1 chain (ch.)
80 chains = 1 mile

Square Measure

Square measure, or surface measure, is used in measuring surfaces. The standard unit of measurement is the square yard. It is a square surface one yard long and one yard wide.

Table of Square Measure

144	square inches (sq. in.)	= 1 square foot (sq. ft.)
9	square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$	square yards	= 1 square rod (sq. rd.)
160	square rods	= 1 acre (A.)
640	acres	= 1 square mile (sq. mi.)
36	square miles	= 1 township or section

The acre is used in measuring land. A square field 209 feet long and 209 feet wide measures about one acre.

Surveyors use the following:

16	square rods	= 1 square chain (sq. ch.)
10	square chains	= 1 acre

Miscellaneous Measures of Length or Surface

4	inches	= 1 hand (used in measuring the height of horses)
6	feet	= 1 fathom (used in measuring the depth of water)
$1\frac{1}{6}$	miles	= 1 knot or nautical mile
100	square feet	= a square (of roof, pavement)

Cubic Measure

Cubic measure is used in measuring solids and other volumes. The standard unit is the cubic foot. It is equal to the volume of a cube one foot in length, one foot in width, and one foot in height or depth.

A body has three measurements or dimensions: length, breadth, and thickness or height.

A cube is a body whose length, breadth, and thickness are equal. It is bounded by six equal square surfaces.

A cubic foot is a cube each of whose dimensions is one foot.

Table of Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet of wood	= 1 cord
231 cubic inches (of liquid)	= 1 gallon
2150.4 cubic inches (of grain, etc.)	= 1 bushel (the standard Winchester bushel)

The standard bushel in England is the Imperial bushel; it contains 2218.192 cu. in.

A cord is a pile of wood 8 feet long, 4 feet wide, and 4 feet high.

Measure of Angles or Circular Measure

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
90 degrees	= 1 right angle (rt. $\measuredangle$) or 1 quadrant
360 angle degrees	= 4 right angles
360 arc degrees	= 1 circumference

Avoirdupois Weight

Avoirdupois weight is used in weighing grain, coal, and other bulky merchandise. The standard unit is the avoirdupois pound containing 7000 Troy grains.

Table of Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
20 hundredweight	= 1 ton
2240 pounds	= 1 long ton
112 pounds	= 1 long hundredweight

Miscellaneous Weights

60 pounds (lb.)	= 1 bushel of potatoes
32 pounds	= 1 bushel of oats
60 pounds	= 1 bushel of wheat
56 pounds	= 1 bushel of corn (shelled)
56 pounds	= 1 bushel of rye
48 pounds	= 1 bushel of barley
80 pounds	= 1 bushel of coal
200 pounds	= 1 barrel of pork
196 pounds	= 1 barrel of flour
56 pounds	= 1 firkin of butter
$62\frac{1}{2}$ pounds	= 1 cubic foot of water

Troy Weight

Troy weight is used in weighing the precious metals such as gold and silver.

Table of Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt. or dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
5760 grains	= 1 pound

An ounce Troy weighs 480 Troy grains.

Dry Measure

Dry measure is used in measuring dry articles such as grains, vegetables, and fruits. The standard unit is the Winchester bushel. It is cylindrical in form, $18\frac{1}{2}$ inches in diameter, and 8 inches deep. It contains 2150.42 cubic inches, or $1\frac{1}{4}$ cubic feet.

Dry Measure		Liquid Measure *	
2 pints (pt.)	= 1 quart (qt.)	4 gills (gi.)	= 1 pint (pt.)
8 quarts	= 1 peck (pk.)	2 pints	= 1 quart (qt.)
4 pecks or 32 quarts	= 1 bushel (bu.)	4 quarts	= 1 gallon (gal.)

* The standard unit of liquid measure is the gallon, containing 231 cubic inches.

Time Measure

Time measure is used in measuring time. The standard unit of time is the day of 24 hours. It is the time required for the earth to make one complete rotation upon its axis.

The year consists of 365 days. It is the time required for the earth to make one complete revolution around the sun.

In telling the time of day, the hours before midday are called forenoon and those after midday are called afternoon. A.M. designates forenoon; P.M. afternoon. Thus, 7 A.M. means 7 o'clock in the morning, or forenoon; 4 P.M. means 4 o'clock in the afternoon; 12 M. indicates noon; 12 P.M. indicates midnight; 9.30 means 30 minutes after 9; 9.50 means 50 minutes after 9, or 10 minutes of or before 10.

Table of Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
365 days	= 1 year (yr.)
366 days	= 1 leap year.
100 years	= 1 century.

In examples involving reduction, 30 days are usually considered a month, and 12 months a year.

Thirty days has September.
April, June, and November, etc.

A **leap year** contains **366 days**; the extra day is added to February (Feb. 29). The system of leap year was devised to make up for the loss of nearly $\frac{1}{4}$ of a day each year, due to the difference between the actual length of a solar year (nearly $365\frac{1}{4}$ days) and the length of a calendar year (365 days).

Years indicated by numbers which are divisible by 4 without a remainder are leap years (*e.g.* 1912, 1916, 1920).

The only exception is that a leap year is not counted at the end of a century that is not exactly divisible by 400, *e.g.*, 1700, 1800, 1900 were not leap years; but 2000 will be a leap year.

Miscellaneous Tables

2 things	= 1 pair	20 things	= 1 score (sc.)
12 things	= 1 dozen (doz.)	24 sheets	= 1 quire (qr.)
12 dozen, 144 things	= 1 gross (gro.)	20 quires, 480 sheets	= 1 ream (rm.)
12 gross, or 1728 things	= 1 great gross (gt. gro.)		

English Money

English money is the currency used in Great Britain and Ireland. The unit is the pound sterling (£), a gold coin, worth \$4.8665, or about \$4.87, in U. S. money. The shilling is worth a little more than 24 cents, and the penny is worth about 2 cents of our money.

Table of English Money

4 farthings (far.)	= 1 penny (*d.*)
12 pence	= 1 shilling (*s.*)
20 shillings	= 1 pound sterling (£)
21 shillings	= 1 guinea

The currency of systems of several other countries o commercial importance are shown in the following table:

Country	Standard	Divisions	U. S. Value
Germany	1 Mark	= 100 Pfennig	= \$ .238
France	1 Franc	= 100 Centimes	= .193
Italy	1 Lira	= 100 Centesimi	= .193
Austria	1 Crown	= 100 Heller	= .203
Russia	1 Ruble	= 100 Kopecks	= .51

Table of Metric System

Metric Table of Length

10 millimeters (mm.)	= 1 centimeter (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 meter (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 kilometer (Km.)

Measurement of Surfaces

The **unit** of **surface measure** in the metric system is the **square meter**; it is the surface of a square, each side of which is one meter in length.

This diagram represents 1 square centimeter.

A **square meter** equals **1.196 square yards.**

Metric Table of Surface Measure

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.)
100 square centimeters	= 1 square decimeter (sq. dm.)
100 square decimeters	= 1 **square meter** (sq. m.).
100 square meters	= 1 square dekameter (sq. Dm.)
100 square dekameters	= 1 square hektometer(sq. Hm.)
100 square hektometers	= 1 **square kilometer** (sq. Km.)

The areas of walls, floors, ceilings, etc., are expressed in square meters; the areas of large surfaces, such as the areas of countries, are expressed in square kilometers (1 square kilometer equals nearly .4 square mile).

The **unit** of the metric table of **land measure** is the **are,** which is the area of a square dekameter; the area of an are, therefore, is

$$100 \times 1.196 \text{ sq. yd.} = 119.6 \text{ sq. yd.}$$

Metric Table of Land Measure

100 centares (ca.) = 1 are (a.)
100 ares = 1 hektare (nearly $2\frac{1}{2}$ acres) (Ha.)

Measurements of Volume

The **unit of measurement of volume** in the Metric System is the **cubic meter**; it is the volume of a cube each edge of which is one meter in length.

A **cubic meter** equals about **1.308 cubic yards**, or $1\frac{1}{3}$ cubic yards.

Metric Table of Volume

1000 cubic millimeters = 1 **cubic centimeter**
1000 cubic centimeters = 1 **cubic decimeter**
1000 cubic decimeters = 1 **cubic meter**

The unit of the metric table of wood measure is the **stere**, which is a cubic meter; the stere is the only unit used.

Measures of Capacity

The **unit** of measurement of **capacity** in the metric system for both dry and liquid measures is the **liter**, which is the volume of a **cubic decimeter.**

A liter is about 1 quart; it is equal to 1.057 qt. (liquid measure) and .908 qt. (dry measure).

Metric Table of Dry and Liquid Measures

10 millimeters = 1 centiliter
10 centiliters = 1 deciliter
10 deciliters = 1 **liter** (l.)
10 liters = 1 dekaliter
10 dekaliters = 1 **hektoliter**

Small quantities, such as are purchased in stores, are measured by the liter; large quantities, those that are measured by the bushel in our system, are measured by the hektoliter (nearly 2.84 bushels).

Measures of Weight

The **unit** of measurement of **weight** in the Metric System is the **gram**; a gram is the weight of 1 cubic centimeter of water at its greatest density. Our nickel (5-cent piece) weighs about 5 grams.

A **kilogram** or **kilo** is the weight of 1000 cubic centimeters of water, or 1 liter of water. A **kilogram** weighs **2.2046 pounds** avoirdupois or 2.2 pounds.

Metric Table of Measures of Weight

10 milligrams (mg.)	= 1 centigram
10 centigrams	= 1 decigram
10 decigrams	= 1 **gram** (gr.)
10 **grams**	= 1 dekagram
10 dekagrams	= 1 hectogram
10 hectograms	= 1 kilogram (kilo) = 2.2046 lb.
1000 kilograms	= 1 metric ton.

The gram is a light weight (about $\frac{1}{28}$ ounce avoirdupois); it is used in weighing precious metals, medicines, etc.; the kilogram is used in weighing groceries as ordinarily bought and sold; the metric ton (about 2205 pounds) is used in weighing heavy or bulky articles, such as coal, iron ore, etc.

THE following pages contain advertisements of books by the same author or on kindred subjects.

The Golden Rule Series

BY E. HERSHEY SNEATH, GEORGE HODGES AND EDWARD LAWRENCE STEVENS

READING WITH A MORAL PURPOSE

This is a new series of Literary Readers containing selections carefully chosen for the vivid portrayal of ideal beauty in human life and character. They emphasize right action in human conduct and suggest the correct moral response to situations within the experience of the child. While avoiding religious instruction, moral precepts and lectures, they teach the simple attainable virtues of school and home life, grade by grade, in an indirect manner, leaving the child to do his own moralizing. This is the first and only series of readers to concentrate attention on the problems of morality and their solution in an indirect manner.

These books are all neatly and strongly bound, printed in large, clear type, on good paper, with appropriate illustration.

I. The Golden Ladder Book, Third Grade
II. The Golden Path Book, Fourth Grade
III. The Golden Door Book, Fifth Grade
IV. The Golden Key Book, Sixth Grade
V. The Golden Word Book, Seventh Grade
VI. The Golden Deed Book, Eighth Grade

THE MACMILLAN COMPANY

64–66 Fifth Avenue, New York

CHICAGO BOSTON SAN FRANCISCO DALLAS ATLANTA